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ONE POINT UNION OF PATHS OF GRAPHS AND THEIR λ-NUMBERS

M. MURUGAN AND P. SRIRAMAN

Abstract. An $L(2,1)$-labeling of a graph G is an assignment f from the vertex set $V(G)$ to the set of non-negative integers such that $|f(x)-f(y)| \geq 2$ if x and y are adjacent, and $|f(x)-f(y)| \geq 1$ if x and y are at distance 2, for all x and y in $V(G)$. A k-$L(2,1)$-labeling is an $L(2,1)$-labeling $f: V(G) \to \{0,1,\ldots,k\}$, and we are interested to find the minimum k among all possible labelings. This invariant, the minimum k, is known as the $L(2,1)$-labeling number or λ-number and is denoted by $\lambda(G)$. In this paper, we determine the λ number of one point union of paths of stars, cycles and Petersen graphs.

1. Introduction

For standard terminology and notation, we follow Bondy and Murty [1] or Murugan [2]. Here, unless otherwise mentioned, a graph denotes simple, finite, connected, undirected graph without loops or multiple edges.

For the channel assignment problem, Hale [3] introduced graph model using vertex labeling/coloring problem in 1980. Vertices of the graph correspond to television, radio stations and edges show the proximity of the stations. In 1991, Roberts [4] proposed a variation of the channel assignment problem considering "close" and "very close" locations. In 2001, Chartrand et al.[5] introduced radio labeling problem in which the diameter of the graph is involved.

The channel assignment problem is a telecommunication problem in which our aim is to assign a channel to each television or radio station so that we do not have any interference in the communication. The level of interference between the television or radio stations correlates with the geographic locations of these stations. The designers of television networks considered close locations and very close locations so that the transmitters at the close locations receive different channels and the transmitters at very close locations are at least two apart for clear communication.

The mathematical abstraction of the above concept is $L(2,1)$ labeling or distance two labeling. An $L(2,1)$-labeling of a graph G is an assignment f from the vertex set $V(G)$ to the set of non-negative integers such that $|f(x)-f(y)| \geq 2$ if x and y are adjacent, and $|f(x)-f(y)| \geq 1$ if x and y are at distance 2, for all x and

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y in $V(G)$. A k-$L(2,1)$-labeling is an $L(2,1)$-labeling $f : V(G) \to \{0, 1, \ldots, k\}$, and we are interested to find the minimum k among all possible assignments. This invariant, the minimum k, is known as the $L(2,1)$-labeling number or λ-number and is denoted by $\lambda(G)$. The generalization of this concept is as below.

For positive integers k, d_1, d_2, a k-$L(d_1, d_2)$-labeling of a graph G is a function $f : V(G) \to \{0, 1, 2, \ldots, k\}$ such that $|f(u) - f(v)| \geq d_i$ whenever the distance between u and v in G, $d_G(u, v) = i$, for $i = 1, 2$. The $L(d_1, d_2)$-number of G, $\lambda_{d_1,d_2}(G)$, is the smallest k such that there exists a k-$L(d_1, d_2)$-labeling of G.

2. Some Existing Results

Distance two labeling or $L(2,1)$-labeling has received the attention of many researchers and here we present some important existing results.

- Griggs and Yeh [6] have discussed $L(2,1)$-labeling for path, cycle, tree and cube. They have derived results for the graphs of diameter 2. They have shown that $\lambda(T)$ for trees with maximum degree $\Delta \geq 1$ is either $\Delta + 1$ or $\Delta + 2$.
- Chang and Kuo [7] provided an algorithm to obtain $\lambda(T)$.
- Vaidya and Bantava [8] have discussed $L(2,1)$-labeling of cacti.
- Vaidya et.al. [9] have discussed $L(2,1)$-labeling in the context of some graph operations.
- Yeh [10] have discussed the $L(2,1)$-labeling on various class of graphs like trees, cycles, chordal graphs, Cartesian products of graphs etc.
- Griggs and Yeh [6] proved that if a graph G contain three vertices of degree Δ such that one of them is adjacent to the other two, then $\lambda(G) \geq \Delta + 2$, where Δ is the maximum degree of G.
- Murugan [11] has discussed $L(2,1)$-labeling for subdivisions of cycle dominated graphs.
- Griggs and Yeh [6] posed a conjecture that $\lambda(G) \leq \Delta^2$ for any graph with $\Delta \geq 2$, where Δ is the maximum degree of G, and they proved that $\lambda(G) \leq \Delta^2 + 2\Delta$ at the same time.
- Chang and Kuo [7] proved that $\lambda(G) \leq \Delta^2 + \Delta$, for any graph with $\Delta \geq 2$, where Δ is the maximum degree of G.
- Kral and Skrekovski [12] proved that $\lambda(G) \leq \Delta^2 + \Delta - 1$, for any graph with $\Delta \geq 2$, where Δ is the maximum degree of G.
- Goncalves [13] proved that $\lambda(G) \leq \Delta^2 + \Delta - 2$, for any graph with $\Delta \geq 2$, where Δ is the maximum degree of G.
- Murugan [14] has discussed radio number for family of graphs with diameter 2,3 and 4.

In spite of all the efforts the conjecture posed by Griggs and Yeh is still open.

3. Concepts

Sze-Chin Shee and Yong-Song Ho [15] denoted the graph obtained from n copies of G by identifying their roots where G is a rooted graph by $G^{(n)}$ and it is named as one point union of n copies of the graph G. Kaneria and Meera Meghpara [16] used the notation P_n^t to denote the one point union of t copies of path P_n.

A graph G is obtained by replacing each vertices of P_n^t except the central vertex by the graphs $G_1, G_2, \ldots, G_{tn}$ is known as one point union of path of graphs. We shall denote such graph G by $P_n^t(G_1, G_2, \ldots, G_{tn})$, where P_n^t is the one point union of t copies of path P_n. Here, if $G_1 = G_2 = \cdots = G_{tn} = H$, then they denote this by $P_n^t(tn.H)$ [16].

An $L(2,1)$-labeling of a graph G is an assignment f from the vertex set $V(G)$ to the set of non-negative integers such that $|f(x) - f(y)| \geq 2$ if x and y are adjacent, and $|f(x) - f(y)| \geq 1$ if x and y are at distance 2, for all x and y in $V(G)$. A k-$L(2,1)$-labeling is an $L(2,1)$-labeling $f : V(G) \to \{0, 1, \ldots, k\}$, and we are interested to find the minimum k among all possible labelings. This invariant, the minimum k, is known as the $L(2,1)$-labeling number or λ-number and is denoted by $\lambda(G)$. Here, we determine the λ number of one point union of paths of stars, cycles and Petersen graphs.

4. Results

Theorem 1. *The λ-number of one point union of paths of stars, $P_n^t(tn.K_{1,m})$, $m \geq 3$, is*

$$\lambda\left(P_n^t(tn.K_{1,m})\right) = \begin{cases} m+4 & \text{if} \quad t \leq m+2 \\ t+1 & \text{if} \quad t \geq m+3 \end{cases}$$

Proof. Consider $P_n^t(tn.K_{1,m})$. Let u be the central vertex (root) of $P_n^t(tn.K_{1,m})$. Let $v_{(\alpha,\beta,i)}$, $\alpha = 1, 2, \ldots, t$, $\beta = 1, 2, \ldots, n$, $i = 0, 1, 2, \ldots, m$ be the vertices of $P_n^t(tn.K_{1,m})$. Clearly, for a fixed α, $v_{(\alpha,\beta,0)}$, $\beta = 1, 2, \ldots, n$ are the vertices of the α^{th} copy P_n^t. We note that for a fixed α and for a fixed β, the end vertices of $K_{1,m}$ at $v_{(\alpha,\beta,0)}$ are $v_{(\alpha,\beta,i)}$, $i = 1, 2, \ldots, m$. Now we define $f : V(P_n^t(tn.K_{1,m})) \to \mathbb{N} \cup \{0\}$ such that f is a distance two labeling.

Case 1: $t \leq m+2$.

If $\alpha \leq m$, then define

$$\begin{aligned} f(u) &= 0 \\ f(v_{\alpha,\beta,0}) &= \begin{cases} \alpha+1 & \text{if} \quad \beta \equiv 1 (\text{mod } 3) \\ \alpha+3 & \text{if} \quad \beta \equiv 2 (\text{mod } 3) \\ 0 & \text{if} \quad \beta \equiv 0 (\text{mod } 3) \end{cases} \end{aligned}$$

$$f(v_{\alpha,\beta,i}) = \begin{cases} i+\alpha+3 \\ \qquad \text{if } i+\alpha+3 \leq m+4 \text{ and } \beta \equiv 1 (\text{mod } 3), \quad 1 \leq i \leq m \\ (i+\alpha+3) - (m+4) \\ \qquad \text{if } i+\alpha+3 > m+4 \text{ and } \beta \equiv 1 (\text{mod } 3), \quad 1 \leq i \leq m \\ i+\alpha+4 \\ \qquad \text{if } i+\alpha+4 \leq m+4 \text{ and } \beta \equiv 2 (\text{mod } 3), \quad 1 \leq i \leq m \\ (i+\alpha+4) - (m+4) \\ \qquad \text{if } i+\alpha+4 > m+4 \text{ and } \beta \equiv 2 (\text{mod } 3), \quad 1 \leq i \leq m \\ i+\alpha+3 \\ \qquad \text{if } i+\alpha+3 \leq m+4 \text{ and } \beta \equiv 0 (\text{mod } 3), \quad 1 \leq i \leq m \\ (i+\alpha+3) - (m+3) \\ \qquad \text{if } i+\alpha+3 > m+4 \text{ and } \beta \equiv 0 (\text{mod } 3), \quad 1 \leq i \leq m \end{cases}$$

If $\alpha = m+1$, then define

$$f(u) = 0$$

$$f\left(v_{\alpha,\beta,0}\right) = \begin{cases} \alpha+1 & \text{if} \quad \beta \equiv 1 (\text{mod } 3) \\ \alpha+3 & \text{if} \quad \beta \equiv 2 (\text{mod } 3) \\ 0 & \text{if} \quad \beta \equiv 0 (\text{mod } 3) \end{cases}$$

$$f\left(v_{\alpha,\beta,i}\right) = \begin{cases} i & \text{if} \quad \beta \equiv 1 (\text{mod } 3), \quad 1 \le i \le m \\ i & \text{if} \quad \beta \equiv 2 (\text{mod } 3), \quad 1 \le i \le m \\ i+1 & \text{if} \quad \beta \equiv 0 (\text{mod } 3), \quad 1 \le i \le m \end{cases}$$

If $\alpha = m+2$, then define

$$f(u) = 0$$

$$f\left(v_{\alpha,\beta,0}\right) = \begin{cases} \alpha+1 & \text{if} \quad \beta \equiv 1 (\text{mod } 3) \\ \alpha-1 & \text{if} \quad \beta \equiv 2 (\text{mod } 3) \\ 0 & \text{if} \quad \beta \equiv 0 (\text{mod } 3) \end{cases}$$

$$f\left(v_{\alpha,\beta,i}\right) = \begin{cases} i & \text{if} \quad \beta \equiv 1 (\text{mod } 3), \quad \text{and} \quad 1 \le i \le m \\ i & \text{if} \quad \beta \equiv 2 (\text{mod } 3), \quad \text{and} \quad 1 \le i \le m-1 \\ m+4 & \text{if} \quad \beta \equiv 2 (\text{mod } 3) \quad \text{and} \quad i = m \\ i+1 & \text{if} \quad \beta \equiv 0 (\text{mod } 3), \quad \text{and} \quad 1 \le i \le m-1 \\ i+2 & \text{if} \quad \beta \equiv 0 (\text{mod } 3), \quad \text{and} \quad i = m. \end{cases}$$

Now we prove that this labeling is an $L(2,1)$ labeling. The label of the edge connecting u with $v_{\alpha,1,0} = \alpha + 1 \ge 2,\ \alpha = 1, 2, \ldots, t$.

If $\alpha \le m$, then, since

$$f\left(v_{\alpha,\beta,0}\right) = \begin{cases} \alpha+1 & \text{if} \quad \beta \equiv 1 (\text{mod } 3) \\ \alpha+3 & \text{if} \quad \beta \equiv 2 (\text{mod } 3) \\ 0 & \text{if} \quad \beta \equiv 0 (\text{mod } 3), \end{cases}$$

the absolute value of the difference of labels at the adjacent vertices in the α^{th} branch of P_n^t is 2 or $\alpha+3$ or $\alpha+1$, which is clearly greater than or equal to 2.

Consider the star $K_{1,m}$ at $v_{\alpha,\beta,0}$. First we consider vertices at distance 1.

If $\beta \equiv 1(\text{mod } 3)$, then

if $i + \alpha + 3 \le m + 4$,

$$|f(v_{\alpha,\beta,0}) - f(v_{\alpha,\beta,i})| = |i + \alpha + 3 - (\alpha + 1)| = |i + 2| \ge 2,$$

if $i + \alpha + 3 > m + 4$,

$$\begin{aligned} |f(v_{\alpha,\beta,0}) - f(v_{\alpha,\beta,i})| &= |(i + \alpha + 3) - (m + 4) - (\alpha + 1)| \\ &= |i - m - 2| \ge 2. \end{aligned}$$

If $\beta \equiv 2(\text{mod } 3)$, then

if $i + \alpha + 4 \le m + 4$,

$$|f(v_{\alpha,\beta,0}) - f(v_{\alpha,\beta,i})| = |i + \alpha + 4 - (\alpha + 3)| = |i + 1| \ge 2,$$

if $i+\alpha+4 > m+4$,

$$\begin{aligned} |f(v_{\alpha,\beta,0}) - f(v_{\alpha,\beta,i})| &= |(i+\alpha+4)-(m+4)-(\alpha+3)| \\ &= |i-m-3| \geq 2. \end{aligned}$$

If $\beta \equiv 0(\text{mod } 3)$, then

if $i+\alpha+3 \leq m+4$,

$$|f(v_{\alpha,\beta,0}) - f(v_{\alpha,\beta,i})| = |i+\alpha+3-0| \geq 2,$$

if $i+\alpha+3 > m+4$,

$$\begin{aligned} |f(v_{\alpha,\beta,0}) - f(v_{\alpha,\beta,i})| &= |(i+\alpha+3)-(m+3)-0| \\ &= |i+\alpha-m| \geq 2. \end{aligned}$$

Also, the vertices at distance two have different labelings and so their label difference are always greater than or equal to one.

If $\alpha = m+1$, then, since

$$f(v_{\alpha,\beta,0}) = \begin{cases} \alpha+1 & \text{if} \quad \beta \equiv 1(\text{mod } 3) \\ \alpha+3 & \text{if} \quad \beta \equiv 2(\text{mod } 3) \\ 0 & \text{if} \quad \beta \equiv 0(\text{mod } 3), \end{cases}$$

the absolute value of the difference of labels at the adjacent vertices in the α^{th} branch of P_n^t is 2 or $\alpha+3$ or $\alpha+1$, which is clearly greater than or equal to 2.

Consider the star $K_{1,m}$ at $v_{\alpha,\beta,0}$. First we consider vertices at distance 1.

If $\beta \equiv 1(\text{mod } 3)$, then

$$|f(v_{\alpha,\beta,0}) - f(v_{\alpha,\beta,i})| = |\alpha+1-i| = |m+2-i| \geq 2.$$

If $\beta \equiv 2(\text{mod } 3)$, then

$$|f(v_{\alpha,\beta,0}) - f(v_{\alpha,\beta,i})| = |(\alpha+3)-i| = |m+4-i| \geq 2.$$

If $\beta \equiv 0(\text{mod } 3)$, then

$$|f(v_{\alpha,\beta,0}) - f(v_{\alpha,\beta,i})| = |0-(i+1)| = |i+1| \geq 2.$$

Also, the vertices at distance two have different labelings and so their label difference are always greater than or equal to one.

If $\alpha = m+2$, then, since

$$f(v_{\alpha,\beta,0}) = \begin{cases} \alpha+1 & \text{if} \quad \beta \equiv 1(\text{mod } 3) \\ \alpha-1 & \text{if} \quad \beta \equiv 2(\text{mod } 3) \\ 0 & \text{if} \quad \beta \equiv 0(\text{mod } 3), \end{cases}$$

the absolute value of the difference of labels at the adjacent vertices in the α^{th} branch of P_n^t is 2 or $\alpha-1$ or $\alpha+1$, which is clearly greater than or equal to 2.

Consider the star $K_{1,m}$ at $v_{\alpha,\beta,0}$. First we consider vertices at distance 1.

If $\beta \equiv 1(\text{mod } 3)$, then

$$|f(v_{\alpha,\beta,0}) - f(v_{\alpha,\beta,i})| = |\alpha+1-i| = |m+3-i| \geq 2.$$

If $\beta \equiv 2(\text{mod } 3)$, and $i \neq m$, then

$$|f(v_{\alpha,\beta,0}) - f(v_{\alpha,\beta,i})| = |(\alpha-1)-i| = |m+1-i| \geq 2.$$

If $\beta \equiv 2 (\text{mod } 3)$, and $i = m$, then

$$|f(v_{\alpha,\beta,0}) - f(v_{\alpha,\beta,i})| = |(\alpha - 1) - (m + 4)| = |-3| \geq 2.$$

If $\beta \equiv 0 (\text{mod } 3)$, and $i \neq m$, then

$$|f(v_{\alpha,\beta,0}) - f(v_{\alpha,\beta,i})| = |0 - (i + 1)| = |i + 1| \geq 2.$$

If $\beta \equiv 0 (\text{mod } 3)$, and $i = m$, then

$$|f(v_{\alpha,\beta,0}) - f(v_{\alpha,\beta,i})| = |0 - (i + 2)| = |i + 2| \geq 2.$$

Also, the vertices at distance two have different labelings and so their label difference are always greater than or equal to one.

Hence this is an $L(2,1)$ labeling. That is, $\lambda(P_n^t(tn.K_{1,m})) \leq m+4$, if $t \leq m+2$.

If a graph G contains three vertices of degree Δ such that one of them is adjacent to the other two, then $\lambda(G) \geq \Delta + 2$, where Δ is the maximum degree of G [6].

Since the maximum degree of $P_n^t(tn.K_{1,m})$ is $m + 2$ and the above condition is satisfied, $\lambda(P_n^t(tn.K_{1,m})) \geq m + 4$. Hence

$$\lambda(P_n^t(tn.K_{1,m})) = m + 4$$

if $t \leq m + 2$.

Case 2: $t \geq m + 3$.

When $\alpha \leq m + 2$, the labeling defined in case 1 is used here and we have proved that this satisfies the conditions of distance two labeling.

If $\alpha \geq m + 3$, define,

$$\begin{aligned} f(u) &= 0 \\ f(v_{\alpha,\beta,0}) &= \begin{cases} \alpha + 1 & \text{if} \quad \beta \equiv 1 (\text{mod } 3) \\ \alpha - 1 & \text{if} \quad \beta \equiv 2 (\text{mod } 3) \\ 0 & \text{if} \quad \beta \equiv 0 (\text{mod } 3) \end{cases} \\ f(v_{\alpha,\beta,i}) &= \begin{cases} i & \text{if} \quad \beta \equiv 1 (\text{mod } 3), \quad 1 \leq i \leq m \\ i & \text{if} \quad \beta \equiv 2 (\text{mod } 3), \quad 1 \leq i \leq m \\ i + 1 & \text{if} \quad \beta \equiv 0 (\text{mod } 3), \quad 1 \leq i \leq m \end{cases} \end{aligned}$$

Now we prove that this labeling is an $L(2,1)$ labeling. The label of the edge connecting u with $v_{\alpha,1,0} = \alpha + 1 \geq 2, \ \alpha = 1, 2, \ldots, t$.

If $\alpha \geq m + 3$, then, since

$$f(v_{\alpha,\beta,0}) = \begin{cases} \alpha + 1 & \text{if} \quad \beta \equiv 1 (\text{mod } 3) \\ \alpha - 1 & \text{if} \quad \beta \equiv 2 (\text{mod } 3) \\ 0 & \text{if} \quad \beta \equiv 0 (\text{mod } 3), \end{cases}$$

the absolute value of the difference of labels at the adjacent vertices in the α^{th} branch of P_n^t is 2 or $\alpha - 1$ or $\alpha + 1$, which is clearly greater than or equal to 2.

Consider the star $K_{1,m}$ at $v_{\alpha,\beta,0}$. First we consider vertices at distance 1.

If $\beta \equiv 1 (\text{mod } 3)$, then

$$|f(v_{\alpha,\beta,0}) - f(v_{\alpha,\beta,i})| = |(\alpha + 1) - i| \geq |m + 4 - i| \geq 2.$$

If $\beta \equiv 2 (\text{mod } 3)$, then

$$|f(v_{\alpha,\beta,0}) - f(v_{\alpha,\beta,i})| = |(\alpha - 1) - i| = |m + 2 - i| \geq 2.$$

If $\beta \equiv 0 (\text{mod } 3)$, then

$$|f(v_{\alpha,\beta,0}) - f(v_{\alpha,\beta,i})| = |0 - (i+1)| = |i+1| \geq 2.$$

Also, the vertices at distance two have different labelings and so their label difference are always greater than or equal to one.

Hence this is an $L(2,1)$ labeling. That is, $\lambda(P_n^t(tn.K_{1,m})) \leq t+1$, if $t \geq m+3$.

Since the maximum degree of the tree $P_n^t(tn.K_{1,m})$ is t, $\lambda(P_n^t(tn.K_{1,m})) \geq t+1$. Hence

$$\lambda(P_n^t(tn.K_{1,m})) = t+1$$

if $t \geq m+3$. □

Theorem 2. *The λ-number of one point union of paths of cycles, $P_n^t(tn.C_m)$, $m \geq 6$, is*

$$\lambda\left(P_n^t(tn.C_m)\right) = \begin{cases} 5 & \text{if} \quad t \leq 4 \\ t+1 & \text{if} \quad t \geq 5 \end{cases}$$

Proof. Consider $P_n^t(tn.C_m)$. Let u be the central vertex (root) of $P_n^t(tn.C_m)$. Let $v_{(\alpha,\beta,i)}$, $\alpha = 1, 2, \ldots, t$, $\beta = 1, 2, \ldots, n$, $i = 1, 2, \ldots, m$ be the vertices of $P_n^t(tn.C_m)$. We note that for a fixed α and for a fixed β, $v_{(\alpha,\beta,i)}$ $i = 1, 2, \ldots, m$ are the vertices of the cycle C_m at the β^{th} position of the branch α of $P_n^t(tn.C_m)$. Now we define $f : V(P_n^t(tn.C_m)) \to \mathbb{N} \cup \{0\}$ such that f is a distance two labeling.

Case 1: $t \leq 4$.

Label the vertex u with the label 5.

Subcase 1: $m \equiv 0 (\text{mod } 3)$

If $\alpha = 1, 3$ and β is odd, then define

$$f\left(v_{\alpha,\beta,i}\right) = \begin{cases} 0 & \text{if} \quad i \equiv 1 (\text{mod } 3) \\ 2 & \text{if} \quad i \equiv 2 (\text{mod } 3) \\ 4 & \text{if} \quad i \equiv 0 (\text{mod } 3), \end{cases}$$

If $\alpha = 1, 3$ and β is even, then define

$$f\left(v_{\alpha,\beta,i}\right) = \begin{cases} 1 & \text{if} \quad i \equiv 1 (\text{mod } 3) \\ 3 & \text{if} \quad i \equiv 2 (\text{mod } 3) \\ 5 & \text{if} \quad i \equiv 0 (\text{mod } 3), \end{cases}$$

If $\alpha = 2$ and β is odd, then define

$$f\left(v_{\alpha,\beta,i}\right) = \begin{cases} 1 & \text{if} \quad i \equiv 1 (\text{mod } 3) \quad \text{and} \quad i \neq m-2, m-1, m \\ 3 & \text{if} \quad i \equiv 2 (\text{mod } 3) \quad \text{and} \quad i \neq m-2, m-1, m \\ 5 & \text{if} \quad i \equiv 2 (\text{mod } 3) \quad \text{and} \quad i \neq m-2, m-1, m \\ 0 & \text{if} \quad i = m-2 \\ 2 & \text{if} \quad i = m-1 \\ 4 & \text{if} \quad i = m \end{cases}$$

If $\alpha = 2$ and β is even, then define

$$f\left(v_{\alpha,\beta,i}\right) = \begin{cases} 0 & \text{if} \quad i \equiv 1 (\text{mod } 3) \\ 2 & \text{if} \quad i \equiv 2 (\text{mod } 3) \\ 4 & \text{if} \quad i \equiv 0 (\text{mod } 3), \end{cases}$$

If $\alpha = 4$ and β is odd, then define

$$f\left(v_{\alpha,\beta,i}\right) = \begin{cases} 1 & \text{if} \quad i \equiv 1 (\text{mod } 6) \\ 3 & \text{if} \quad i \equiv 2 (\text{mod } 6) \\ 0 & \text{if} \quad i \equiv 3 (\text{mod } 6) \\ 4 & \text{if} \quad i \equiv 4 (\text{mod } 6) \\ 2 & \text{if} \quad i \equiv 5 (\text{mod } 6) \\ 5 & \text{if} \quad i \equiv 0 (\text{mod } 6) \end{cases}$$

If $\alpha = 4$ and β is even, then define

$$f\left(v_{\alpha,\beta,i}\right) = \begin{cases} 0 & \text{if} \quad i \equiv 1 (\text{mod } 3) \\ 2 & \text{if} \quad i \equiv 2 (\text{mod } 3) \\ 4 & \text{if} \quad i \equiv 0 (\text{mod } 3), \end{cases}$$

For each $\alpha \neq 2$ and each β, join $v_{\alpha,\beta,m}$ to $v_{\alpha,\beta+1,1}$. For $\alpha = 2$, if β is odd join $v_{\alpha,\beta,2}$ to $v_{\alpha,\beta+1,1}$ and if β is even join $v_{\alpha,\beta,m}$ to $v_{\alpha,\beta+1,1}$.
Subcase 2: $m \equiv 1 (\text{mod } 3)$

If $\alpha = 1, 3, 4$ and β is odd, then define

$$f\left(v_{\alpha,\beta,i}\right) = \begin{cases} 0 & \text{if} \quad i \equiv 1 (\text{mod } 3) \quad \text{and} \quad i \neq m, m-1, m-2, m-3 \\ 2 & \text{if} \quad i \equiv 2 (\text{mod } 3) \quad \text{and} \quad i \neq m, m-1, m-2, m-3 \\ 4 & \text{if} \quad i \equiv 0 (\text{mod } 3) \quad \text{and} \quad i \neq m, m-1, m-2, m-3 \\ 0 & \text{if} \quad i = m-3 \\ 3 & \text{if} \quad i = m-2 \\ 1 & \text{if} \quad i = m-1 \\ 4 & \text{if} \quad i = m \end{cases}$$

If $\alpha = 1, 3, 4$ and β is even, then define

$$f\left(v_{\alpha,\beta,i}\right) = \begin{cases} 1 & \text{if} \quad i \equiv 1 (\text{mod } 3) \quad \text{and} \quad i \neq m, m-1, m-2, m-3 \\ 3 & \text{if} \quad i \equiv 2 (\text{mod } 3) \quad \text{and} \quad i \neq m, m-1, m-2, m-3 \\ 5 & \text{if} \quad i \equiv 0 (\text{mod } 3) \quad \text{and} \quad i \neq m, m-1, m-2, m-3 \\ 1 & \text{if} \quad i = m-3 \\ 4 & \text{if} \quad i = m-2 \\ 2 & \text{if} \quad i = m-1 \\ 5 & \text{if} \quad i = m \end{cases}$$

If $\alpha = 2$ and β is odd, then define

$$f\left(v_{\alpha,\beta,i}\right) = \begin{cases} 1 & \text{if} \quad i \equiv 1 (\text{mod } 3) \quad \text{and} \quad i \neq m, m-1, m-2, m-3 \\ 3 & \text{if} \quad i \equiv 2 (\text{mod } 3) \quad \text{and} \quad i \neq m, m-1, m-2, m-3 \\ 5 & \text{if} \quad i \equiv 0 (\text{mod } 3) \quad \text{and} \quad i \neq m, m-1, m-2, m-3 \\ 1 & \text{if} \quad i = m-3 \\ 4 & \text{if} \quad i = m-2 \\ 2 & \text{if} \quad i = m-1 \\ 5 & \text{if} \quad i = m \end{cases}$$

If $\alpha = 2$ and β is even, then define

$$f\left(v_{\alpha,\beta,i}\right) = \begin{cases} 0 & \text{if} \quad i \equiv 1 (\text{mod } 3) \quad \text{and} \quad i \neq m, m-1, m-2, m-3 \\ 2 & \text{if} \quad i \equiv 2 (\text{mod } 3) \quad \text{and} \quad i \neq m, m-1, m-2, m-3 \\ 4 & \text{if} \quad i \equiv 0 (\text{mod } 3) \quad \text{and} \quad i \neq m, m-1, m-2, m-3 \\ 0 & \text{if} \quad i = m-3 \\ 3 & \text{if} \quad i = m-2 \\ 1 & \text{if} \quad i = m-1 \\ 4 & \text{if} \quad i = m \end{cases}$$

For each $\alpha \neq 2$, if β is odd, join $v_{\alpha,\beta,1}$ to $v_{\alpha,\beta+1,2}$ and if β is even, join $v_{\alpha,\beta,3}$ to $v_{\alpha,\beta+1,2}$. For $\alpha = 2$, if β is odd join $v_{\alpha,\beta,3}$ to $v_{\alpha,\beta+1,2}$ and if β is even join $v_{\alpha,\beta,1}$ to $v_{\alpha,\beta+1,2}$.

Subcase 3: $m \equiv 2(\text{mod } 3)$

For all α and odd β define

$$f\left(v_{\alpha,\beta,i}\right) = \begin{cases} 0 & \text{if} \quad i \equiv 1 (\text{mod } 3) \quad \text{and} \quad i \neq m, m-1 \\ 2 & \text{if} \quad i \equiv 2 (\text{mod } 3) \quad \text{and} \quad i \neq m, m-1 \\ 4 & \text{if} \quad i \equiv 0 (\text{mod } 3) \quad \text{and} \quad i \neq m, m-1 \\ 1 & \text{if} \quad i = m-1 \\ 3 & \text{if} \quad i = m \end{cases}$$

For all α and even β define

$$f\left(v_{\alpha,\beta,i}\right) = \begin{cases} 1 & \text{if} \quad i \equiv 1 (\text{mod } 3) \quad \text{and} \quad i \neq m, m-1 \\ 3 & \text{if} \quad i \equiv 2 (\text{mod } 3) \quad \text{and} \quad i \neq m, m-1 \\ 5 & \text{if} \quad i \equiv 0 (\text{mod } 3) \quad \text{and} \quad i \neq m, m-1 \\ 2 & \text{if} \quad i = m-1 \\ 5 & \text{if} \quad i = m \end{cases}$$

For α, if β is odd, join $v_{\alpha,\beta,4}$ to $v_{\alpha,\beta+1,2}$ and if β is even, join $v_{\alpha,\beta,3}$ to $v_{\alpha,\beta+1,2}$.

In all the above 3 cases, join u to the vertex of label $\alpha - 1$ of the first copy of α^{th} branch of $P_n^t(tn.C_m)$, say v, such that degree of v is 3.

Now we prove that this labeling is an $L(2,1)$ labeling.

Since any 2 adjacent vertices receive labels with difference 2 and the labels at any 2 vertices at distance 2 are all different, this labeling f is an $L(2,1)$ labeling.

That is, $\lambda\left(P_n^t(tn.C_m)\right) \leq 5$, if $t \leq 4$.

Since $\lambda(C_m) = 4$ [6], $\lambda(P_n^t(tn.C_m)) \geq 4$. Suppose $\lambda(P_n^t(tn.C_m)) = 4$, then the labels 0,1,2,3,4 are sufficient to give a $L(2,1)$ labeling to $P_n^t(tn.C_m)$. If u receives the label x, $0 \leq x \leq 4$, then the adjacent vertices of u cannot receive labels $x, x-1, x+1$. So only 2 labels are available to the adjacent vertices of u (when u receives label 0 or 4, 3 labels are available to the adjacent vertices of u). But u has more than 2 adjacent vertices. This is a contradiction. That is $\lambda\left(P_n^t(tn.C_m)\right) \geq 5$. Hence, $\lambda\left(P_n^t(tn.C_m)\right) = 5$, if $t \leq 4$.

Case 2: $t \geq 5$.

Label the vertex u with the label 0.

Subcase 1: $m \equiv 0(\text{mod } 3)$

If $\alpha \leq 5$ is odd and β is odd, then define

$$f\left(v_{\alpha,\beta,i}\right) = \begin{cases} 2 & \text{if} \quad i \equiv 1 (\text{mod } 3) \\ 4 & \text{if} \quad i \equiv 2 (\text{mod } 3) \\ 6 & \text{if} \quad i \equiv 0 (\text{mod } 3), \end{cases}$$

If $\alpha \leq 5$ is odd and β is even, then define

$$f\left(v_{\alpha,\beta,i}\right) = \begin{cases} 1 & \text{if} \quad i \equiv 1 (\text{mod } 3) \\ 3 & \text{if} \quad i \equiv 2 (\text{mod } 3) \\ 5 & \text{if} \quad i \equiv 0 (\text{mod } 3), \end{cases}$$

If $\alpha \leq 5$ is even and β is odd, then define

$$f\left(v_{\alpha,\beta,i}\right) = \begin{cases} 1 & \text{if} \quad i \equiv 1 (\text{mod } 3) \\ 3 & \text{if} \quad i \equiv 2 (\text{mod } 3) \\ 5 & \text{if} \quad i \equiv 0 (\text{mod } 3), \end{cases}$$

If $\alpha \leq 5$ is even and β is even, then define

$$f\left(v_{\alpha,\beta,i}\right) = \begin{cases} 2 & \text{if} \quad i \equiv 1 (\text{mod } 3) \\ 4 & \text{if} \quad i \equiv 2 (\text{mod } 3) \\ 6 & \text{if} \quad i \equiv 0 (\text{mod } 3), \end{cases}$$

For all $\alpha \geq 6$, keep the above labeling as it is except for $v_{\alpha,1,m}$ and the new label is $f\left(v_{\alpha,1,m}\right) = \alpha + 1$.

For each α and β, join $v_{\alpha,\beta,1}$ to $v_{\alpha,\beta+1,m}$.

Subcase 2: $m \equiv 1(\text{mod } 3)$

If $\alpha \leq 5$ and if β is odd, then define

$$f\left(v_{\alpha,\beta,i}\right) = \begin{cases} 2 & \text{if} \quad i \equiv 1 (\text{mod } 3) \quad \text{and} \quad i \neq m, m-1, m-2, m-3 \\ 4 & \text{if} \quad i \equiv 2 (\text{mod } 3) \quad \text{and} \quad i \neq m, m-1, m-2, m-3 \\ 6 & \text{if} \quad i \equiv 0 (\text{mod } 3) \quad \text{and} \quad i \neq m, m-1, m-2, m-3 \\ 2 & \text{if} \quad i = m-3 \\ 5 & \text{if} \quad i = m-2 \\ 3 & \text{if} \quad i = m-1 \\ 6 & \text{if} \quad i = m \end{cases}$$

If $\alpha \leq 5$ and if β is even, then define

$$f\left(v_{\alpha,\beta,i}\right) = \begin{cases} 0 & \text{if} \quad i \equiv 1 (\text{mod } 3) \quad \text{and} \quad i \neq m, m-1, m-2, m-3 \\ 2 & \text{if} \quad i \equiv 2 (\text{mod } 3) \quad \text{and} \quad i \neq m, m-1, m-2, m-3 \\ 4 & \text{if} \quad i \equiv 0 (\text{mod } 3) \quad \text{and} \quad i \neq m, m-1, m-2, m-3 \\ 0 & \text{if} \quad i = m-3 \\ 3 & \text{if} \quad i = m-2 \\ 1 & \text{if} \quad i = m-1 \\ 4 & \text{if} \quad i = m \end{cases}$$

For all $\alpha \geq 6$, keep the above labeling as it is except for $v_{\alpha,1,m}$ and the new label is $f\left(v_{\alpha,1,m}\right) = \alpha + 1$.

For all α, if β is odd, join $v_{\alpha,\beta,m-3}$ to $v_{\alpha,\beta+1,m}$ and if β is even, join $v_{\alpha,\beta,m-3}$ to $v_{\alpha,\beta+1,m-3}$.

Subcase 3: $m \equiv 2(\text{mod } 3)$

For all $\alpha \leq 5$, if β is odd, then define

$$f\left(v_{\alpha,\beta,i}\right)=\begin{cases} 2 & \text{if} \quad i\equiv 1(\text{mod } 3) \quad \text{and} \quad i\neq m,m-1\\ 4 & \text{if} \quad i\equiv 2(\text{mod } 3) \quad \text{and} \quad i\neq m,m-1\\ 6 & \text{if} \quad i\equiv 0(\text{mod } 3) \quad \text{and} \quad i\neq m,m-1\\ 3 & \text{if} \quad i=m-1\\ 5 & \text{if} \quad i=m \end{cases}$$

For all $\alpha \leq 5$, if β is even, then define

$$f\left(v_{\alpha,\beta,i}\right)=\begin{cases} 0 & \text{if} \quad i\equiv 1(\text{mod } 3) \quad \text{and} \quad i\neq m,m-1\\ 2 & \text{if} \quad i\equiv 2(\text{mod } 3) \quad \text{and} \quad i\neq m,m-1\\ 4 & \text{if} \quad i\equiv 0(\text{mod } 3) \quad \text{and} \quad i\neq m,m-1\\ 1 & \text{if} \quad i=m-1\\ 3 & \text{if} \quad i=m \end{cases}$$

For all $\alpha \geq 6$, keep the above labeling as it is except for $v_{\alpha,1,3}$ and the new label is $f\left(v_{\alpha,1,3}\right)=\alpha+1$.

For all α, if β is odd, join $v_{\alpha,\beta,6}$ to $v_{\alpha,\beta+1,1}$ and if β is even, join $v_{\alpha,\beta,n-1}$ to $v_{\alpha,\beta+1,n}$.

In all the above 3 cases, join u to the vertex of label $\alpha+1$ of the first copy of α^{th} branch of $P_n^t(tn.C_m)$, say v, such that degree of v is 3.

Now we prove that this labeling is an $L(2,1)$ labeling.

Since any 2 adjacent vertices receive labels with difference 2 and the labels at any 2 vertices at distance 2 are all different, this labeling f is an $L(2,1)$ labeling.

That is, $\lambda\left(P_n^t(tn.C_m)\right) \leq t+1$, if $t \geq 5$.

When $t \geq 5$, since the maximum degree Δ is t, $\lambda\left(P_n^t(tn.C_m)\right) \geq t+1$. Hence $\lambda\left(P_n^t(tn.C_m)\right) = t+1$ if $t \geq 5$. □

Theorem 3. *The λ-number of one point union of paths of Peterson graphs P, $P_n^t(tn.P)$ is*

$$\lambda\left(P_n^t(tn.P)\right)=\begin{cases} 9 & \text{if} \quad t\leq 5\\ 10 & \text{if} \quad 6\leq t\leq 9\\ t+1 & \text{if} \quad t\geq 10. \end{cases}$$

Proof. Consider $P_n^t(tn.P)$. Let u be the central vertex (root) of $P_n^t(tn.P)$. Let $v_{\alpha,\beta,i}$, $\alpha=1,2,\ldots,t$, $\beta=1,2,\ldots,n$, $i=1,2,\ldots,10$ be the vertices of $P_n^t(tn.P)$. We note that, for a fixed α and fixed β, $v_{\alpha,\beta,i}$, $i=1,2,\ldots,10$ are the vertices of P at the β^{th} position of the branch α of $P_n^t(tn.P)$. Now we define $f: V(P_n^t(tn.P)) \to \mathbb{N}\cup\{0\}$ such that f is a distance two labeling.

Label the vertex u with 0.

Here, the Petersen graphs consist of the outer cycle

$$v_{\alpha,\beta,1}, v_{\alpha,\beta,2}, v_{\alpha,\beta,3}, v_{\alpha,\beta,4}, v_{\alpha,\beta,5}, v_{\alpha,\beta,1},$$

the inner cycle

$$v_{\alpha,\beta,6}, v_{\alpha,\beta,7}, v_{\alpha,\beta,8}, v_{\alpha,\beta,9}, v_{\alpha,\beta,10}, v_{\alpha,\beta,6},$$

and the edges

$$(v_{\alpha,\beta,1}, v_{\alpha,\beta,6}), (v_{\alpha,\beta,2}, v_{\alpha,\beta,9}), (v_{\alpha,\beta,3}, v_{\alpha,\beta,7}), (v_{\alpha,\beta,4}, v_{\alpha,\beta,10}), (v_{\alpha,\beta,5}, v_{\alpha,\beta,8}),$$

at the β^{th} position of the branch α of $P_n^t(tn.P)$.

Case 1: $t \leq 5$.

For all α and β, define

$$f(v_{\alpha,\beta,1}) = 0, \quad f(v_{\alpha,\beta,2}) = 2, \quad f(v_{\alpha,\beta,3}) = 5, \quad f(v_{\alpha,\beta,4}) = 1, \quad f(v_{\alpha,\beta,5}) = 3,$$
$$f(v_{\alpha,\beta,6}) = 4, \quad f(v_{\alpha,\beta,7}) = 7, \quad f(v_{\alpha,\beta,8}) = 9, \quad f(v_{\alpha,\beta,9}) = 6, \quad f(v_{\alpha,\beta,10}) = 8.$$

If $\alpha = 1$, and β is odd, join the vertex with label 5 of β^{th} copy to the vertex of label 8 of $\beta + 1^{\text{th}}$ copy and if β is even, join the vertex with label 8 of β^{th} copy to the vertex of label 0 of $\beta + 1^{\text{th}}$ copy of Petersen graph.

If $\alpha = 2$, and β is odd, join the vertex with label 6 of β^{th} copy to the vertex of label 3 of $\beta + 1^{\text{th}}$ copy and if β is even, join the vertex with label 3 of β^{th} copy to the vertex of label 5 of $\beta + 1^{\text{th}}$ copy of Petersen graph.

If $\alpha = 3$, and β is odd, join the vertex with label 7 of β^{th} copy to the vertex of label 3 of $\beta + 1^{\text{th}}$ copy and if β is even, join the vertex with label 3 of β^{th} copy to the vertex of label 5 of $\beta + 1^{\text{th}}$ copy of Petersen graph.

If $\alpha = 4$, and β is odd, join the vertex with label 8 of β^{th} copy to the vertex of label 5 of $\beta + 1^{\text{th}}$ copy and if β is even, join the vertex with label 5 of β^{th} copy to the vertex of label 0 of $\beta + 1^{\text{th}}$ copy of Petersen graph.

If $\alpha = 5$, and β is odd, join the vertex with label 9 of β^{th} copy to the vertex of label 5 of $\beta + 1^{\text{th}}$ copy and if β is even, join the vertex with label 5 of β^{th} copy to the vertex of label 0 of $\beta + 1^{\text{th}}$ copy of Petersen graph.

Join u to the vertex of label $\alpha + 4$ of the first copy of the branch α. Clearly this is a distance two labeling and so, $\lambda(P_n^t(tn.P)) \leq 9$, if $t \leq 5$. But we know that the λ-number of Petersen graph is 9 [17] and so, $\lambda(P_n^t(tn.P)) \geq 9$. Hence $\lambda(P_n^t(tn.P)) = 9$, if $t \leq 5$.

Case 2: $6 \leq t \leq 9$.

For all α and odd β, define

$$f(v_{\alpha,\beta,1}) = 1, \quad f(v_{\alpha,\beta,2}) = 3, \quad f(v_{\alpha,\beta,3}) = 6, \quad f(v_{\alpha,\beta,4}) = 2, \quad f(v_{\alpha,\beta,5}) = 4,$$
$$f(v_{\alpha,\beta,6}) = 5, \quad f(v_{\alpha,\beta,7}) = 8, \quad f(v_{\alpha,\beta,8}) = 10, \quad f(v_{\alpha,\beta,9}) = 7, \quad f(v_{\alpha,\beta,10}) = 9.$$

Define for all α and even β,

$$f(v_{\alpha,\beta,1}) = 0, \quad f(v_{\alpha,\beta,2}) = 2, \quad f(v_{\alpha,\beta,3}) = 5, \quad f(v_{\alpha,\beta,4}) = 1, \quad f(v_{\alpha,\beta,5}) = 3,$$
$$f(v_{\alpha,\beta,6}) = 4, \quad f(v_{\alpha,\beta,7}) = 7, \quad f(v_{\alpha,\beta,8}) = 9, \quad f(v_{\alpha,\beta,9}) = 6, \quad f(v_{\alpha,\beta,10}) = 8.$$

For all α, if β is odd, join the vertex with label 10 of β^{th} copy to the vertex of label 2 of $\beta + 1^{\text{th}}$ copy and if β is even, join the vertex with label 9 of β^{th} copy to the vertex of label 1 of $\beta + 1^{\text{th}}$ copy of Petersen graph.

Join u to the vertex of label $\alpha + 1$ of the first copy of each branch α. Clearly this is a distance two labeling and so, $\lambda(P_n^t(tn.P)) \leq 10$, if $6 \leq t \leq 9$.

Suppose $\lambda(P_n^t(tn.P)) \leq 9$ then the labels $0, 1, \ldots, 9$ are sufficient to give a distance two labeling for $P_n^t(tn.P)$. Since λ-number of Petersen graph is 9 and it has 10 vertices, all the 10 labels between 0 and 9 are used on the Petersen graph. Suppose the label x, $0 \leq x \leq 9$, is given to u then the labels at the 3 adjacent vertices of the vertex with label x in the Petersen graph and the labels $x-1, x, x+1$ cannot be given to the adjacent vertices of u; that is, these 6 labels between 0 and 9 cannot be given to the adjacent vertices of u (in the case of 0 and 9, 5 labels between 0 to 9 cannot be given to the adjacent vertices of u). That is, only 4 labels between 0 to 9 can be given to the adjacent vertices of u (in the case of 0 and 9, only 5 labels from 0 to 9 can be given to the adjacent vertices of u). But since there are 9 branches, there are 9 adjacent vertices for u. This is a contradiction. Hence $\lambda(P_n^t(tn.P)) \geq 10$, if $6 \leq t \leq 9$. Hence $\lambda(P_n^t(tn.P)) = 10$, if $6 \leq t \leq 9$.

Case 3: $t \geq 10$.

Define for all α and odd β,

$$f(v_{\alpha,\beta,1}) = 1, \quad f(v_{\alpha,\beta,2}) = 3, \quad f(v_{\alpha,\beta,3}) = 6, \quad f(v_{\alpha,\beta,4}) = 2, \quad f(v_{\alpha,\beta,5}) = 4,$$
$$f(v_{\alpha,\beta,6}) = 5, \quad f(v_{\alpha,\beta,7}) = 8, \quad f(v_{\alpha,\beta,8}) = 10 \text{ if } \alpha \leq 9, \quad \alpha + 1 \text{ if } \alpha \geq 10,$$
$$f(v_{\alpha,\beta,9}) = 7, \quad f(v_{\alpha,\beta,10}) = 9.$$

Define for all α and even β,

$$f(v_{\alpha,\beta,1}) = 0, \quad f(v_{\alpha,\beta,2}) = 2, \quad f(v_{\alpha,\beta,3}) = 5, \quad f(v_{\alpha,\beta,4}) = 1, \quad f(v_{\alpha,\beta,5}) = 3,$$
$$f(v_{\alpha,\beta,6}) = 4, \quad f(v_{\alpha,\beta,7}) = 7, \quad f(v_{\alpha,\beta,8}) = 9, \quad f(v_{\alpha,\beta,9}) = 6, \quad f(v_{\alpha,\beta,10}) = 8.$$

For all $\alpha \leq 9$, if β is odd, join the vertex with label 10 of β^{th} copy to the vertex of label 2 of $\beta + 1^{\text{th}}$ copy and if β is even, join the vertex with label 9 of β^{th} copy to the vertex of label 1 of $\beta + 1^{\text{th}}$ copy of Petersen graph.

For all $\alpha \geq 10$, if β is odd, join the vertex with label $\alpha + 1$ of β^{th} copy to the vertex of label 2 of $\beta + 1^{\text{th}}$ copy and if β is even, join the vertex with label 9 of β^{th} copy to the vertex of label 1 of $\beta + 1^{\text{th}}$ copy of Petersen graph.

Join u to the vertex of label $\alpha+1$ of the first copy of each branch α. Clearly this is a distance two labeling and so, $\lambda(P_n^t(tn.P)) \leq t+1$, if $t \geq 10$. The maximum degree of $P_n^t(tn.P)$ is t and so $\lambda(P_n^t(tn.P)) \geq t + 1$, if $t \geq 10$. Hence $\lambda(P_n^t(tn.P)) = t + 1$, if $t \geq 10$. □

5. Conclusion

Assignment of television and radio frequency become a very important problem because determining frequencies increases day by day due to installation of more stations. Here, interference is a basic problem and so we have to concentrate neighboring stations and stations at a distance two. This will make the quality of broadcast better. Here comes, the distance two labeling. We believe that this work will motivate many more researchers towards this area.

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M. MURUGAN
Department of Mathematics, School of Science, Tamil Nadu Open University, Chennai, INDIA
E-mail address: muruganganesan@yahoo.in

P. Sriraman
Research Scholar, Department of Mathematics, School of Science, Tamil Nadu Open University, Chennai, INDIA
E-mail address: psriraman@gmail.com