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FIXED POINT THEOREMS FOR GENERALIZED RATIONAL $\alpha-\psi-$ GERAGHTY CONTRACTION TYPE MAPPINGS IN METRIC SPACE

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Abstract. In this paper, we introduce the notion of generalized rational $\alpha-\psi-$ Geraghty contraction type mappings in the context of metric space and establish some fixed point theorems for such mappings. An example is also given to illustrate our result.

1. Introduction

The Banach contraction principle is one of the most fundamental results in fixed point theory. Due to its usefulness and applications in many disciplines, several authors have improved, generalized and extended this basic result of Banach by defining new contractive conditions and replacing the metric space by more general abstract spaces. In 1973, Geraghty [8] generalized the Banach contraction principle by considering an auxiliary function.In 2010, Amini-Harandi and Emami [1] characterized the result of Geraghty in the context of a partially ordered complete metric space. Caballero et al. [5] discussed the existence of a best proximity point of Geraghty contraction. Gordji et al. [9] defined the notion of $\psi-$Geraghty type contraction and obtained results extending the results of Amini-Harandi and Emami [1]. Samet et al.[18] defined the notion of $\alpha-\psi-$contractive mappings and obtained remarkable fixed point results. Karapinar and Samet [12] introduced the concept of generalized $\alpha-\psi-$ contractive mappings and obtained fixed point results for such mappings. Recently in 2013, Cho et al. [7] defined the concept of generalized $\alpha-$Geraghty contraction type maps and $\alpha-$Geraghty contraction type maps in the setting of a metric space and proved some fixed point results for such maps in the context of a complete metric space. Then in 2014, Erdal Karapinar [15] introduced the concept of generalized $\alpha-\psi-$Geraghty contraction type maps and $\alpha-\psi-$Geraghty contraction type maps and proved fixed point results generalizing the results obtained by Cho et al.[7]. Very recently in 2017, Muhammad Arshad

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and Aftab Hussain [17] defined generalized rational $\alpha-$Geraghty contraction type mappings and proved some fixed point results.

In this paper, motivated by the developments above, we define generalized rational $\alpha-\psi-$Geraghty contraction type mapping in the setting of metric space and obtain the existence and uniqueness of a fixed point of such mappings. We also give an example to illustrate our result.

We recall some basic definitions and related results on the topic in the literature.

Let $\mathbb{F}$ be the family of all functions $\beta:[0,\infty)\to[0,1)$ which satisfies the condition

$$\lim_{n\to\infty}\beta(t_n)=1\Rightarrow\lim_{n\to\infty}t_n=0$$

By using such a map, Geraghty proved the following interesting result.

Theorem 1.1[8] Let (X,d) be a complete metric space and let T be a mapping on X. Suppose there exists $\beta\in\mathbb{F}$ such that for all $x,y\in X$,

$$d(Tx,Ty)\leq\beta((d(x,y))d(x,y).$$

Then T has a unique fixed point $x_*\in X$ and $\{T^nx\}$ converges to x_* for each $x\in X$.

Definition 1.2[18] Let $T:X\to X$ be a map and $\alpha:X\times X\to\mathbb{R}$ be a function. Then T is said to be $\alpha-$admissible if $\alpha(x,y)\geq1$ implies $\alpha(Tx,Ty)\geq1$.

Definition 1.3[10] A map $T:X\to X$ is said to be triangular $\alpha-$admissible if

(T1) T is $\alpha-$admissible,
(T2) $\alpha(x,z)\geq1$ and $\alpha(z,y)\geq1$ imply $\alpha(x,y)\geq1$.

Lemma 1.4[10] Let $T:X\to X$ be a triangular $\alpha-$admissible map. Assume that there exists $x_1\in X$ such that $\alpha(x_1,Tx_1)\geq1$. Define a sequence $\{x_n\}$ by $x_{n+1}=Tx_n$. Then we have $\alpha(x_n,x_m)\geq1$ for all $m,n\in\mathbb{N}$ with $n<m$.

Erdal Karapinar [15] defined the following class of auxiliary functions.

Let Ψ denote the class of functions $\psi:[0,\infty)\to[0,\infty)$ which satisfy the following conditions:

(a) ψ is nondecreasing;
(b) ψ is subadditive, that is, $\psi(s+t)\leq\psi(s)+\psi(t)$;
(c) ψ is continuous;
(d) $\psi(t)=0\Leftrightarrow t=0$

2. Main Results

We now state and prove our main results.

First we introduce the following new definitions.

Let Ω be the family of all functions $\theta:[0,\infty)\to[0,1]$ which satisfy the following conditions

(1) $\theta(t)<1$ for $t>0$ and
(2) $\lim_{n\to\infty}\theta(t_n)=1$ implies $\lim_{n\to\infty}t_n=0$.

Remark 2.1 Here instead of the family $\mathbb{F}$ we are introducing a slightly extended family Ω.

Definition 2.2 Let (X,d) be a metric space and $\alpha:X\times X\to\mathbb{R}$ be a function. Then the mapping $T:X\to X$ is called a generalized rational $\alpha-\psi-$Geraghty contraction type mapping if there exists $\theta\in\Omega$ such that for all $x,y\in X$,
$\alpha(x,y)\psi(d(Tx,Ty))\leq\theta(\psi(N(x,y)))\psi(N(x,y))$ where
$N(x,y)=max\{d(x,y),\frac{d(x,Tx)d(y,Ty)}{1+d(x,y)},\frac{d(x,Tx)d(y,Ty)}{1+d(Tx,Ty)}\}$ and $\psi\in\Psi$.

If we take $\psi(t)=t$ in definition 2.2, then T is called generalized rational $\alpha-$Geraghty contraction type mapping [17].

Theorem 2.3 Let (X,d) be a complete metric space, $\alpha : X \times X \to \mathbb{R}$ be a function and let $T : X \to X$ be a mapping. Suppose that the following conditions hold:

(i) T is a generalized rational $\alpha-\psi-$Geraghty contraction type mapping,
(ii) T is triangular $\alpha-$admissible,
(iii) there exists $x_1 \in X$ such that $\alpha(x_1, Tx_1) \geq 1$,
(iv) T is continuous.

Then T has a fixed point $x^* \in X$ and $\{T^n x_1\}$ converges to x^*.

Proof: Let $x_1 \in X$ be such that $\alpha(x_1, Tx_1) \geq 1$. We construct a sequence of points $\{x_n\}$ in X by $x_{n+1} = Tx_n$ for $n \in \mathbb{N}$. If $x_{n_0} = x_{n_0+1}$ for some $n_0 \in \mathbb{N}$, then $x_{n_0}$is clearly a fixed point of T and the proof is complete. Hence, we suppose that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}$.

By hypothesis, $\alpha(x_1, x_2) \geq 1$ and T is triangular $\alpha-$admissible. Therefore by Lemma 1.4., we have $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \mathbb{N}$.

Then, we have

$$\begin{aligned} \psi(d(x_{n+1}, x_{n+2})) &= \psi(d(Tx_n, Tx_{n+1})) \leq \alpha(x_n, x_{n+1})\psi(d(Tx_n, Tx_{n+1})) \\ &\leq \theta(\psi(N(x_n, x_{n+1})))\psi(N(x_n, x_{n+1})), \forall\, n \in \mathbb{N} \end{aligned} \tag{1}$$

Here we have

$$\begin{aligned} N(x_n, x_{n+1}) &= max\Big\{d(x_n, x_{n+1}), \frac{d(x_n, Tx_n)d(x_{n+1}, Tx_{n+1})}{1 + d(x_n, x_{n+1})}, \frac{d(x_n, Tx_n)d(x_{n+1}, Tx_{n+1})}{1 + d(Tx_n, Tx_{n+1})}\Big\} \\ &= max\Big\{d(x_n, x_{n+1}), \frac{d(x_n, x_{n+1})d(x_{n+1}, x_{n+2})}{1 + d(x_n, x_{n+1})}, \frac{d(x_n, x_{n+1})d(x_{n+1}, x_{n+2})}{1 + d(x_{n+1}, x_{n+2})}\Big\} \\ &\leq max\Big\{d(x_n, x_{n+1}), d(x_{n+1}, x_{n+2})\Big\}. \end{aligned}$$

If $max\Big\{d(x_n, x_{n+1}), d(x_{n+1}, x_{n+2})\Big\} = d(x_{n+1}, x_{n+2})$ i.e. $d(x_{n+1}, x_{n+2}) \geq d(x_n, x_{n+1})$, then from (1) and the definition of θ, we have
$\psi(d(x_{n+1}, x_{n+2})) < \psi(d(x_{n+1}, x_{n+2}))$, which is a contradiction.
Therefore, we have
$d(x_{n+1}, x_{n+2}) < d(x_n, x_{n+1})$ for all $n \in \mathbb{N}$.
Thus the sequence $\{d(x_n, x_{n+1})\}$ is nonnegative and nonincreasing and also we have

$$N(x_n, x_{n+1}) = d(x_n, x_{n+1}).$$

Now, we prove that $d(x_n, x_{n+1}) \to 0$ as $n \to \infty$.
It is clear that $\{d(x_n, x_{n+1})\}$ is a decreasing sequence which is bounded from below.
Therefore there exists $r \geq 0$ such that $\lim_{n\to\infty} d(x_n, x_{n+1}) = r$. We show that $r = 0$. And we suppose on the contrary that $r > 0$.
Then, we have $\frac{\psi(d(x_{n+1}, x_{n+2}))}{\psi(d(x_n, x_{n+1}))} \leq \theta(\psi(d(x_n, x_{n+1}))) < 1$.
Now by taking limit $n \to \infty$, we have

$$\lim_{n\to\infty} \theta(\psi(d(x_n, x_{n+1}))) = 1$$

By the property of θ , we have
$\lim_{n\to\infty} \psi(d(x_n, x_{n+1})) = 0 \Rightarrow \lim_{n\to\infty} d(x_n, x_{n+1}) = 0$, which is a contradiction.

Hence

$$r = 0 \tag{2}$$

Now we show that the sequence $\{x_n\}$ is a Cauchy sequence. Let us suppose on the contrary that $\{x_n\}$ is not a Cauchy sequence. Then there exists $\varepsilon > 0$ such that, for all positive integers k, there exist $m_k > n_k > k$ with

$$d(x_{m_k}, x_{n_k}) \geq \varepsilon \tag{3}$$

Let m_k be the smallest number satisfying the conditions above. Then we have

$$d(x_{m_k-1}, x_{n_k}) < \varepsilon \tag{4}$$

By (3) and (4), we have

$$\begin{aligned} \varepsilon &\leq d(x_{m_k}, x_{n_k}) \\ &\leq d(x_{m_k}, x_{m_k-1}) + d(x_{m_k-1}, x_{n_k}) \\ &< d(x_{m_k-1}, x_{m_k}) + \varepsilon \end{aligned}$$

that is,

$$\varepsilon \leq d(x_{m_k}, x_{n_k}) < \varepsilon + d(x_{m_k-1}, x_{m_k}), \forall\ k \in \mathbb{N} \tag{5}$$

Then in view of (2) and (5), we have

$$\lim_{k\to\infty} d(x_{m_k}, x_{n_k}) = \varepsilon \tag{6}$$

Again, we have,

$$\begin{aligned} d(x_{m_k}, x_{n_k}) &\leq d(x_{m_k}, x_{m_k-1}) + d(x_{n_k}, x_{m_k-1}) \\ &\leq d(x_{m_k}, x_{m_k-1}) + d(x_{n_k}, x_{n_k-1}) + d(x_{m_k-1}, x_{n_k-1}) \end{aligned}$$

and $d(x_{m_k-1}, x_{n_k-1}) \leq d(x_{m_k-1}, x_{m_k}) + d(x_{n_k-1}, x_{n_k}) + d(x_{m_k}, x_{n_k})$
Taking limit as $k \to \infty$ and using (2) and (6), we obtain

$$\lim_{k\to\infty} d(x_{m_k-1}, x_{n_k-1}) = \varepsilon \tag{7}$$

By Lemma 1.4.,we get $\alpha(x_{n_k-1}, x_{m_k-1}) \geq 1$. Therefore, we have

$$\begin{aligned} \psi(d(x_{m_k}, x_{n_k})) &= \psi(d(Tx_{m_k-1}, Tx_{n_k-1})) \\ &\leq \alpha(x_{n_k-1}, x_{m_k-1})\psi(d(Tx_{n_k-1}, Tx_{m_k-1})) \\ &\leq \theta(\psi(N(x_{n_k-1}, x_{m_k-1})))\psi(N(x_{n_k-1}, x_{m_k-1})) \end{aligned}$$

Here we have

$$\begin{aligned} N(x_{n_k-1}, x_{m_k-1}) = max\Big\{&d(x_{n_k-1}, x_{m_k-1}), \frac{d(x_{n_k-1}, Tx_{n_k-1})d(x_{m_k-1}, Tx_{m_k-1})}{1 + d(x_{n_k-1}, x_{m_k-1})}, \\ &\frac{d(x_{n_k-1}, Tx_{n_k-1})d(x_{m_k-1}, Tx_{m_k-1})}{1 + d(Tx_{n_k-1}, Tx_{m_k-1})}\Big\} \\ = max\Big\{d(x_{n_k-1}, x_{m_k-1}), &\frac{d(x_{n_k-1}, x_{n_k})d(x_{m_k-1}, x_{m_k})}{1 + d(x_{n_k-1}, x_{m_k-1})}, \frac{d(x_{n_k-1}, x_{n_k})d(x_{m_k-1}, x_{m_k})}{1 + d(x_{n_k}, x_{m_k})}\Big\} \end{aligned}$$

And we see that

$$\lim_{k\to\infty} N(x_{n_k-1}, x_{m_k-1}) = \varepsilon$$

Now we have

$$\frac{\psi(d(x_{n_k}, x_{m_k})}{\psi(N(x_{n_k-1}, x_{m_k-1}))} \leq \theta(\psi(N(x_{n_k-1}, x_{m_k-1}))) < 1.$$

By using (6) and taking limit as $k \to \infty$ in the above inequality, we obtain

$$\lim_{k\to\infty} \theta(\psi(N(x_{n_k-1}, x_{m_k-1}))) = 1.$$

So, $\lim_{k\to\infty} \psi(N(x_{n_k-1}, x_{m_k-1})) = 0 \Rightarrow \lim_{k\to\infty} N(x_{n_k-1}, x_{m_k-1}) = 0 = \varepsilon$, which is a contradiction.

Hence $\{x_n\}$ is a Cauchy sequence. Since X is complete, there exists $x^* \in X$ such that $x_n \to x^*$. As T is continuous, we have $Tx_n \to Tx^*$ i.e. $\lim_{n\to\infty} x_{n+1} = Tx^*$ and so $x^* = Tx^*$. Hence x^* is a fixed point of T.

For the uniqueness of a fixed point of a generalized rational $\alpha - \psi-$Geraghty contraction type mapping, we consider the following hypothesis:

(**G**) For any two fixed points x and y of T, there exists $z \in X$ such that $\alpha(x, z) \geq 1, \alpha(y, z) \geq 1$ and $\alpha(z, Tz) \geq 1$.

Theorem 2.4 Adding condition (**G**) to the hypotheses of Theorem 2.3., we obtain that x^* is the unique fixed point of T.

Proof: Due to Theorem 2.3., we obtain that $x^* \in X$ is a fixed point of T. Let $y^* \in X$ be another fixed point of T. Then by hypothesis (**G**), there exists $z \in X$ such that $\alpha(x^*, z) \geq 1, \alpha(y^*, z) \geq 1$ and $\alpha(z, Tz) \geq 1$.

Since T is $\alpha-$ admissible we get $\alpha(x^*, T^n z) \geq 1$ and $\alpha(y^*, T^n z) \geq 1$ for all $n \in \mathbb{N}$

Then we have

$$\begin{aligned} \psi(d(x^*, T^{n+1}z)) &\leq \alpha(x^*, T^n z)\psi(d(Tx^*, TT^n z)) \\ &\leq \theta(\psi(N(x^*, T^n z)))\psi(N(x^*, T^n z)), \forall\ n \in \mathbb{N}. \end{aligned}$$

Here we have

$$\begin{aligned} N(x^*, T^n z) &= max\{d(x^*, T^n z), \frac{d(x^*, Tx^*)d(T^n z, TT^n z)}{1 + d(x^*, T^n z)}, \frac{d(x^*, Tx^*)d(T^n z, TT^n z)}{1 + d(Tx^*, TT^n z)}\} \\ &= d(x^*, T^n z) \end{aligned}$$

By Theorem 2.3. we deduce that the sequence $\{T^n z\}$ converges to a fixed point $z^* \in X$

Then taking limit $n \to \infty$ in the above equality, we get $\lim_{n\to\infty} N(x^*, T^n z) = d(x^*, z^*)$. And let us suppose that $z^* \neq x^*$. Then we have

$$\frac{\psi(d(x^*, T^{n+1}z))}{\psi(N(x^*, T^n z))} \leq \theta(\psi(N(x^*, T^n z))) < 1.$$

And taking limit $n \to \infty$, we get $\lim_{n\to\infty} \theta(\psi(N(x^*, T^n z))) = 1$. Therefore we have $\lim_{n\to\infty} \psi(N(x^*, T^n z)) = 0$. This implies $\lim_{n\to\infty} N(x^*, T^n z) = 0$ i.e. $d(x^*, z^*) = 0$, which is a contradiction. Therefore we must have $z^* = x^*$. Similarly, we get $z^* = y^*$. Thus we have $x^* = y^*$. Hence x^* is the unique fixed point of T.

Here we give an example to illustrate Theorem 2.3.

Example 2.5 Let $X = \{1, 2, 3\}$ with the metric d defined as $d(1, 1) = d(2, 2) = d(3, 3) = 0$, $d(1, 2) = d(2, 1) = 1$ and $d(1, 3) = d(3, 1) = d(2, 3) = d(3, 2) = \frac{1}{2}$. Then (X, d) is a complete metric space. And let $\theta(t) = \frac{1}{1+t}$ for all $t \geq 0$. Then $\theta \in \Omega$. Also let the function $\psi : [0, \infty) \to [0, \infty)$ be defined as $\psi(t) = \frac{t}{2}$. Then we have $\psi \in \Psi$.

Let a mapping $T : X \to X$ be defined by $T(1) = T(3) = 1, T(2) = 3$.

And let a function $\alpha : X \times X \to \mathbb{R}$ be defined by $\alpha(x, y) = \begin{cases} 1, & \text{(x=y);} \\ \frac{1}{2}, & \text{otherwise.} \end{cases}$

Then, T is triangular $\alpha-$admissible, which is condition (ii) of Theorem 2.3.

Condition (iii) of Theorem 2.3. is satisfied with $x_1 = 1$. And condition (iv) of

Theorem 2.3. is satisfied because T is continuous. We finally show that condition (i) is also satisfied i.e.

$$\alpha(x,y)\psi(d(Tx,Ty)) \le \theta(\psi(N(x,y)))\psi(N(x,y)).$$

If $(x,y)=(1,1)$or$(2,2)$or(3,3),then $d(Tx,Ty)=0$. Therefore condition (i) is obviously satisfied.
If $(x,y)=(1,3)$or$(3,1)$, then $d(Tx,Ty)=d(1,1)=0$. Therefore condition (i) is satisfied.
If $(x,y)=(1,2)$, then we have
$\alpha(x,y)\psi(d(Tx,Ty))=\alpha(1,2)\psi(d(T(1),T(2)))=\frac{1}{2}\frac{d(1,3)}{2}=\frac{1}{8}$. And

$$\begin{aligned} N(x,y) &= N(1,2) \\ &= max\{d(1,2), \frac{d(1,T(1))d(2,T(2))}{1+d(1,2)}, \frac{d(1,T(1))d(2,T(2))}{1+d(T(1),T(2))}\} \\ &= max\{1,0,0\} \\ &= 1 \end{aligned}$$

Therefore, $\theta(\psi(N(x,y)))\psi(N(x,y))=\frac{\psi(N(x,y))}{1+\psi(N(x,y))}=\frac{\frac{N(x,y)}{2}}{1+\frac{N(x,y)}{2}}=\frac{\frac{1}{2}}{1+\frac{1}{2}}=\frac{1}{3}$
Thus condition (i) is satisfied. Similarly, we see that condition (i) is satisfied for $(x,y)=(2,1)$.
If $(x,y)=(2,3)$, then we have
$\alpha(x,y)\psi(d(Tx,Ty))=\alpha(2,3)\psi(d(T(2),T(3)))=\frac{1}{2}\frac{d(3,1)}{2}=\frac{1}{8}$.

$$\begin{aligned} N(x,y) &= N(2,3) \\ &= max\{d(2,3), \frac{d(2,T(2))d(3,T(3))}{1+d(2,3)}, \frac{d(2,T(2))d(3,T(3))}{1+d(T(2),T(3))}\} \\ &= max\{\frac{1}{2}, \frac{\frac{1}{2}\times\frac{1}{2}}{1+\frac{1}{2}}, \frac{\frac{1}{2}\times\frac{1}{2}}{1+\frac{1}{2}}\} \\ &= max\{\frac{1}{2}, \frac{1}{6}, \frac{1}{6}\} \\ &= \frac{1}{2} \end{aligned}$$

Therefore, $\theta(\psi(N(x,y)))\psi(N(x,y))=\frac{\psi(N(x,y))}{1+\psi(N(x,y))}=\frac{\frac{N(x,y)}{2}}{1+\frac{N(x,y)}{2}}=\frac{\frac{1}{4}}{1+\frac{1}{4}}=\frac{1}{5}$.
Thus condition (i) is satisfied. Similarly, we see that condition (i) is satisfied for $(x,y)=(3,2)$. Hence all the conditions of Theorem 2.3. are satisfied and T has a unique fixed point $x^*=1$.

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