

## COMMON FIXED POINT THEOREM IN Menger SPACE USING $(CLR_g)$ PROPERTY

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ABSTRACT. The object of this paper is to establish a common fixed point theorem for semi-compatible pair of self maps by using CLR<sub>g</sub> Property in Menger space.

### 1. INTRODUCTION

There have been a number of generalizations of metric space. One such generalization is Menger space initiated by Menger [2]. It is a probabilistic generalization in which we assign to any two points  $x$  and  $y$ , a distribution function  $F_{x,y}$ . Schweizer and Sklar [6] studied this concept and gave some fundamental results on this space. Sehgal and Bharucha-Reid [7] obtained Banach contraction principle in a complete Menger space, which is a milestone in developing fixed point theory in Menger space. Sessa [8] initiated the tradition of improving commutativity in fixed point theorems by introducing the notion of weakly commuting maps in metric spaces. Jungck [1] soon enlarged this concept to compatible maps. The notion of compatible mapping in a Menger space has been introduced by Mishra [3]. Pant [4] introduced the notion of reciprocal continuity of mappings in metric spaces. Popa [5] proved theorem for weakly compatible non-continuous mapping using implicit relation. Singh and Jain [9] have been introduced semi-compatible, compatible and weak compatible maps in Menger space.

B. Singh et. al. [10] introduced the notion of semi compatible maps in fuzzy metric space. In 2011, Sintunayarat and Kuman [11] introduced the concept of common limit in the range property. Chouhan et. al. [12] utilize the notion of common limit range property to prove fixed point theorems for weakly compatible mapping in fuzzy metric space.

In 2012, Jain et al. [13] extended the concept of CLR<sub>g</sub> property in the coupled case and also established a common fixed point theorem for weakly compatible

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mappings in fuzzy metric spaces. Most recently, Hierro and Sintunavarat [14] generalized the results in [13] by using the generalized contractive conditions and the CLRg property in fuzzy metric spaces.

## 2. PRELIMINARIES

**Definition 2.1** A mapping  $F : R \rightarrow R^+$  is called a distribution if it is non-decreasing left continuous with  $\inf\{F(t) : t \in R\} = 0$  and  $\sup\{F(t) : t \in R\} = 1$ . We shall denote by  $L$  the set of all distribution functions while  $H$  will always denote the specific distribution function defined by

$$H(x) = \begin{cases} 0, & x \leq 0 \\ 1, & x > 0. \end{cases}$$

**Definition 2.2** A Probabilistic metric space (PM-space) is an ordered pair  $(X, F)$ , where  $X$  is an abstract set of elements and  $F : X \times X \rightarrow L$ , defined by  $(p, q) \rightarrow F_{p,q}$ , where  $L$  is the set of all distribution functions i.e.  $L = \{F_{p,q/p,q} \in X\}$ , if the functions  $F_{p,q}$  satisfy.

- (a)  $F_{p,q}(x) = 1$ , for all  $x > 0$ , if and only if  $p = q$ ;
- (b)  $F_{p,q}(0) = 0$ ;
- (c)  $F_{p,q} = F_{q,p}$ ;
- (d) If  $F_{p,q}(x) = 1$  and  $F_{q,r}(y) = 1$  then  $F_{p,r}(x + y) = 1$ .

**Definition 2.3** A mapping  $t : [0, 1] \times [0, 1] \rightarrow [0, 1]$  is called a  $t$ -norm if

- (a)  $t(a, 1) = a$ ;
  - (b)  $t(a, b) = t(b, a)$ ;
  - (c)  $t(c, d) \geq t(a, b)$  for  $c \geq a, d \geq b$ ;
  - (d)  $t(t(a, b), c) = t(a, t(b, c))$ ,
- for all  $a, b, c, d \in [0, 1]$ .

**Definition 2.4** A Menger space is a triplet  $(X, F, t)$  where  $(X, F)$  is PM-space and  $t$  is a  $t$ -norm such that  $\forall p, q, r \in X$  and  $\forall x, y \geq 0$

$$F_{p,r}(x + y) \geq t(F_{p,q}(x), F_{q,r}(y)).$$

Schweizer and Sklar [6] proved that if  $(X, F, t)$  is a Menger space with  $\sup_{0 < x < 1} t(x, x) = 1$ , then  $(X, F, t)$  is a Hausdorff topological space in the topology induced by the family of  $(\varepsilon, \lambda)$ -neighborhoods,  $\{U_p(\varepsilon, \lambda) : p \in X, \varepsilon > 0, \lambda > 0\}$ , where  $U_p(\varepsilon, \lambda) = \{x \in X : F_{x,p}(\varepsilon) > 1 - \lambda\}$ .

**Definition 2.5** Let  $(X, F, t)$  be a Menger space with  $\sup_{0 < x < 1} t(x, x) = 1$ . A sequence  $\{p_n\}$  in  $X$  is said to converge to a point  $p$  in  $X$  (written as  $p_n \rightarrow p$ ) if for every  $\varepsilon > 0$  and  $\lambda > 0$ ,  $\exists$  an integer  $M(\varepsilon, \lambda)$  such that  $F_{p_n,p}(\varepsilon) > 1 - \lambda$ ,  $\forall n \geq M(\varepsilon, \lambda)$ . Further, the sequence is said to be a Cauchy sequence if for each  $\varepsilon > 0$  and  $\lambda > 0$ ,  $\exists$ , an integer  $M(\varepsilon, \lambda)$  such that  $F_{p_n,p_m}(\varepsilon) > 1 - \lambda$ ,  $\forall n, m \geq M(\varepsilon, \lambda)$ . A Menger space  $(X, F, t)$  is said to be complete if every Cauchy sequence in it converges to a point of it.

A complete metric space can be treated as a complete Menger space in the following way.

**Proposition 2.6** If  $(X, d)$  is a metric space then the metric  $d$  induces a mapping  $X \times X \rightarrow L$ , defined by  $F_{p,q}(x) = H(x - d(p, q))$ ,  $\forall p, q \in X$  and  $x \in R$ . Further, if  $t : [0, 1] \times [0, 1] \rightarrow [0, 1]$  is defined by  $t(a, b) = \min\{a, b\}$ , then  $(X, F, t)$  is a Menger space. It is complete if  $(X, d)$  is complete. Then space  $(X, F, t)$  so obtained is called the induced Menger space.

**Proposition 2.7** In a Menger space  $(X, F, t)$ , if  $t(x, x) \geq x$ ,  $\forall x \in [0, 1]$  then  $t(a, b) = \min\{a, b\}$ ,  $\forall a, b \in [0, 1]$ .

**Definition 2.8** Self mappings  $A$  and  $S$  of a Menger space  $(X, F, t)$  are said to be weak compatible if they commute at their coincidence points i.e.  $Ax = Sx$  for  $x \in X$  implies  $ASx = SAx$ .

**Definition 2.9** Self mappings  $A$  and  $S$  of a Menger space  $(X, F, t)$  are called compatible if  $F_{ASp_n, SAp_n}(x) \rightarrow 1$ ,  $\forall x > 0$ , whenever  $\{p_n\}$  is a sequence in  $X$  such that  $A_{p_n}, S_{p_n} \rightarrow u$ , for some  $u \in X$ , as  $n \rightarrow \infty$ .

**Definition 2.10** Self mappings  $A$  and  $S$  of a Menger space  $(X, F, t)$  are called semi-compatible if  $F_{ASp_n, Su}(x) \rightarrow 1$ ,  $\forall x > 0$ , whenever  $\{p_n\}$  is a sequence in  $X$  such that  $A_{p_n}, S_{p_n} \rightarrow u$ , for some  $u \in X$ , as  $n \rightarrow \infty$ .

**Proposition 2.11** If self mappings  $A$  and  $S$  of a Menger space  $(X, F, t)$  are semi-compatible then they are weak compatible.

**Proposition 2.12** Let  $S$  and  $T$  be two self maps on a Menger space  $(X, F, t)$  with  $t(a, a) \geq a$ ,  $\forall a \in [0, 1]$  of which  $T$  is continuous. Then  $(S, T)$  is semi-compatible if and only if  $(S, T)$  is compatible.

**Lemma 2.13** Let  $\{p_n\}$  be a sequence in a Menger space  $(X, F, t)$  with continuous  $t$ -norm  $t(x, x) \geq x$ ,  $\forall x \in [0, 1]$ . If  $k \in (0, 1)$  such that for all  $x > 0$  and  $n \in N$ ,  $F_{p_n, p_{n+1}}(kx) \geq F_{p_{n-1}, p_n}(x)$ . Then  $\{p_n\}$  is a Cauchy sequence in  $X$ .

**Definition 2.14** Let  $(X, F, t)$  be a menger space with continuous  $t$ -norm  $t(x, x) \geq x$ ,  $\forall x \in [0, 1]$ . Then two mappings  $f, g : X \rightarrow X$  are said to have the  $CLRg$  property if there exist a sequence  $\{x_n\}$  in  $X$  and a point  $z$  in  $X$  such that

$$\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gx_n = gz.$$

**Definition 2.15** Two pairs  $(A, S)$  and  $(B, T)$  of self mappings of a menger space  $(X, F, t)$  are said to satisfy the  $(CLR_{ST})$  property if there exist two sequence  $\{x_n\}$  and  $\{y_n\}$  in  $X$  such that

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} By_n = \lim_{n \rightarrow \infty} Ty_n = Sz, \text{ for some } z \in S(X) \text{ and } z \in T(X).$$

**Definition 2.16** Two pairs  $(A, S)$  and  $(B, T)$  of self mappings of a menger space  $(X, F, t)$  are said to share  $CLRg$  of  $S$  property if there exist two sequence  $\{x_n\}$  and

$\{y_n\}$  in  $X$  such that

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} By_n = \lim_{n \rightarrow \infty} Ty_n = Sz, \text{ for some } z \in X.$$

**Example 2.17** Let  $X = [0, \infty)$  be the usual metric space. Define  $g, h : X \rightarrow X$  by  $gx = x + 3$  and  $hx = 4x$ , for all  $x \in X$ . We consider the sequence  $\{x_n\} = \{1 + 1/n\}$ . Since,  $\lim_{n \rightarrow \infty} gx_n = \lim_{n \rightarrow \infty} hx_n = 4 = h(1) \in X$ . Therefore  $g$  and  $h$  satisfy the  $(CLRg)$  property.

**Definition 2.18** We will apply an implicit relation as, Let  $\Phi$  be set of all real continuous functions  $\phi : (R^+)^4 \rightarrow R$ , nondecreasing in first argument and satisfying the following conditions:

- (i) For  $u, v \geq 0$ ,  $\phi(u, v, v, u) \geq 0$  or  $\phi(u, v, u, v) \geq 0$  imply  $u \geq v$ .
- (ii)  $\phi(u, u, 1, 1) \geq 0$  implies  $u \geq 1$ .

**Example 2.19** Define  $\phi(t_1, t_2, t_3, t_4) = 15t_1 - 13t_2 + 5t_3 - 7t_4$ . Then  $\phi \in \Phi$ .

### 3. MAIN RESULT

In the following theorem we replace the continuity condition by using  $(CLRg)$  property.

**Theorem 3.1** Let  $A, B, S$  and  $T$  be self mapping on a complete menger space  $(X, F, t)$ , satisfying

- (a)  $A(X) \subseteq T(X)$ ,  $B(X) \subseteq S(X)$ ,
- (b)  $(B, T)$  is semi compatible,
- (c) For some  $\phi \in \Phi$ , there exists  $k \in (0, 1)$  such that for all  $x, y \in X$  and  $t > 0$ ,

$$\phi\left(F_{Ax, By}(kt), F_{Sx, Ty}(t), F_{Ax, Sx}(t), F_{By, Ty}(kt)\right) \geq 0 \quad (1)$$

$$\phi\left(F_{Ax, By}(kt), F_{Sx, Ty}(t), F_{Ax, Sx}(kt), F_{By, Ty}(t)\right) \geq 0 \quad (2)$$

If the pair  $(A, S)$  and  $(B, T)$  share the common limit in the range of  $S$  property, then  $A, B, S$  and  $T$  have a unique common fixed point.

*Proof.* Let  $x_0$  be an arbitrary point in  $X$ . Since  $A(X) \subset T(X)$  and  $B(X) \subset S(X)$ , there exist  $x_1, x_2 \in X$  such that  $Ax_0 = Tx_1$  and  $Bx_1 = Sx_2$ . Inductively, we construct the sequences  $\{y_n\}$  and  $\{x_n\}$  in  $X$  such that

$$y_{2n+1} = Ax_{2n} = Tx_{2n+1}, \quad y_{2n+2} = Bx_{2n+1} = Sx_{2n+2}$$

for  $n = 0, 1, 2, \dots$ . Now putting in (1)  $x = x_{2n}, y = x_{2n+1}$ , we obtain

$$\phi\left(F_{Ax_{2n}, Bx_{2n+1}}(kt), F_{Sx_{2n}, Tx_{2n+1}}(t), F_{Ax_{2n}, Sx_{2n}}(t), F_{Bx_{2n+1}, Tx_{2n+1}}(kt)\right) \geq 0$$

that is

$$\phi\left(F_{y_{2n+1}, y_{2n+2}}(kt), F_{y_{2n}, y_{2n+1}}(t), F_{y_{2n+1}, y_{2n}}(t), F_{y_{2n+2}, y_{2n+1}}(kt)\right) \geq 0$$

Using (i), we get

$$F_{y_{2n+2}, y_{2n+1}}(kt) \geq F_{y_{2n+1}, y_{2n}}(t) \quad (3)$$

Analogously, putting  $x = x_{2n+2}$ ,  $y = x_{2n+1}$  in (2), we have

$$\begin{aligned} \phi\left(F_{Ax_{2n}, Bx_{2n+1}}(kt), F_{Sx_{2n+2}, Tx_{2n+1}}(t), F_{Ax_{2n+2}, Sx_{2n+2}}(kt), F_{Bx_{2n+1}, Tx_{2n+1}}(t)\right) &\geq 0 \\ \phi\left(F_{y_{2n+3}, y_{2n+2}}(kt), F_{y_{2n+2}, y_{2n+1}}(t), F_{y_{2n+3}, y_{2n+2}}(kt), F_{y_{2n+2}, y_{2n+1}}(t)\right) &\geq 0 \end{aligned}$$

Using (i), we get

$$F_{y_{2n+3}, y_{2n+2}}(kt) \geq F_{y_{2n+1}, y_{2n+2}}(t) \quad (4)$$

Thus, from (3) and (4), for  $n$  and  $t$ , we have

$$F_{y_n, y_{n+1}}(kt) \geq F_{y_{n-1}, y_n}(t)$$

Hence, by Lemma 2.13,  $\{y_n\}$  is a Cauchy sequence in  $X$ , which is complete. Therefore,  $\{y_n\}$  converges to  $z$  in  $X$ . That is  $\{Ax_{2n}\}$ ,  $\{Tx_{2n+1}\}$ ,  $\{Bx_{2n+1}\}$  and  $\{Sx_{2n}\}$  also converges to  $z$  in  $X$ .

Since the pair  $(A, S)$  and  $(B, T)$  share the common limit in the range of  $S$  property, then there exist two sequences  $\{x_n\}$  and  $\{y_n\}$  in  $X$  such that

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} By_n = \lim_{n \rightarrow \infty} Ty_n = Sz, \text{ for some } z \in X$$

First we prove that  $Az = Sz$

By (1), putting  $x = z$  and  $y = y_n$ , we get

$$\phi\left(F_{Az, By_n}(kt), F_{Sz, Ty_n}(t), F_{Az, Sz}(t), F_{By_n, Ty_n}(kt)\right) \geq 0$$

Taking limit  $n \rightarrow \infty$ , we get

$$\phi\left(F_{Az, Sz}(kt), F_{Sz, Sz}(t), F_{Az, Sz}(t), F_{Sz, Sz}(kt)\right) \geq 0$$

As  $\phi$  is non-decreasing in first argument, we have

$$\phi\left(F_{Az, Sz}(t), 1, F_{Az, Sz}(t), 1\right) \geq 0$$

using (ii), we have

$$F_{Az, Sz}(t) \geq 1 \text{ for all } t > 0$$

$$\text{which gives } F_{Az, Sz}(t) = 1, \text{ that is } Az = Sz \quad (5)$$

Since,  $A(X) \subseteq T(X)$ , therefore there exist  $u \in X$ , such that  $Az = Tu$

$$(6)$$

Again by inequality (1), putting  $x = z$  and  $y = u$ , we get

$$\phi\left(F_{Az, Bu}(kt), F_{Sz, Tu}(t), F_{Az, Sz}(t), F_{Bu, Tu}(kt)\right) \geq 0$$

using (5) and (6), we get

$$\phi\left(F_{Tu, Bu}(kt), F_{Az, Az}(t), F_{Sz, Sz}(t), F_{Bu, Tu}(kt)\right) \geq 0$$

$$\phi\left(F_{Tu, Bu}(kt), 1, 1, F_{Bu, Tu}(kt)\right) \geq 0$$

using (i), we have  $F_{Tu, Bu}(kt) \geq 1$  for all  $t > 0$ ,

$$\text{which gives } F_{Tu, Bu}(kt) = 1. \text{ Thus } Tu = Bu \quad (7)$$

$$\text{Thus from (5), (6), (7), we get } Az = Sz = Tu = Bu \quad (8)$$

Now we will prove that  $Az = z$

By inequality (1), putting  $x = z$  and  $y = x_{2n+1}$ ,

$$\phi\left(F_{Az, Bx_{2n+1}}(kt), F_{Sz, Tx_{2n+1}}(t), F_{Az, Sz}(t), F_{Bx_{2n+1}, Tx_{2n+1}}(kt)\right) \geq 0$$

taking limit  $n \rightarrow \infty$ , using (i) we get

$$\phi\left(F_{Az, z}(kt), F_{Sz, z}(t), F_{Az, Sz}(t), F_{z, z}(kt)\right) \geq 0$$

$$\phi\left(F_{Az, z}(kt), F_{Az, z}(t), F_{Az, Az}(t), F_{z, z}(kt)\right) \geq 0$$

$$\phi\left(F_{Az, z}(kt), F_{Az, z}(t), 1, 1\right) \geq 0$$

as  $\phi$  is non-decreasing in first argument, we have

$$\phi\left(F_{Az, z}(t), F_{Az, z}(t), 1, 1\right) \geq 0$$

using (ii), we have  $F_{Az, z}(t) \geq 1$  for all  $t > 0$ , which gives  $F_{Az, z}(t) = 1$ . Thus  $Az = z$ .

Therefore from (8), we get  $z = Tu = Bu$

Now Semicompatibility of  $(B, T)$  gives  $BTy_{2n+1} \rightarrow Tz$ , i. e.  $Bz = Tz$  Now putting  $x = z$  and  $y = z$  in inequality (1), we get

$$\phi\left(F_{Az, Bz}(kt), F_{Sz, Tz}(t), F_{Az, Sz}(t), F_{Bz, Tz}(kt)\right) \geq 0$$

$$\phi\left(F_{Az, Bz}(kt), F_{Az, Bz}(t), F_{Az, Az}(t), F_{Bz, Bz}(kt)\right) \geq 0$$

$$\phi\left(F_{Az, Bz}(kt), F_{Az, Bz}(t), 1, 1\right) \geq 0$$

as  $\phi$  is non-decreasing in first argument, we have

$$\phi\left(F_{Az, Bz}(t), F_{Az, Bz}(t), 1, 1\right) \geq 0$$

using (ii), we have  $F_{Az, Bz}(t) \geq 1$  for all  $t > 0$ , which gives  $F_{Az, Bz}(t) = 1$ . Thus  $Az = Bz$  and hence  $Az = Bz = z$ . combining all results, we get  $z = Az = Bz = Sz = Tz$ .

From this we conclude that  $z$  is a common fixed point of  $A, B, S$  and  $T$ .

**Uniqueness.** Let  $z_1$  be another common fixed point of  $A, B, S$  and  $T$ . Then  $z_1 = Az_1 = Bz_1 = Sz_1 = Tz_1$  and  $z = Az = Bz = Sz = Tz$  then by inequality (1), putting  $x = z$  and  $y = z_1$ , we get

$$\phi\left(F_{Az, Bz_1}(kt), F_{Sz, Tz_1}(t), F_{Az, Sz}(t), F_{Bz_1, Tz_1}(kt)\right) \geq 0$$

$$\phi\left(F_{z, z_1}(kt), F_{z, z_1}(t), F_{z, z}(t), F_{z_1, z_1}(kt)\right) \geq 0$$

$$\phi\left(F_{z, z_1}(kt), F_{z, z_1}(t), 1, 1\right) \geq 0$$

as  $\phi$  is non-decreasing in first argument, we have

$$\phi\left(F_{z, z_1}(t), F_{z, z_1}(t), 1, 1\right) \geq 0$$

using (ii), we have  $F_{z, z_1}(t) \geq 1$  for all  $t > 0$ , which gives  $F_{z, z_1}(t) = 1$ . Thus  $z = z_1$ . Thus  $z$  is the unique common fixed point of  $A, B, S$  and  $T$ .

If we increase the number of self maps from four to six then we have the following.

**Corollary 3.2** Let  $A, B, S, T, I$  and  $J$  be self mappings on a complete menger space  $(X, F, t)$ , satisfying

- (a)  $AB(X) \subseteq J(X)$  and  $ST(X) \subseteq I(X)$ ,
- (b)  $(ST, J)$  is semi compatible,
- (c) For some  $\phi \in \Phi$ , there exists  $k \in (0, 1)$  such that for all  $x, y \in X$  and  $t > 0$ ,

$$\phi\left(F_{ABx,STy}(kt), F_{Ix,Jy}(t), F_{ABx,Ix}(t), F_{STy,Jy}(kt)\right) \geq 0 \quad (1)$$

$$\phi\left(F_{ABx,STy}(kt), F_{Ix,Jy}(t), F_{ABx,Ix}(kt), F_{STy,Jy}(t)\right) \geq 0 \quad (2)$$

If the pair  $(AB, I)$  and  $(ST, J)$  share the common limit in the range of  $I$  property, then  $AB, ST, I$  and  $J$  have a unique common fixed point. Furthermore, if the pairs  $(A, B), (A, I), (B, I), (S, T), (S, J)$  and  $(T, J)$  are commuting mapping then  $A, B, S, T, I$  and  $J$  have a unique common fixed point.

*Proof.* From theorem 3.1,  $z$  is the unique common fixed point of  $AB, ST, I$  and  $J$ . Finally, we need to show that  $z$  is also a common fixed point of  $A, B, S, T, I$ , and  $J$ . For this, let  $z$  be the unique common fixed point of both the pairs  $(AB, I)$  and  $(ST, J)$ . Then, by using commutativity of the pair  $(A, B), (A, I)$  and  $(B, I)$ , we obtain

$$Az = A(ABz) = A(BAz) = AB(Az), \quad Az = A(Iz) = I(Az),$$

$$Bz = B(ABz) = B(A(Bz)) = BA(Bz) = AB(Bz), \quad Bz = B(Iz) = I(Bz), \quad (3)$$

which shows that  $Az$  and  $Bz$  are common fixed point of  $(AB, I)$ , yielding thereby

$$Az = z = Bz = Iz = ABz \quad (4)$$

in the view of uniqueness of the common fixed point of the pair  $(AB, I)$ . Similarly, using the commutativity of  $(S, T), (S, J), (T, J)$ , it can be shown that

$$Sz = Tz = Jz = STz = z. \quad (5)$$

Now, we need to show that  $Az = Sz (Bz = Tz)$  also remains a common fixed point of both the pairs  $(AB, I)$  and  $(ST, I)$ . For this, put  $x = z$  and  $y = z$  in (1) and using (4) and (5), we get

$$\phi\left(F_{ABz,STz}(kt), F_{Iz,Jz}(t), F_{ABz,Iz}(t), F_{STz,Jz}(kt)\right) \geq 0$$

that is,

$$\phi\left(F_{Az,Sz}(kt), F_{Az,Sz}(t), F_{Az,Az}(t), F_{Sz,Sz}(kt)\right) \geq 0$$

as  $\phi$  is nondecreasing in first argument, we have

$$\phi\left(F_{Az,Sz}(t), F_{Az,Sz}(t), F_{Az,Az}(t), F_{Sz,Sz}(kt)\right) \geq 0$$

$$\phi\left(F_{Az,Sz}(kt), F_{Az,Sz}(t), 1, 1\right) \geq 0$$

using (ii), we obtain  $F_{Az,Sz}(t) \geq 1$  for all  $t > 0$ , which gives  $F_{Az,Sz}(t) = 1$ , that is,  $Az = Sz$ . Similarly, it can be shown that  $Bz = Tz$ . Thus,  $z$  is the unique common fixed point of  $A, B, S, T, I$ , and  $J$ .

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