

## COMMON FIXED POINT THEOREM FOR PAIR OF MAPPINGS SATISFYING COMMON $(E.A)$ -PROPERTY IN COMPLETE METRIC SPACES WITH APPLICATION

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**ABSTRACT.** In this paper, we establish a common fixed point theorem for quadruple of mappings satisfying common  $(E.A)$ -property in the setup of complete metric spaces. We employ an example to substantiate the utility of our established result. We provide an application of our main result for four finite families of mappings.

### 1. INTRODUCTION

In 1922, Banach [4] proved his celebrated fixed point theorem, which assures the existence and uniqueness of a fixed point under certain conditions. It is very effectively utilized for solving various applied problems in mathematical sciences and engineering. Many authors extended and generalized Banach fixed point theorem in different ways.

In 1976, Jungck [7] generalized this theorem by using the notion of commuting mappings under the assumption that one of the maps must be continuous. In 1982, Sessa [18] proved some fixed point theorems by introducing the concept of weak commutativity for a pair of self maps. He also showed that weakly commuting mappings are commuting but the converse need not to be true. Later, Jungck [8] introduced the concept of compatible mappings which is more general than weakly commuting mappings and showed that weak commuting maps are compatible but the converse need not be true. In 1996, Jungck [9] defined weakly compatible mappings if they commute at their coincidence points. Many authors established fixed points results for different classes of mappings on metric spaces. ([6],[12],[15])

The study of common fixed points for non compatible mappings was initiated by Pant [14]. In 2002, Aamri and El Moutawakil [1] introduced a new property called  $(E.A)$ -property for pair of mappings which is a generalization of non compatible mappings and they proved some common fixed point theorems. Some authors showed that the notion of weakly compatible mappings and mappings satisfying  $(E.A)$ -property are independent [15, 16].

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In 2005, Liu et al. [11] defined the notion of common  $(E.A)$  property. Many authors established common fixed point theorems by using common  $(E.A)$  property in the setup of metric spaces and variants of metric spaces [3, 5, 6, 13]. Recently, R. Tiwari and S. Gupta [19] proved some new common fixed point theorems in metric spaces for weakly compatible mappings satisfying an implicit relation involving quadratic terms. Most recently, Neog et al. [12] presented some common fixed point theorems for  $A_\varphi$ -contraction mappings.

## 2. PRELIMINARIES

We give some important definitions, which are useful to establish our main result.

**Definition 1** [17] Consider the class of functions  $\Phi = \{\varphi | \varphi : \mathbb{R}_+ \rightarrow \mathbb{R}_+\}$ , which satisfy the following assertions. Where  $\mathbb{R}_+$  denote the set of all non negative real numbers.

- (1)  $t_1 \leq t_2$  implies  $\varphi(t_1) \leq \varphi(t_2)$ ,
- (2)  $(\varphi^n(t))_{n \in \mathbb{N}}$  converges to 0 for all  $t > 0$ ,
- (3)  $\sum \varphi^n(t)$  converges for all  $t > 0$ .

If conditions (1 – 2) hold then  $\varphi$  is called a comparison function and if the comparison function satisfies (3) then  $\varphi$  is called a strong comparison function.

**Remark 1** [17] If  $\varphi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$  is a comparison function, then  $\varphi(t) < t$ , for all  $t > 0$ ,  $\varphi(0) = 0$  and  $\varphi$  is right continuous at 0.

**Definition 2** [2] Suppose  $\mathbb{R}_+$  is the set of all non negative real numbers and  $A$  be the collection of all functions  $\alpha : \mathbb{R}_+^3 \rightarrow \mathbb{R}_+$  which satisfies the following conditions:

- (1)  $\alpha$  is continuous on  $\mathbb{R}_+^3$  (with respect to the Euclidean metric on  $\mathbb{R}_+^3$ ),
- (2)  $a \leq kb$  for some  $k \in [0, 1)$  whenever  $a \leq \alpha(a, b, b)$  or  $a \leq \alpha(b, a, b)$  or  $a \leq \alpha(b, b, a)$  for all  $a, b$ .

**Definition 3** [2] Let  $(X, d)$  be a metric space and  $T$  a self map on  $X$ .  $T$  is said to be a  $A$ -contraction if

$$d(Tx, Ty) \leq \alpha(d(x, y), d(x, Tx), d(y, Ty)),$$

for all  $x, y \in X$  and some  $\alpha \in A$ .

**Definition 4** [10] Let  $(X, d)$  be a metric space and  $T, S$  are two self maps on  $X$ .  $T$  and  $S$  are said to be weakly compatible if for all  $x \in X$  the equality  $Tx = Sx$  implies  $TSx = STx$ .

**Definition 5** [1] Let  $S$  and  $T$  be two self mappings of a metric space  $(X, d)$ . We say that  $S$  and  $T$  satisfy  $(E.A)$ -property if there exists a sequence  $\{x_n\}$  in  $X$  such that

$$\lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Tx_n = t,$$

for some  $t \in X$ .

**Definition 6** [11] Two pairs  $(A, S)$  and  $(B, T)$  of self mappings of a metric space  $(X, d)$  are said to satisfy common  $(E.A)$ -property if there exists two sequences  $\{x_n\}$

and  $\{y_n\}$  in  $X$  such that

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} By_n = \lim_{n \rightarrow \infty} Ty_n = t,$$

for some  $t \in X$ .

In the present paper, we prove a common fixed point theorem for quadruple of weakly compatible mappings satisfying common  $(E.A)$ -property in the setup of complete metric spaces. An example is furnished which demonstrates the validity of our main result. We also provide an application of our main result for four finite families of mappings.

### 3. MAIN RESULT

In this section, we introduce a function  $\alpha$  which extends the corresponding function  $\alpha$  given in [12] as follows:

**Definition 7** Let  $\mathbb{R}_+$  be the set of all non-negative real numbers and  $A_\varphi$  be the collection of all functions  $\alpha : \mathbb{R}_+^4 \rightarrow \mathbb{R}_+$  which satisfy the following conditions:

- (1)  $\alpha$  is continuous on  $\mathbb{R}_+^4$  (with respect to the Euclidean metric on  $\mathbb{R}_+^4$ ),
- (2) for all  $u, v \in \mathbb{R}_+$ , if
  - (2<sub>a</sub>)  $u \leq \alpha(u, v, v, v)$  or
  - (2<sub>b</sub>)  $u \leq \alpha(v, u, v, v)$  or
  - (2<sub>c</sub>)  $u \leq \alpha(v, v, u, v)$ ,

then  $u \leq \varphi(v)$ , where  $\varphi$  is a strong comparison function. If  $\varphi(t) = kt$  for  $k \in [0, 1)$  and for all  $t > 0$ , then we have  $\alpha \in A_\varphi$ .

Now, we state and prove our main result as follows:

**Theorem 1** Let  $(X, d)$  be a complete metric space and  $E$  be a nonempty subset of  $X$ . Let  $T, S : E \rightarrow E$  be self maps. If there exists some  $\alpha \in A_\varphi$  such that for all  $x, y \in X$

$$d(Tx, Sy) \leq \alpha \left( d(Ax, By), d(Ax, Tx), d(Sy, By), \frac{d(Sy, By)d(By, Tx)}{1 + d(Ax, Sy)} \right), \quad (1)$$

where  $A, B : E \rightarrow X$  satisfying the following assertions:

- (1)  $T(E) \subseteq B(E)$  and  $S(E) \subseteq A(E)$ ,
- (2) The pairs  $(T, A)$  and  $(S, B)$  satisfying common  $(E.A)$ -property,
- (3) The pairs  $(T, A)$  and  $(S, B)$  are weakly compatible mappings.

Also, assume that  $A(E)$  or  $B(E)$  is closed in  $X$ . Then  $T, A, B$  and  $S$  have a unique common fixed point.

**Proof.** Since  $(T, A)$  and  $(S, B)$  satisfy the common  $(E.A)$ -property, there exist two sequences  $\{x_n\}$  and  $\{y_n\}$  in  $E$  such that

$$\lim_{n \rightarrow \infty} Tx_n = \lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sy_n = \lim_{n \rightarrow \infty} By_n = z,$$

for some  $z \in E$ . Since  $B(E)$  is closed subset subset of  $X$ , there exists  $u \in E$  such that  $Bu = z$ . we assert that  $Su = z$ . From (1) we obtain

$$\begin{aligned} d(z, Su) &= d(Tx_n, Su) \\ &\leq \alpha \left( d(Ax_n, Bu), d(Ax_n, Tx_n), d(Su, Bu), \frac{d(Su, Bu)d(Bu, Tx_n)}{1 + d(Ax_n, Su)} \right), \end{aligned}$$

taking  $n \rightarrow \infty$ , we obtain

$$\begin{aligned} d(z, Su) &\leq \alpha \left( d(z, z), d(z, z), d(Su, z), \frac{d(Su, z)d(z, z)}{1 + d(z, Su)} \right) \\ &\leq \alpha(0, 0, d(z, Su), 0), \end{aligned}$$

by definition of  $(2_c)$ , we get  $d(z, Su) \leq \varphi(0) = 0$ . Which implies  $d(z, Su) = 0$ , it means  $Su = z$ . Therefore  $Su = Bu = z$ . Hence  $z$  is a coincidence point of  $(S, B)$ . Now, since  $S(E) \subseteq A(E)$ , there exists  $v \in E$  such that  $Av = z$ . We assert that  $Tv = z$ . From (1) we obtain

$$\begin{aligned} d(Tv, z) &= d(Tv, Sy_n) \\ &\leq \alpha \left( d(Av, By_n), d(Av, Tv), d(Sy_n, By_n), \frac{d(Sy_n, By_n)d(By_n, Tv)}{1 + d(Av, Sy_n)} \right), \end{aligned}$$

taking  $n \rightarrow \infty$ , we obtain

$$\begin{aligned} d(Tv, z) &\leq \alpha \left( d(z, z), d(z, Tv), d(z, z), \frac{d(z, z)d(z, Tv)}{1 + d(z, z)} \right) \\ &\leq \alpha(0, d(Tv, z), 0, 0), \end{aligned}$$

by definition of  $(2_b)$ , we get  $d(Tv, z) \leq \varphi(0) = 0$ . Which implies  $d(Tv, z) = 0$ , it means  $Tv = z$ . Therefore  $Tv = Av = z$ . Hence  $z$  is a coincidence point of  $(T, A)$ . Thus  $Su = Bu = Tv = Av = z$  and by weak compatibility of  $(S, B)$  and  $(T, A)$ , we deduce that  $Sz = Bz$  and  $Tz = Az$ . Now, we show that  $z$  is a fixed point of  $T$ . By (1), we obtain

$$\begin{aligned} d(Tz, z) &= d(Tz, Su) \\ &\leq \alpha \left( d(Az, Bu), d(Az, Tz), d(Su, Bu), \frac{d(Su, Bu)d(Bu, Tz)}{1 + d(Az, Su)} \right) \\ &\leq \alpha \left( d(Tz, z), d(Tz, Tz), d(z, z), \frac{d(z, z).d(z, Tz)}{1 + d(Tz, z)} \right) \\ &\leq \alpha(d(Tz, z), 0, 0, 0), \end{aligned}$$

by definition of  $(2_a)$ , we get  $d(Tz, z) \leq \varphi(0) = 0$ . Which implies  $(Tz, z) = 0$ , this means  $Tz = z$ . Hence  $z$  is a fixed point of  $T$ . Since  $Az = Tz = z$  we conclude that  $z$  is a fixed point of  $A$ .

Now, we show that  $z$  is a fixed point of  $S$ . By using (1), we obtain

$$\begin{aligned} d(z, Sz) &= d(Tv, Sz) \\ &\leq \alpha\left(d(Av, Bz), d(Av, Tv), d(Sz, Bz), \frac{d(Sz, Bz)d(Bz, Tv)}{1 + d(Av, Sz)}\right) \\ &\leq \alpha\left(d(z, Sz), d(z, z), d(Bz, Bz), \frac{d(Bz, Bz)d(Sz, z)}{1 + d(z, Sz)}\right) \\ &\leq \alpha(d(z, Sz), 0, 0, 0), \end{aligned}$$

by definition of (2<sub>a</sub>), we get  $d(z, Sz) \leq \varphi(0) = 0$ . Which implies  $(z, Sz) = 0$ , this means  $Sz = z$ . Hence  $z$  is a fixed point of  $S$ . Since  $Bz = Sz = z$  we conclude that  $z$  is a fixed point of  $B$ . Thus  $z$  is a common fixed point of  $T, A, B$  and  $S$ .

If there exists another common fixed point  $v \in E$  such that  $Sv = Bv = Tv = Av = v$ , then we have

$$\begin{aligned} d(z, v) &= d(Tz, Sv) \\ &\leq \alpha\left(d(Az, Bv), d(Az, Tz), d(Sv, Bv), \frac{d(Sv, Bv)d(Bv, Tz)}{1 + d(Az, Sv)}\right) \\ &\leq \alpha\left(d(z, v), d(z, z), d(v, v), \frac{d(v, v)d(v, z)}{1 + d(z, v)}\right) \\ &\leq \alpha(d(z, v), 0, 0, 0), \end{aligned}$$

By definition of (2<sub>a</sub>), we get  $d(z, v) \leq \varphi(0) = 0$ . Which implies  $d(z, v) = 0$ , this means  $z = v$ . Thus  $A, B, S$  and  $T$  have a unique common fixed point. This completes the proof.

**Remark 2** For  $T = S$  and  $B = A$  in Theorem 1, we get the following result.

**Corollary 1** Let  $(X, d)$  be a complete metric space and  $E$  be a nonempty subset of  $X$ . Let  $S : E \rightarrow E$  be self maps. If there exists some  $\alpha \in A_\varphi$  such that for all  $x, y \in X$ .

$$d(Sx, Sy) \leq \alpha\left(d(Ax, Ay), d(Ax, Sx), d(Sy, Ay), \frac{d(Sy, Ay)d(Ay, Sx)}{1 + d(Ax, Sy)}\right),$$

where  $A : E \rightarrow X$  satisfying the following assertion:

- (1)  $S(E) \subseteq A(E)$ ,
- (2) The pair  $(S, A)$  satisfying  $(E.A)$ -property,
- (3) The pair  $(S, A)$  is weakly compatible.

Also, assume that  $A(E)$  is closed subset of  $X$ . Then  $S$  and  $A$  have a unique common fixed point.

**Remark 3** If  $B = A$  and  $\alpha(t_1, t_2, t_3, t_4) = \alpha(t_1, t_2, t_3)$  in Theorem 1, then we get the following result.

**Corollary 2** Let  $(X, d)$  be a complete metric space and  $E$  be a nonempty subset of  $X$ . Let  $T, S : E \rightarrow E$  be self maps. If there exists some  $\alpha \in A_\varphi$  such that for all  $x, y \in X$ .

$$d(Tx, Sy) \leq \alpha\left(d(Ax, Ay), d(Ax, Tx), d(Sy, Ay)\right),$$

where  $A : E \rightarrow X$  satisfying the following assertion:

- (1)  $T(E) \subseteq A(E)$  and  $S(E) \subseteq A(E)$ ,
- (2) The pairs  $(T, A)$  or  $(S, A)$  satisfying  $(E.A)$ -property,
- (3) The pairs  $(T, A)$  and  $(S, A)$  are weakly compatible.

Also, assume that  $A(E)$  is closed subset of  $X$ . Then  $T, A$  and  $S$  have a unique common fixed point.

Now, we give an example to demonstrate the validity of Theorem 1.

**Example 1** Consider  $X = [3, 15]$  and  $d(x, y) = \max\{x, y\}$ ,  $\alpha(x, y, z, t) = \max\{x, y, z, t\}$ . Define the mappings  $T, A, S$  and  $B$  on  $X$  such that

$$Tx = \begin{cases} 3, & \text{if } x \in \{3\} \cup (5, 15]; \\ 4, & \text{if } x \in (3, 5]; \end{cases} \quad Sx = \begin{cases} 3, & \text{if } x \in \{3\} \cup (5, 15]; \\ 5, & \text{if } x \in (3, 5]; \end{cases}$$

$$Ax = \begin{cases} 3, & \text{if } x = 3; \\ 5-x, & \text{if } x \in (3, 5]; \\ \frac{x+1}{2}, & \text{if } x \in (5, 15]; \end{cases} \quad Bx = \begin{cases} 3, & \text{if } x = 3; \\ 10, & \text{if } x \in (3, 5]; \\ \frac{x+1}{2}, & \text{if } x \in (5, 15]. \end{cases}$$

We take the sequence  $x_n = \{5 + \frac{1}{n}\}$  and  $y_n = \{3\}$ . We have

$$\lim_{n \rightarrow \infty} Tx_n = \lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sy_n = \lim_{n \rightarrow \infty} By_n = 3 \in X.$$

Therefore, both pairs  $(T, A)$  and  $(S, B)$  satisfy the common  $(E.A)$ -property. We see that mappings  $(T, A)$  and  $(S, B)$  commute at 3 which is the coincidence point.

Also,

$$T(x) = \{3, 4\} \subseteq [3, 8] \cup \{10\} = B(X) \text{ and } S(X) = \{3, 5\} \subseteq [3, 8] \cup (8, 10] = A(X).$$

We can verify the contraction condition (1) by a simple calculation for the case  $x, y \in [3, 5]$ ,

$$d(Tx, Sy) = 5, \quad d(Ax, By) = 10, \quad d(Ax, Tx) = 10,$$

$$d(Sy, By) = 10, \quad \frac{d(Sy, By)d(By, Tx)}{1 + d(Ax, Sy)} = \frac{100}{11},$$

which yields  $5 \leq \alpha(10, 10, 10, \frac{100}{11})$ , where  $\alpha(x, y, z, t) = \max\{x, y, z, t\}$  and  $\varphi(t) = \frac{2t}{3}$ . Thus

$$d(Tx, Sy) \leq \alpha\left(d(Ax, By), d(Ax, Tx), d(Sy, By), \frac{d(Sy, By)d(By, Tx)}{1 + d(Ax, Sy)}\right).$$

Similarly, we can verify for other cases. Thus all the conditions of Theorem 1 are satisfied and 3 is the unique common fixed point of the mappings  $T, A, S$  and  $B$ .

**Remark 4** As an application of Theorem 1, we establish a common fixed point theorem for finite families of mappings as follows:

**Corollary 3** Let  $(X, d)$  be a complete metric space and  $E$  be a nonempty subset of  $X$ . Let  $\{T_1, T_2, \dots, T_m\}$  and  $\{S_1, S_2, \dots, S_n\}$  be two finite families of self mappings on  $E$  with  $T = T_1 T_2 \dots T_m$  and  $S = S_1 S_2 \dots S_n$  and two finite families  $\{A_1, A_2, \dots, A_r\} : E \rightarrow X$  and  $\{B_1, B_2, \dots, B_s\} : E \rightarrow X$ . There exists some  $\alpha \in A_\varphi$

such that for all  $x, y \in X$

$$d(Tx, Sy) \leq \alpha \left( d(Ax, By), d(Ax, Tx), d(Sy, By), \frac{d(Sy, By)d(By, Tx)}{1 + d(Ax, Sy)} \right),$$

where  $A = A_1A_2\dots A_r$  and  $B = B_1B_2\dots B_s$  satisfying the conditions (1), (2), (3) of Theorem 1.

Also, assume that  $A(E)$  or  $B(E)$  is closed subset of  $X$ . Then  $(T, A)$  and  $(S, B)$  have a point of coincidence.

Moreover, if  $T_tT_u = T_uT_t$ ,  $T_tA_i = A_iT_t$ ,  $S_pS_q = S_qS_p$ , and  $S_pB_k = B_kS_p$ ,  $A_iA_j = A_jA_i$  and  $B_kB_l = B_lB_k$ , for all  $t, u \in I_1 = \{1, 2, \dots, m\}$ ,  $i, j \in I_2 = \{1, 2, \dots, r\}$ ,  $p, q \in I_3 = \{1, 2, \dots, n\}$  and  $k, l \in I_4 = \{1, 2, \dots, s\}$ , then (for all  $t \in I_1$ ,  $i \in I_2$ ,  $p \in I_3$  and  $k \in I_4$ )  $T_t$ ,  $A_i$ ,  $S_p$  and  $B_k$  have a common fixed point.

**Proof.** The conclusions are immediate as  $T$ ,  $A$ ,  $B$  and  $S$  satisfy all the conditions of Theorem 1. Now appealing to component wise commutativity of various pairs, one can immediately prove that  $TA = AT$  and  $SB = BS$  and hence, obviously both pairs  $(T, A)$  and  $(S, B)$  are coincidentally commuting. Note that all the conditions of Theorem 1 (for mappings  $T$ ,  $A$ ,  $B$  and  $S$ ) are satisfied ensuring the existence of a unique common fixed point, say  $z$ . Now, we show that  $z$  remains the fixed point of all the component maps. For this we consider

$$\begin{aligned} T(T_tz) &= ((T_1T_2\dots T_m)T_t)z \\ &= (T_1T_2\dots T_{m-1})(T_mT_t)z \\ &= (T_1T_2\dots T_{m-1})(T_tT_m)z \\ &= (T_1T_2\dots T_{m-2})(T_{m-1}T_t(T_mz)) \\ &= (T_1T_2\dots T_{m-2})(T_tT_{m-1}(T_mz)) \\ &= T_1T_t(T_2T_3\dots T_mz) \\ &= T_tT_1(T_2T_3\dots T_mz) = T_t(Tz) = T_tz. \end{aligned}$$

Similarly, we show that

$$T(A_iz) = A_i(Tz) = A_iz, \quad A(A_iz) = A_i(Az) = A_iz,$$

$$A(T_tz) = T_t(Az) = T_tz, \quad S(S_pz) = S_p(Sz) = S_pz,$$

$$S(B_kz) = B_k(Sz) = B_kz, \quad B(B_kz) = B_k(Bz) = B_kz,$$

$$B(S_pz) = S_p(Bz) = S_pz,$$

which show that (for all  $t, i, p$  and  $k$ )  $T_t z$  and  $A_i z$  are other fixed points of the pair  $(T, A)$  whereas  $S_p z$  and  $B_k z$  are other fixed points of the pair  $(S, B)$ . Now appealing to the uniqueness of common fixed points of both pairs separately, we get

$$z = T_t z = A_i z = S_p z = B_k z,$$

which shows that  $z$  is a common fixed point of  $T_t, A_i, S_p$  and  $B_k$  for all  $t, i, p$  and  $k$ .

**Remark 5** By setting  $T = T_1 = T_2 = \dots = T_m$ ,  $A = A_1 = A_2 = \dots = A_r$ ,  $S = S_1 = S_2 = \dots = S_n$  and  $B = B_1 = B_2 = \dots = B_s$ , we get the following corollary.

**Corollary 4** Let  $(X, d)$  be a complete metric space and  $E$  be a nonempty subset of  $X$ . Let  $T, S : E \rightarrow E$  be self maps and  $A, B : E \rightarrow X$  such that  $T^m, A^r, S^n$  and  $B^s$  satisfying the conditions (1), (2) and inequality (1). Also, assume that  $A^r(E)$  or  $B^s(E)$  are closed subset of  $X$ . Then  $T, A, B$  and  $S$  have a unique common fixed point provided  $TA = AT$  and  $SB = BS$ .

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