Electronic Journal of Mathematical Analysis and Applications
Vol. 9(2) July 2021, pp. 77-87.
ISSN: 2090-729X(online)
http://math-frac.org/Journals/EJMAA/

K-BANHATTI INDICES, K-HYPER BANHATTI INDICES, FORGOTTEN INDEX, FIRST HYPER ZAGREB INDEX OF GENERALIZED TRANSFORMATION GRAPHS

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Abstract. Let G be a simple connected graph. The K-Banhatti indices and K-Hyper Banhatti indices are introduced by Kulli in 2016. The Zagreb indices were introduced in 1972. In this paper, we established the expressions for the K-Banhatti indices, K-Hyper Banhatti indices, Zagreb indices, first Hyper Zagreb indices and Forgotten index of the generalized transformation graphs G^{xy} and their complement graphs are obtained in terms of some parameters of a graph.

1. Introduction

The graphs considered here are finite, undirected without loops and multiple edges. Let G be a connected graph with n vertices and m edges. The degree $d_G(u)$of a vertex u. The edge connecting the vertices u and v will be denoted by uv. Let $d_G(e)$ denote the degree of an edge $e = uv$ in G. Which is denoted by $d_G(e) = d_G(u) + d_G(v) - 2$.

Topological indices are useful tool for modeling physical and chemical properties of molecules for design of pharmacologically active compounds for recognizing environmentally hazardons materials. A number of chemical applications especially to multiple quantum NMR Spectroscopy. Chemical graph theory is the topology branch of Mathematical Chemistry which applies graph theory to mathematical modelling of chemical phenomena.

A chemical graph [3] is a graph in which the vertices correspond to atoms and edges to the bonds of a chemical structure. A single number that can be finding from the chemical graph and used to characterize some property of the underlying chemical is said to be a topological index or molecular structure descriptor. Lot of such descriptors have been considered in theoretical chemistry and have some applications especially in QSPR/QSAR fields of research see ([5], [7]).

The first and second Banhatti indices are first introduced by Kulli [6] and are

2010 *Mathematics Subject Classification.* 05C10; 05C76; 97M60.
Key words and phrases. Banhatti indices; Forgotten index; first Hyper-Zagreb indices; Transformation Graphs.
Submitted May 7, 2020. Revised Aug. 14, 2020.

denoted and defined as below

$$B_1(G) = \sum_{ue}[d_G(u) + d_G(e)] \quad \text{and} \quad B_2(G) = \sum_{ue} d_G(u).d_G(e).$$

Where ue means that the vertex u and edge e are incident in G.
In [7, 8] Kulli introduced the first and second K-Hyper Banhatti Indices and are defined as
$HB_1(G) = \sum_{ue}[d_G(u) + d_G(e)]^2$ and $HB_2(G) = \sum_{ue}[d_G(u)d_G(e)]^2$.
The first and second Zagreb indices of a graph G are defined as,
see [5, 10, 11]
$M_1(G) = \sum_{uv\in E(G)}[d_G(u) + d_G(v)]$ and $M_2(G) = \sum_{uv\in E(G)}[d_G(u)d_G(v)]$.
In [2], Furtula et al., introduced the forgotten topological index F, defined as $F(G) = \sum_{uv\in E(G)}[d_G(u)^2 + d_G(v)^2]$.
In [13], Shirdel et al., introduced the first Hyper Zagreb index of G and defined as $HM_1(G) = \sum_{uv\in E(G)}[d_G(u) + d_G(v)]^2$.
The generalized transformation graph G^{xy} introduced recently by Basavanagoud et al [1]., is a graph whose vertex set is $V(G) \cup E(G)$ and $\alpha, \beta \in V(G^{xy})$ the vertices α, β are adjacent in G^{xy} if and only if (i) and (ii) satisfied.
(i) $\alpha, \beta \in V(G)$, α, β are adjacent in G if $x = +$, and α, β are not adjacent in G if $x = -$.
(ii) $\alpha \in V(G)$ and $\beta \in E(G)$, α, β are incident in G if $y = +$, and α, β are not incident in G if $y = -$.
One can obtain the four graphical transformations of graphs as G^{++}, G^{+-}, G^{-+}, G^{--}. An example of generalized transformation graphs and their complements are depicted in the Fig.1. Note that G^{++} is just the semitotal point graph of G, which was introduced by Sampathkumar and Chikkodimath [12]. The vertex u of G^{xy} corresponding to a vertex u of G is referred to as a point vertex. The vertex e of G^{xy} corresponding to an edge of G is referred to as a line vertex.

Lemma 1.1. [1] *Let G be a graph with n vertices and m edges , Let $u \in V(G)$ and $e \in E(G)$ then the degree of point and line vertices in G^{xy} are,*
(i) $d_G^{++}(u) = 2d_G(u)$ *and* $d_G^{++}(e) = 2$
(ii) $d_G^{+-}(u) = m$ *and* $d_G^{+-} = n - 2$
(iii) $d_G^{-+}(u) = n - 1$ *and* $d_G^{-+} = 2$
(iv) $d_G^{--} = n + m - 1 - 2d_G(u)$ *and* $d_G^{--}(e) = n - 2$.

The complement of G will be denoted by $\overline{G}$. If G has n vertices and m edges then the number of vertices of G^{xy} is $n+m$. By Lemma 1.1 and taking into account that $d_{\overline{G}}(u) = n - 1 - d_G(u)$. We have following Lemma.

Lemma 1.2. [1] *Let G be a graph with n vertices and m edges, Let $u \in V(G)$ and $e \in E(G)$ then the degree of point and line vertices in $\overline{G^{xy}}$ are,*
(i) $d_{\overline{G^{++}}}(u) = 2d_G(u)$ *and* $d_{\overline{G^{++}}}(e) = 2$
(ii) $d_{\overline{G^{+-}}}(u) = m$ *and* $d_{\overline{G^{+-}}}(e) = n - 2$
(iii) $d_{\overline{G^{-+}}}(u) = n - 1$ *and* $d_{\overline{G^{-+}}}(e) = 2$
(iv) $d_{\overline{G^{--}}}(u) = n + m - 1 - 2d_G(u)$ *and* $d_{\overline{G^{--}}}(e) = n - 2$.

Lemma 1.3. *Let G be a graph with $n \geq 3$ vertices and m edges, then*

$$B_2(G) = \sum_{uv \in E(G)} [d_G(u) + d_G(v)]^2 - 2[d_G(u) + d_G(v)].$$

In this paper, we obtain the expressions for the K-Banhatti indices, K-Hyper Banhatti indices, Zagreb indices, Forgotten index and first Hyper Zagreb index of generalized transformation graphs G^{xy} and of their complements $\overline{G^{xy}}$ in terms of some parameters of a graph.

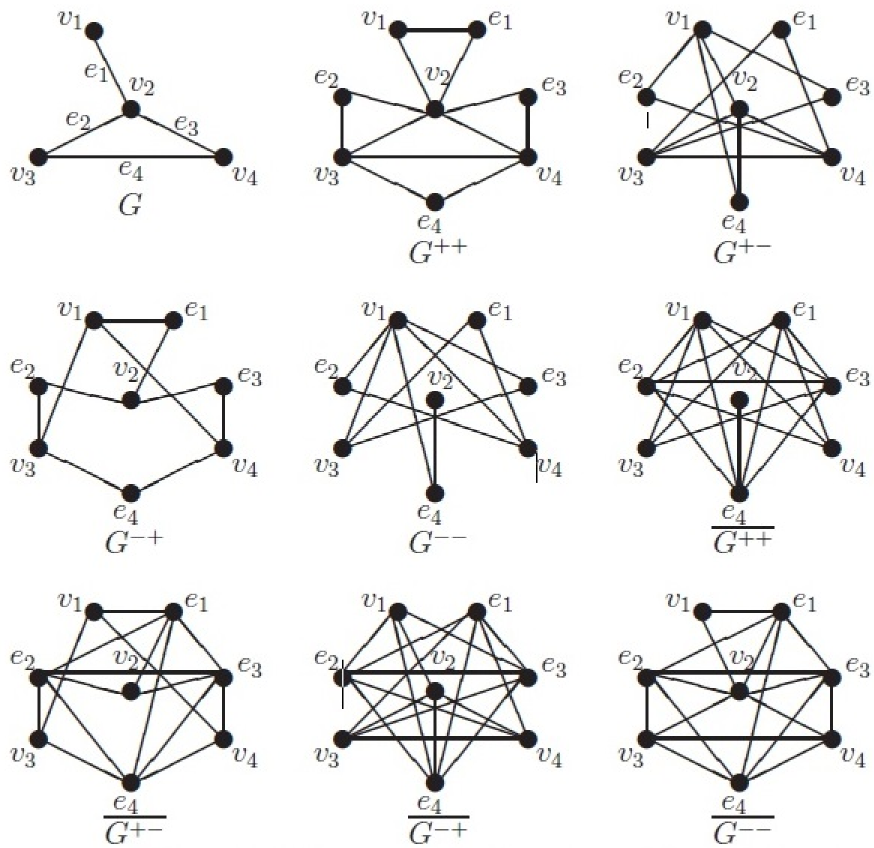

Fig.1 : the Graph G, its generalized transformations G^{xy} and their complements $\overline{G^{xy}}$.

2. The First Banhatti Index of G^{xy}

Theorem 2.1. *Let G be a graph with n vertices and m edges, then*
$B_1(G^{++}) = 6M_1(G) - 4m + \sum_{u \in V(G)} 2d_G(u)[3d_G(u) + 1]$.

Proof. Partition the edge set $E(G^{++})$ into subsets E_1 and E_2.
Where $E_1 = \{uv/uv \in E(G)\}$ and $E_2 = \{ue$ / the vertex u is incident to the egde e in G $\}$. It is easy to check that $|E_1| = m$ and $|E_2| = 2m$.
By Lemma 1.1, if $u \in V(G)$ then $d_G^{++}(u) = 2d_G(u)$ and if $e \in E(G)$ then

$d_G^{++}(e) = 2$. Therefore,

$$\begin{aligned} B_1(G^{++}) &= \sum_{ue} [d_G^{++}(u) + d_G^{++}(e)] \\ &= \sum_{uv \in E(G^{++})} [3d_G^{++}(u) + 3d_G^{++}(v) - 4] \\ &= 6M_1(G) - 4m + \sum_{ue \in E_2} [6d_G(u) + 2]. \end{aligned}$$

In the second part of above eqution, the quantity $[6d_G(u)+2]$ appears $d_G(u)$ times, hence above expression can be written as,
$B_1(G^{++}) = 6M_1(G) - 4m + \sum_{u \in V(G)} 2d_G(u)[3d_G(u) + 1]$. □

Theorem 2.2. *Let G be a graph with n vertices and m edges, then*
$B_1(G^{+-}) = 2m(3m-2) + m(n-2)(3m+3n-10)$.

Proof. Partition the edge set $E(G^{+-})$ into subsets E_1 and E_2.
Where $E_1 = \{uv/uv \in E(G)\}$ and $E_2= \{ue$ / the vertex u is not incident to the edge e in G $\}$. It is easy to check that $|E_1| = m$ and $|E_2| = m(n-2)$.
By Lemma 1.1, if $u \in V(G)$ then $d_G^{+-}(u) = m$ and if $e \in E(G)$ then $d_G^{++}(e) = n-2$. Therefore,

$$\begin{aligned} B_1(G^{+-}) &= \sum_{ue} [d_G^{+-}(u) + d_G^{+-}(e)] \\ &= \sum_{uv \in E(G^{+-})} [3d_G^{+-}(u) + 3d_G^{+-}(v) - 4] \\ &= m(6m-4) + \sum_{ue \in E_2} (3m+3n-10) \\ &= 2m(3m-2) + m(n-2)(3m+3n-10). \end{aligned}$$

□

Theorem 2.3. *Let G be a graph with n vertices and m edges, then*
$B_1(G^{-+}) = n(n-1)(3n-5) + 8m$.

Proof. Partition the edge set $E(G^{-+})$ into subsets E_1 and E_2.
Where $E_1 = \{uv/uv \notin E(G)\}$ and $E_2= \{ue$ / the vertex u is incident to the edge e in G $\}$. It is easy to check that $|E_1| = \begin{pmatrix} n \\ 2 \end{pmatrix} - m$ and $|E_2| = 2m$.
By Lemma 1.1, if $u \in V(G)$ then $d_G^{-+}(u) = n-1$ and if $e \in E(G)$ then $d_G^{-+}(e) = 2$. Therefore,

$$\begin{aligned}
B_1(G^{-+}) &= \sum_{ue}[d_G^{-+}(u) + d_G^{-+}(e)] \\
&= \sum_{uv\in E(G^{-+})} [3d_G^{-+}(u) + 3d_G^{-+}(v) - 4] \\
&= [\binom{n}{2} - m](6m-4) + 2m(3n-1) \\
&= n(n-1)(3n-5) + 8m.
\end{aligned}$$

□

Theorem 2.4. *Let G be a graph with n vertices and m edges, then* $B_1(G^{--}) = \sum_{uv\notin E(G)}(6m+6n-10) - 2\overline{M_1(G)} + \sum_{ue\in E_2}[2n+m-7-2d_G(u)]$.

Proof. Partition the edge set $E(G^{--})$ into subsets E_1 and E_2.
Where $E_1 = \{uv \ / \ uv \notin E(G) \ \}$ and $E_2 = \{ue \ / $ the vertex u is not incident to the edge e in $G \ \}$. It is easy to check that $|E_1| = \binom{n}{2} - m$ and $|E_2| = m(n-2)$. By Lemma 1.1, if $u \in V(G)$ then $d_G^{--}(u) = n+m-1-2d_G(u)$ and if $e \in E(G)$ then $d_G^{--}(e) = n-2$. Therefore,

$$\begin{aligned}
B_1(G^{--}) &= \sum_{ue}[d_G^{--}(u) + d_G^{--}(e)] \\
&= \sum_{uv\in E(G^{--})} [3d_G^{--}(u) + 3d_G^{--}(v) - 4] \\
&= \sum_{uv\notin} 6(n+m) - 10 - 2[d_G(u) + d_G(v)] + \sum_{ue\in E_2} [2n+m-7-2d_G(u)] \\
&= \sum_{uv\notin E(G)} (6n+6m-10) - 2\overline{M_1(G)} + \sum_{ue\in E_2} [2n+m-7-2d_G(u)]
\end{aligned}$$

.

□

Note 1 : In Theorems 2.2 and 2.3, graphs G having same number of vertices and edges, then $B_1(G^{+-})$ and $B_1(G^{-+})$ are same.

3. The First Banhatti Index of $\overline{G^{xy}}$

Theorem 3.1. *Let G be a graph with n vertices and m edges, then* $B_1(\overline{G^{++}}) = \sum_{uv\notin E(G)} 2(3n+3m-5) - 2\overline{M_1}(G) + \sum_{u\in V(G)} 2[m-d_G(u)][3n+3m-7-d_G(u)] + m(m-1)(3n+3m-11)$.

Proof. Partition the edge set $E(\overline{G^{++}})$ into subsets E_1, E_2 and E_3. Where $E_1 = \{uv \ / \ uv \notin E(G)\}$, $E_2 = \{ue \ /$ the vertex u is not incident to the edge e in G $\}$ and $E_3 = \{ef \ / \ e, f \in E(G) \ \}$. It is easy to check that $|E_1| = \binom{n}{2} - m$, $|E_2| = m(n-2)$ and $|E_3| = \binom{m}{2}$. By Lemma 1.2, If $u \in V(G)$ then $d_{\overline{G^{++}}}(u) = n+m-1-2d_G(u)$

and If $e \in E(G)$ then $d_{\overline{G^{++}}}(e) = n + m - 3$.
Therefore,

$$\begin{aligned}
B_1(\overline{G^{++}}) &= \sum_{ue}[d_{\overline{G^{++}}}(u) + d_{\overline{G^{++}}}(e)] \\
&= \sum_{uv \in E(\overline{G^{++}})} [3d_{\overline{G^{++}}}(u) + 3d_{\overline{G^{++}}}(v) - 4] \\
&= \sum_{uv \notin E(G)} (6n + 6m - 10) - 2\overline{M_1}(G) + \sum_{ue \in E_2} [6n + 6m - 14 - 2d_G(u)] \\
&+ \sum_{ef \in E_3} [6n + 6m - 22] \\
&= \sum_{uv \notin E(G)} 3(3n + 3m - 5) - 2\overline{M_1}(G) + \sum_{u \in V(G)} 2[m - d_G(u)][3n + 3m - 7 - d_G(u)] \\
&+ m(m-1)(3n + 3m - 11).
\end{aligned}$$

□

Theorem 3.2. *Let G be a graph with n vertices and m edges, then*
$B_1(\overline{G^{+-}})(u) = n(n-1)(3n-5) - 2m(3n-5) + 2m(3n+3m-4) + m(m-1)(3m+1)$.

Proof. Partition the edge set $E(\overline{G^{+-}})$ into subsets E_1, E_2 and E_3 Where E_1 ={uv /$uv \notin E(G)$}, E_2 ={ ue / the vertex u is incident to the edge e in G } and E_3 ={ef / $ef \in E(G)$ }.
It is easy to check that $|E_1| = \binom{n}{2} - m$, $|E_2| = 2m$ and $|E_3| = \binom{m}{2}$ By Lemma 1.2, If $u \in V(G)$ then $d_{\overline{G^{+-}}}(u) = n - 1$ and If $e \in E(G)$ then $d_{\overline{G^{+-}}}(e) = m + 1$.
Therefore,

$$\begin{aligned}
B_1(\overline{G^{+-}}) &= \sum_{ue}[d_{\overline{G^{+-}}}(u) + d_{\overline{G^{+-}}}(e)] \\
&= \sum_{uv \in E(\overline{G^{+-}})} [3d_{\overline{G^{+-}}}(u) + 3d_{\overline{G^{+-}}}(v) - 4] \\
&= [\binom{n}{2} - m] + 2m(3n + 3m - 4) + \binom{m}{2} (6m + 2) \\
&= n(n-1)(3n-5) - 2m(3n-5) + 2m(3n + 3m - 4) + m(m-1)(3m+1).
\end{aligned}$$

□

Theorem 3.3. *Let G be a graph with n vertices and m edges, then*
$B_1(\overline{G^{-+}}) = m(6m-4) + m(n-2)(n+2m-7) + m(m-1)(3n+3m-11)$.

Proof. Partition the edge set $E(\overline{G^{+-}})$ into subsets E_1, E_2 and E_3. Where E_1 ={uv /$uv \in E(G)$ }, E_2={ue / the vertex u is not incident to the edge e in G } and E_3 ={ef /$ef \in E(G)$}.
It is easy to check that $|E_1| = m$, $|E_2| = (n-2)m$ and $|E_3| = \binom{m}{2}$. By Lemma 1.2, If $u \in V(G)$ then $d_{\overline{G^{-+}}}(u) = m$ and If $e \in E(G)$ then $d_{\overline{G^{-+}}}(e) = n + m - 3$.

Therefore,

$$
\begin{aligned}
B_1(\overline{G^{-+}}) &= \sum_{ue}[d_{\overline{G^{-+}}}(u) + d_{\overline{G^{-+}}}(e)] \\
&= \sum_{uv\in E(\overline{G^{-+}})} [3d_{\overline{G^{-+}}}(u) + 3d_{\overline{G^{-+}}}(v) - 4] \\
&= \sum_{uv\in E(G)} (6m-4) + \sum_{ue\in E_2} (n+2m-7) + \sum_{ef\in E_3} (6n+6m-22) \\
&= m(6m-4) + m(n-2)(n+2m-7) + m(m-1)(3n+3m-11).
\end{aligned}
$$

□

Theorem 3.4. *Let G be a graph with n vertices and m edges, then* $B_1(\overline{G^{--}}) = 6M_1(G) - 4m + \sum_{u\in E(G)}[6d_G(u) + 3m - 1] + m(m-1)(3m+1)$.

Proof. Partition the edge set $E(\overline{G^{--}})$ into subsets E_1, E_2 and E_3. Where $E_1 = \{uv\ /uv \in E(G)\}$, $E_2 = \{ue\ /$ the vertex u is incident to the edge e in $G\ \}$ and $E_3 = \{ef\ /ef \in E(G)\}$. It is easy to check that $|E_1| = m$, $|E_2| = 2m$ and $|E_3| = \begin{pmatrix} m \\ 2 \end{pmatrix}$. By Lemma 1.2, If $u \in V(G)$ then $d_{\overline{G^{--}}}(u) = 2d_G(u)$ and If $e \in E(G)$ then $d_{\overline{G^{--}}}(e) = m+1$. Therefore,

$$
\begin{aligned}
B_1(\overline{G^{--}}) &= \sum_{ue}[d_{\overline{G^{--}}}(u) + d_{\overline{G^{--}}}(e)] \\
&= \sum_{uv\in E(\overline{G^{--}})} [3d_{\overline{G^{--}}}(u) + 3d_{\overline{G^{--}}}(v) - 4] \\
&= 6\sum_{uv\in E(G)} [d_G(u) + d_G(v)] - 4\sum_{uv\in E(G)} + \sum_{ue\in E_2} [6d_G(u) + 3m - 1] + m(m-1)(3m+1) \\
&= 6M_1(G) - 4m + \sum_{u\in E(G)} [6d_G(u) + 3m - 1] + m(m-1)(3m+1).
\end{aligned}
$$

□

Note 2: In Theorems 3.2 and 3.3, graphs G having same number of vertices and edges, then $B_1(\overline{G^{+-}})$ and $B_1(\overline{G^{-+}})$ are same.

4. The Second Banhatti Index, K Hyper Banhatti Indices, Forgotten Index and First Hyper-Zagreb Index of G^{xy} and $\overline{G^{xy}}$

In this section, we present some results without proof on G^{xy} and its complement related to some indices. Because the proof technique adopted here similar to the previous theorems.

Theorem 4.1. *Let G be a graph with n vertices and m edges, then*

$$i) B_2(G^{++}) = 4HM_1(G) - 4M_1(G) + 4\sum_{u\in V(G)} d_G(u)^2[d_G(u)+1].$$

$$ii) B_2(G^{+-}) = 4m^2(m-1) + m(m-2)(m+n-2)(m+n-4).$$

$$iii) B_2(G^{-+}) = 2n(n-1)^2(n-2) - 4m(n-1)(n-2) + 2m(n-1)(n+1).$$

$$iv) B_2(G^{--}) = \sum_{uv\notin E(G)} 4[n+m-1-d_G(u)-d_G(v)][n+m-3-d_G(u)-d_G(v)] + \sum_{u\in V(G)} [m-d_G(u)][2n+m-3-2d_G(u)][2n+m-5-2d_G(u)].$$

$$v) B_2(\overline{G^{++}}) = \sum_{uv\notin E(G)} 4[n+m-1-d_(u)-d_G(v)][n+m-2-d_G(u)-d_G(v)] + \sum_{u\in V(G)} 4[m-d_G(u)][n+m-2-d_G(u)][n+m-3-d_G(u)] + 2m(m-1)(n+m-4).$$

$$vi) B_2(\overline{G^{+-}}) = 2n(n-1)^2(n-2) - 4m(n-1)(n-2) + 2m(n+m)(n+m-2) + 2m^2(m-1)(m+1).$$

$$vii) B_2(\overline{G^{-+}}) = 4m^2(m-1) + m(n-2)(n+2m-3)(n+2m-5) + 2m(m-1)(n+m-3)(n+m-4).$$

$$viii) B_2(\overline{G^{--}}) = 4HM_1(G) - 4M_1(G) + \sum_{u\in V(G)} d_G(u)[2d_G(u)+m+1][2d_G(u)+m-1] + 2m^2(m-1)(m+1).$$

Theorem 4.2. *Let G be a graph with n vertices and m edges, then*

$$i) HB_1(G^{++}) = m[6d_G(u)+6d_G(v)-4]^2 + 8m[3d_G(u)+1]^2.$$

$$ii) HB_1(G^{+-}) = 4m(3m-2)^2 + m(n-2)(3m+3n-10)^2.$$

$$iii) HB_1(G^{-+}) = 2n(n-1)(3n-5)^2 - 4m(3n-5)^2 + 2m(3n-1)^2.$$

$$iv) HB_1(G^{--}) = \sum_{uv\notin E(G)} [6(n+m-d_G(u)-d_G(v))-10]^2 + [m-d_G(u)][6(n+m-d_G(u)-d_G(v)-10]^2.$$

$$v) HB_2(G^{++}) = \sum_{uv\in E(G)} 64d_G(u)^2[2d_G(u)-1]^2 + \sum_{ue\in E_2} 4d_G(u)^2[2d_G(u)+2]^2.$$

$$vi) HB_2(G^{+-}) = 16m^3(m-1)^2 + m(n-2)(m+n-2)^2(m+n-4)^2.$$

$$vii) HB_2(G^{-+}) = 8n(n-1)^3(n-2)^2 - 16m(n-1)^2(n-2)^2 + 2m(n-1)^2(n+1)^2.$$

$$viii) HB_2(G^{--}) = \sum_{uv\notin E(G)} 16[n+m-1-d_G(u)-d_G(v)]^2[n+m-2-d_G(u)-d_G(v)]^2 + m[n-2][2n+m-3-2d_G(u)]^2[2n+m-5-2d_G(u)]^2.$$

Theorem 4.3. *Let G be a graph with n vertices and m edges, then*

$$i)HB_1(\overline{G^{++}}) = \sum_{uv\notin E(G)} \{6[n+m-1-d_G(u)-d_G(v)]-4\}^2 + \sum_{u\in V(G)} [m-d_G(u)][6(n+m-1-d_G(u))-4]^2 + m(m-1)(3n+3m-11)^2.$$

$$ii)HB_1(\overline{G^{+-}}) = 2n(n-1)(3n-5)^2 - 4m(3n-5)^2 + 2m(3n+3m-4)^2 + 2m(m-1)(3m+1)^2.$$

$$iii)HB_1(\overline{G^{-+}}) = m(6m-4)^2 + m(m-2)(3n+6m-13)^2 + m(m-1)(3n+3m-11)^2.$$

$$iv)HB_1(\overline{G^{--}}) = \sum_{uv\in E(G)} [6d_G(u)+6d_G(v)-4]^2 + \sum_{ue\in E_2} [6d_G(u)+3m-1]^2 + 2m(m-1)(3m+1)^2.$$

$$v)HB_2(\overline{G^{++}}) = \sum_{uv\notin} 16[n+m-1-d_G(u)-d_G(v)]^2[n+m-2-d_G(u)-d_G(v)]^2 + \sum_{u\in V(G)} [m-d_G(u)][2n+2m-4-2d_G(u)]^2[2n+2m-6-2d_G(u)]^2 + 8m(m-1)(n+m-3)^2(n+m-7).$$

$$vi)HB_2(\overline{G^{+-}}) = \sum_{uv\notin E(G)} 16(n-1)^2(n-2)^2 + 2m(n+m)^2(n+m-2)^2 + 8m(m-1)(m+1)^2(m-3)^2.$$

$$vii)HB_2(\overline{G^{-+}}) = 16m^3(m-1)^2 + m(n-2)(n+2m-3)^2(n+2m-5)^2 + 8m(m-1)(n+m-3)^2(n+m-7)^2.$$

$$viii)HB_2(\overline{G^{--}}) = \sum_{uv\notin E(G)} 16[d_G(u)+d_G(v)]^2[d_G(u)+d_G(v)-1]^2 + \sum_{ue\in E_2} [2d_G(u)+m+1]^2[2d_G(u)+m-1]^2 + 8m^3(m-1)(m+1)^2.$$

Theorem 4.4. *Let G be a graph with n vertices and m edges, then*

$$i)F(G^{++}) = 4F(G) + 4\sum_{ue\in E_2} [d_G(u)^2+1].$$

$$ii)F(G^{+-}) = 2m^3 + m(n-2)[m^2+(n-2)^2]$$

$$iii)F(G^{-+}) = n(n-1)^3 + 8m.$$

$$iv)F(G^{--}) = \sum_{uv\notin E(G)} 2[n+m-1-2d_G(u)]^2 + \sum_{ue\in E_2} \{[n+m-1-2d_G(u)]^2 + (n-2)^2\}$$

$$v)HM_1(G^{++}) = 4HM_1(G) + 4\sum_{ue\in E_2}[d_G(u)+1]^2.$$
$$vi)HM_1(G^{+-}) = 4m^3 + m(n-2)(m+n-2)^2.$$
$$vii)HM_1(G^{-+}) = 2n(n-1)^3 - 4m(n-1)^2 + 2m(n+1)^2.$$
$$viii)HM_1(G^{--}) = \sum_{uv\notin} 4[n+m-1-2d_G(u)]^2 + \sum_{ue\in E_2}[2n+m-3-2d_G(u)]^2.$$

Theorem 4.5. *Let G be a graph with n vertices and m edges, then*

$$\begin{aligned} i)F(\overline{G^{++}}) &= \sum_{uv\notin E(G)}\{[n+m-1-2d_G(u)]^2+[n+m-1-2d_G(v)]^2\} \\ &+ \sum_{u\in V(G)}[m-d_G(u)]\{[n+m-1-2d_G(u)]^2+[n+m-3]^2\} \\ &+ m(m-1)(m+n-3)^2. \end{aligned}$$
$$ii)F(\overline{G^{+-}}) = n(n-1)^3 - 2m(n-1)^2 + 2m[(n-1)^2+(m+1)^2] + m(m-1)(m+1)^2.$$
$$iii)F(\overline{G^{-+}}) = 2m^3 + m(n-2)[m^2+(n+m-3)^2] + m(m-1)(n+m-3)^2.$$
$$iv)F(\overline{G^{--}}) = 4F(G) + \sum_{u\in V(G)} 2d_G(u)[4d_G(u)^2+(m+1)^2] + m(m-1)(m+1)^2.$$
$$\begin{aligned} v)HM_1(\overline{G^{++}}) &= 4\sum_{uv\notin E(G)}[n+m-1-d_G(u)-d_G(v)]^2 + \\ &4\sum_{u\in V(G)}[m-d_G(u)][n+m-2-d_G(u)]^2 + 2m(m-1)(n+m-3)^2. \end{aligned}$$
$$vi)HM_1(\overline{G^{+-}}) = 2n(n-1)^3 - 4m(n-1)^2 + 2m(n+m)^2 + 2m(m-1)(m+1)^2.$$
$$vii)HM_1(\overline{G^{-+}}) = 4m^3 + m(n-2)(n+2m-3)^2 + 2m(m-1)(n+m-3)^2.$$
$$viii)HM_1(\overline{G^{--}}) = 4HM_1(G) + \sum_{u\in V(G)} 2d_G(u)[2d_G(u)+m+1]^2 + 2m(m-1)(m+1)^2.$$

Acknowledgements. The authors thankful to refrees for constructive comments to improve the manuscript.

Conflicts of interest: The authors have no conflicts of interest.

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