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SARIMA MODELING OF CONSUMER PRICE INDEX OF KUWAIT

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ABSTRACT

The purpose of this study is to identify and estimates a statistical model for the consumer price Index (CPI) in Kuwait. Modeling is based on the autoregression of the previous values of the CPI series, and the moving average of it's error terms. Box-Jenkins techniques are used to identify, estimate, and diagnosticaly check the best SARIMA model for CPI, before it is finalized and used for forecasting future values.

Keywords: Consumer Price Index, Box-Jenkins Methodology, Seaconal Autoregressive Integrated Moving Average.

1. INTRODUCTION

Consumer Price Index is an important economic indicator. It is used in social and economic planning, and it affects the standard of living. It is also used as a mean of measuring and adjusting real values of economic variables such as: income, wages, cost of living, and real value of nominal money. Furthermore, CPI is very important for all studies concerning prices, and price trends.

CPI affects a number of economic variables, and at the same it is influenced by economic variables, some of which are international, such as imported inflations.

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There exist many techniques and approaches for formulating forecasting models, all of which could be grouped into two basic kinds: causal models and univariate models.

The use of the causal models requires the identification of other variables that are related to the concerned variable to be predicted. Once these variables have been identified, a suitable statistical model that describes the relationship between them and the variable to be forecasted must be developed. The derived relationship is then statistically estimated, and then used to forecast the variable of interest (i.e. the dependent variable in the model).

In the business world causal models are very common. However, they have several disadvantages, firt they are difficult to develop. Also, they require the data for all related variables included in the model. Beside that, the ability of the estimated model to predict values of the dependent variable depends mainly on the accuracy of predicting the future values for the independent variables.

Univariate models are also common, such a model predicts future values of one variable mainly on the basis of the information available from the past values of the same variable. When such a model is used, historical data are analyzed to identify a pattern in the data. Then this pattern is extrapolated in order to produce forecast.

The Seasonal Autoregressive Integrated Moving Average models described by Box and Jenkins (1976) are examples of the univariate or time series models. Although time series models have been studied for many years, Box and Jenkins have popularized their use.

The Box-Jenkins technique has many appealing features. It allows someone who only has data on past years of one variable to forecast future values of that variable without having to search for other explanatory variables. Beside, literature shows that this technique has been proven to be very powerful and reliable especially for short term forecasting.

The Box-Jenkins technique has been used in many studies. The following are examples of these studies; Thompson and Tio (1971) in their early case study analyzed telephone data with a seasonal pattern using Box-Jenkins technique. Chatfield and Prothero (1973) used the Box-Jenkins technique to study the monthly sales figure of an engineering product. Geurts and Ibrahim (1975) used the technique to study the number of tourists visiting Hawaii. Ledolter (1976) applied the technique in meteorology to model hydrologic sequences. Leskinen and Terasvirta (1976) applied Box-Jenkins approach to study and forecast the consumption of alcoholic beverages in Finland. Helmer and Johansson (1977) used this technique to a marketing case of advertising-sales relationship. Dunstan (1982) applied it in the detection of breast cancer. Hillmer (1982) used this forecasting technique with trading day variation. Harvey and Todd (1983) used a case study to forecast economic time series with structural and Box-Jenkins models.

The CPI has not been studies for Kuwait for modeling or forecasting purpose. To studies the behavior pattern of the CPI series and use it for forecasting the future values of the series, The author has applied the Box-Jenkins technique because of it's feature and power in short term prediction.

This study is divided into four sections and one appendix, in section 2 the data for the CPI series is presented, along with the plot and table of annual changes in the CPI series. Section covers the methodology, in which the different stages of the Box-Jenkins technique applied to the CPI series is shown. Starting with the identification stage in section 3.1, the estimation stage in 3.2, the paper presents diagnostic checking for the possible models in section 3.3, cross validation for best models in section 3,4 and the final model and forecasting results in section 3.5. Section 4 contains the results and conclusions. The appendix contains tables and graphs of the results of the different stages.

2. DATA

The Publication, Research and Training Department of the Central Statistical Office in the Ministry of Planning in Kuwait publishes the Consumer Price index numbers on a monthly bases. Table 1 shows the CPI time series from January 1979 to April 1988. Details on the scope and methodof construction of the CPI are available in a separate publication on "consumer Price Index Numbers, scope and method of construction" issued by the Central Statistical Office of the Ministry of Planning.

CPI has increased largely and was affected by the oil price boom from the end of 1979 to 1982. It was also affected by the "Almanak" (stock market) crisis in August of 1982, and by the drop in prices of crude oil in 1986 and 1987. This is clearly visible from figure (1) of the CPI time series, and in table (2) which presents the annual changes in CPI from 1979 to 1988.

	1979	1980	1981	1982	1983
Jan	106.8	111.1	119.7	127.4	137.4
Feb	105.9	111.5	120.1	128.0	138.0
Mar	105.0	111.7	120.5	128.4	138.8
Apr	105.7	113.1	123.5	132.0	139.7
May	106.5	113.8	122.9	131.6	137.0
Jun	106.9	114.0	121.8	132.9	138.5
Jul	106.9	114.5	122.3	133.8	138.4
Aug	107.6	113.9	122.9	133.5	139.0
Sep	108.1	116.4	124.3	134.3	139.6
Oct	108.6	118.2	125.2	135.1	139.5
Nov	107.8	117.1	125.2	135.4	138.8
Dec	108.8	118.2	126.5	137.2	139.9
	1984	1985	1986	1987	1988
Jan	140.2	141.8	143.7	145.3	145.2
Feb	139.1	142.2	144.0	144.1	145.1
Mar	139.8	142.8	143.8	144.3	146.2
Apr	140.7	143.3	143.8	144.6	147.7
May	139.2	141.9	143.1	144.8	
Jun	141.2	142.1	144.3	145.6	
Jul	140.3	141.6	143.6	145.0	
Aug	140.3	141.7	143.6	145.4	
Sep	141.1	143.4	144.9	145.5	
Oct	139.8	142.9	144.0	143.8	
Nov	140.5	142.3	142.8	143.4	
Dec	142.0	143.3	144.0	145.0	

Table (1) monthly consumer price index series 1979-1988 (base year 1978).

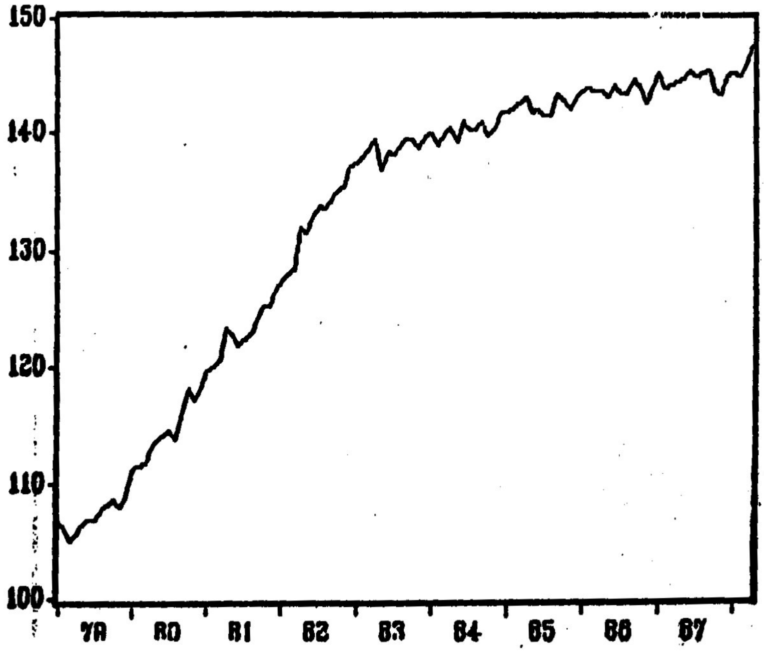

Figure (1) monthly CPI series 1979-1988.

Year	Annual change
1979	2.0
1980	7.1
1981	6.8
1982	9.8
1983	2.5
1984	1.8
1985	1.5
1986	0.3
1987	-0.3
1988	2.5

Table (2) Annual Change in CPI 1979 - 1987

3. METHODOLOGY

The estimation process of a Seasonal Autoregressive Integrated Moving Average (SARIMA) models using the Box-Jenkins technique follows a sequence of logical steps, it starts with an identification approach for the proper SARIMA models that could appropriately used to fit the concerned series of study. The second step is to estimate these models using either the least squares or the maximum likelihood method. Once these models have been estimated, it is then necessary to choose the best one (or ones) among them and to verify whether we can improve upon it or not. This is the third stage of the technique known as diagnostic checking. The final stage comes when a fitted model has been judged as adequately representing the process governing the series, which could be used then, to forecast values of the future periods for the series.

3.1. Identification

The approach of The Box-Jenkins technique makes use of three linear filters, namely the autoregressive, the integration, and the moving average filters. The identification stage of the technique provides a unified approach for identifying which filters are most likely to be appropriate for the series understudy. The first step in nay time eseries analysis should be to plot the series against time. Figure (1) of the CPI series clearly shows that the series does not fluctuate around a constant mean, i,e that the CPI is a non-stationary series.

Stationarity condition is one of the requirement of the time series analysis, and could be achieved by determining the appropriate degree of differencing the original series. But before that one should decide whether to work with the original series or some proper transformation of it; that would achieve that the variance of the process be constant over time, which itself is another condition for stationarity. Table (3) and figure (2) do not show a clear relative trend in the yearly standard deviations versus means which indicates that the data do not need to be transformed even though we will investigate the effect of using the inverse transformation of the CPI series.

Working with the original data, it becomes clear that the CPI series needs some sort of differencing transformation to remove the trend from the data. Figure (3) and table (4) present the graphs and the table of standard deviations of the original CPI series, along with different seasonal and non-seasonal differencing (Dij) series created from CPI, where i is the degree of non-seasonal differencing, and j is the degree of the seasonal one. From figure (3) one can see that the trend in the data is removed by introducting one or two seasonal and non-seasonal differencing, at the most, for the CPI series. But seasonal differencing alone would not be adequate. Also, differencing of the first, or at the most, the second order gave smaller standard deviations than those of the higher ones as table (4) shows.

Year	Mean	Standard Deviation
1979	107.05	1.1251
1980	114.46	2.4012
1981	122.91	2.0670
1982	132.47	2.9926
1983	138.72	0.8754
1984	140.35	0.7995
1985	142.44	0.6383
1986	143.80	0.5132
1987	144.73	0.6774
1988	146.05	1.0452

Table (3) yearly means and standard deviations of CPI.

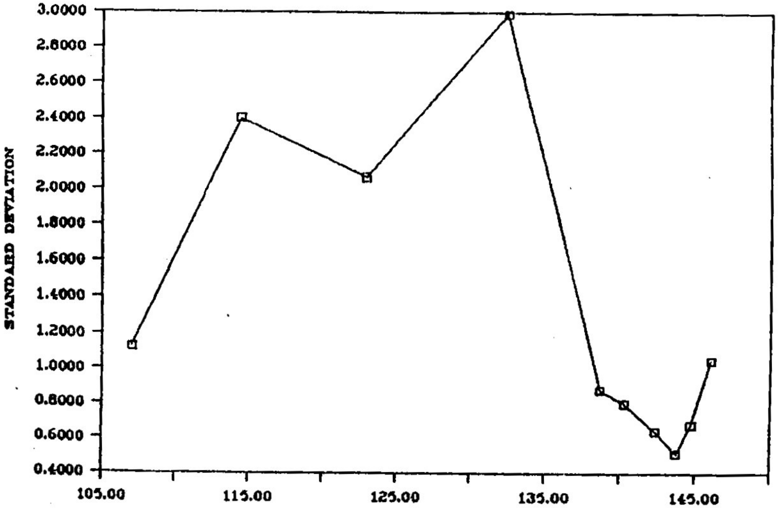

Figure (2) yearly means and standard deviations of CPI.

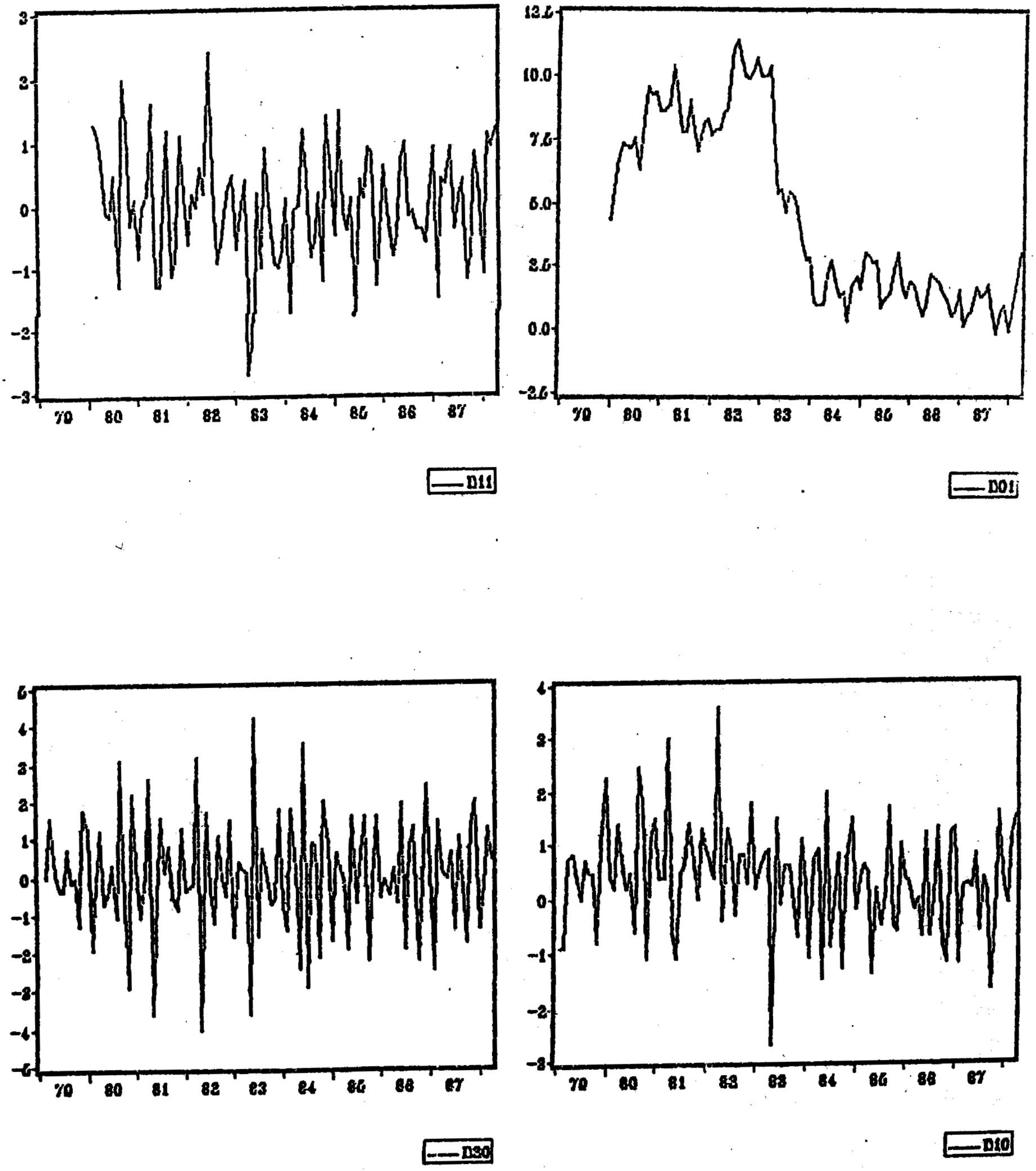

Figure (3) Plot of D10, D20, D01 and D11 series versus time.

Therefore, the concentration would be given on the four case: (1,0), (1,1), (1,2), and (2,1), where the first number refers to the degree of non-seasonal and the second to the seasonal differencing.

seasonal difference	regular difference			
	0	1	2	3
0	13.2579	0.9938	1.5192	2.6427
1	3.5789	0.9167	1.2643	2.1238
2	2.9461	1.4713	1.9951	3.2953
3	4.9252	2.6515	3.4486	5.5824

Table (4) Standard deviations of the original CPI series and seasonal and non-seasonal differencing up to the third order.

Box-Jenkins forecasting models are tentatively idenfitifed by examining the pattern of the autocorrelation function (AC), and the partial autocorrelation function (PAC) for the sampled values of a stationary time series Z_k; K=c, c+1, c+2,...,n. Here Z_k may be the original time series or some transformed series values, and c is the order of the first value in this series.

Tables (A1) to (A5) in the appendix present the values and plots of the AC and PAC functions for the original CPI series and differenced series D10, D11, D21, and D12. The AC function of the CPI series confirm that it is not stationary. Table (A3) of the AC and PAC function of the D11 series does not reveal any reasonable good suggestion for a SARIMA that would fit the series.

The autocorrelation and partial autocorrelation functions of the series D10, D21, and D21 in tables (A2), (A4), and (A5) reveal the possibility of the following four models:

Model 1

If we define:

$$D10 = Z_t,$$

then

$$Z_t = (1-B)\ CPI_t,$$

where:

B is the first backshift operator that shifts the subcript of a time series observation backward in time by one period.

That is,

$$Z_t = CPI_t - CPI_{t-1}.$$

Then a possible SARIMA model for D10 is:

$$(1 - a_1\ B - a_2\ B^2)\ (1 - a_{1,12}\ B^{12})\ Z_t = e_t.$$

Where:

a_1 and a_2 are the first and the second ordinary AR parameters.

B^2 is the second backshift operator that shifts the subscript of a time series observation backward in time by two periods.

B^{12} is the first seasonal backshift operator that shifts the subscript of a time series observation in time by 12 periods. i.e one seasonal backshift.

and,

e_t is the error term for period t.

Model 2

If we define:

$$D10 = Z_t,$$

and

$$Z_t = (1 - B)\ CPI_t.$$

Then another possible SARIMA model for the same series D10 is:

$$(1 - a_{1,12}\ B^{12})\ Z_t = (1 - b_{1,12}\ B^{12})\ e_t.$$

Where:

$a_{1,12}$ is the first seasonal AR parameter,

$b_{1,12}$ is the first seasonal MA parameter.

Model 3

If we define:

$$D12 = Z_t,$$

then

$$\begin{aligned} Z_t &= (1 - B)(1 - B^{12})^2\ CPI_t \\ &= (1 - B)(1 - 2\ B^{12} + B^{24})\ CPI_t \\ &= (1 - B - 2\ B^{12} + 2\ B^{13} + B^{24} - B^{25})\ CPI_t \end{aligned}$$

i.e.

$$Z_t = (CPI_t - CPI_{t-1}) - 2\,(CPI_{t-12} - CPI_{t-13}) + (CPI_{t-24} - CPI_{t-25}).$$

The possible model for D12 series is:

$$Z_t = (1 - b_{1,12}\,B^{12} - b_{2,12}\,B^{24})\,e_t.$$

where:

$b_{2,12}$ is the second seasonal MA parameter.

and

B^{24} is the second seasonal backshift operator that shifts the subscript of a time series observation in time by 24 periods, i.e two seasonal backshifts.

Model 4

If we define:

$$D21 = Z_t,$$

then

$$Z_t = (1 - B)^2\,(1 - B^{12})\,CPI_t$$
$$= (1 - 2B + B^2)\,(1 - B^{12})\,CPI_t$$
$$= (1 - 2B + B^2 - B^{12} + 2\,B^{13} - B^{14})\,CPI_t$$

i.e.

$$Z_t = (CPI_t - CPI_{t-12}) - 2\,(CPI_{t-1} - CPI_{t-13}) + (CPI_{t-2} - CPI_{t-14}).$$

The possible model for D21 series is:

$$Z_t = (1 - b_1\,B)\,(1 - b_{1,12}\,B^{12} - b_{2,12}\,B^{24} - b_{3,12}\,B^{36})\,e_t.$$

3.2. Estimation

The above four models are estimated using the non-linear least squares method using the SPSS-X statistical package. Tables (A6) to (A9) in the appendix summarize the results of these estimations for model (1) through model (4). These four models estimates are as follows:

Model 1

The initial estimate for model (1) indicated the insignificance of the first AR parameter. The modified version for model (1) estimate is:

$$(1 + 0.16094\ B^{2})\ (1 - 0.70681\ B^{12})\ Z_t = e_t .$$

Model 2

$$(1 - 0.875\ B^{12})\ Z_t = (1 - 0.37344\ B^{12})\ e_t .$$

Model 3

$$Z_t = (1 - 1.0313\ B^{12} + 0.175\ B^{24})\ e_t .$$

Model 4

$$Z_t = (1 - 0.9172\ B)\ (1 - 0.5891\ B^{12} - 0.0265\ B^{24} - 0.0781\ B^{36})\ e_t .$$

For the above models most of the estimated parameters are highly significant except the 2nd and 3rd seasonal MA parameters in model (4). But elimination of these two parameters will harm this model by substantially increasing the standard deviation of the residual series.

Model (1) has the lowest standard deviation among the four estimated models, and model (2) is next best alternative.

3.3. Diagnostic Checking

A good way to choose the best SARIMA model is to check the overall adequacy of the estimated models throught the analysis of the residuals obtained from each.

Tables (A10) through (A13) present the results of the overall adequacy of each of the estimated models, applying The Box-Ljung statistics for lag 6, 12, 18, 24, 30, and 36. The greater is the observed prob-value (i.e. the observed significant level), the more we believe in the adequacy of the model.

Tables (A10) through (A13) also display the residual AC functions for the stimated models, which could be utilized to make necessary improvements for an overall non-rejected model.

It is clear from tables (A12) and (A13) that the observed significant levels (OSL) of the Box-Ljung Chi-Square statistics for model (3) and model (4) are very low for all different values of lags, compared to the OSL values for the first two models. Further that, some of the OSL values for the last two models are not sifnificant at 0.1 or 0.05 level. This indicates that models (3) and (4) are inadequare. Hence, they are rejected.

The individual sample autocorrelations of the residuals for model (1) and (2) are all within the tolerance limits, which does not indicate any necessary improvements on any of these models.

We have to choose a final SARIMA model to best fit the CPI series between these two qualified models.

3.4. Cross Validation

One way to choose between good estimated models is to observe their behavior and stability when they are re-estimated using part of the data of the concerned series, while comparing their forecasted values to the original ones in the second part of the same series.

Using the observed values of the CPI series from January 1979 to December 1986 as the first part of the series to re-estimate model (2), and the modified version of model (1). The following estimates are obtained:

Model 1

$$(1 + 0.14531\ B^2)\ (1 - 0.72187\ B^{12})\ Z_t = e_t.$$

Model 2

$$(1 - 0.86419\ B^{12})\ Z_t = (1 - 0.31719\ B^{12})\ e_t.$$

Comparing these estimates to the estimates using the full sample of the CPI series, we observe that the two models perform well, and the new estimates for the parameters in the two model were very close to the original estimates using the full sample. Even though, the r-estimated parameters for the first model were closer.

Tables (A14) and (A15) in the appendix present the original values for the CPI series and the forecasted ones for model (1) and (2) for twelve periods starting from January 1987, along with the 95 per cent confidence limits, standard errors, and plots for both models.

Both of the forecasted values of models (1) and (2) are very close to the real values of the CPI series. But the standard errors of the leading forecasted values for the first model are better than the second model. Hence, the lenght of the confidence intervals for these leading forecasted values are shorter for the first model, which makes it it better choice.

3.5. Final Model and Forecasting

The final and best Box-Jenkins SARIMA model for modeling and forecasting the consumer price index series is the above modified model (1). The estimated model for that using the full data of tne CPI series is:

$$(1 + 0.16094\ B^2)\ (1 - 0.70681\ B^{12})\ Z_t = e_t$$

i.e.

$$(1 + 0.16094\ B^2)\ (1 - 0.70681\ B^{12})\ (1 - B)\ CPI_t = e_t$$

hence,

$$(1+0.16094\ B^2-0.70681\ B^{12}+0.113754\ B^{14})(1-B)CPI_t = e_t.$$

The final model could then be written as:

$$\begin{aligned} CPI_t = CPI_{t-1} &+ 0.16094\ (CPI_{t-2} - CPI_{t-3}) \\ &- 0.70681\ (CPI_{t-12} - CPI_{t-13}) \\ &+ 0.113754\ (CPI_{t-14} - CPI_{t-15}) + e_t. \end{aligned}$$

Note that the portion of this model determined by the autoregressive and differencing operators says that the basic nature of the forecast of the CPI equals

1. CPI_{t-1}, the value of the CPI time series for the previous month, plus
2. $+ 0.16094\ (CPI_{t-2} - CPI_{t-3})$
   $- 0.70681\ (CPI_{t-12} - CPI_{t-13})$
   $+ 0,113754(CPI_{t-14} - CPI_{t-15})$

which is a linear combination of the stochastic month to month difference variation component for the same year ($CPI_{t-2} - CPI_{t-3}$), and for the previous year ($CPI_{t-12} - CPI_{t-13}$), and ($CPI_{t-14} - CPI_{t-15}$).

This model can be utilized to predict the future balues of the CPI series using the previous values of the same series, or their previously predicted ones.

Table (5) presents the forecasted values for the CPI from May 1988 to December 1989.

OBS. NUM.	YEAR	MONTH	FORECASTED CPI
123	1988	May	147.69
124	1988	Jun	148.05
125	1988	Jul	147.65
126	1988	Aug	147.96
127	1988	Sep	148.03
128	1988	Oct	146.82
129	1988	Nov	146.54
130	1988	Dec	147.67
131	1989	Jan	147.81
132	1989	Feb	147.74
133	1989	Mar	148.52
134	1989	Apr	149.58
135	1989	May	149.57
136	1989	Jun	149.82
137	1989	Jul	149.54
138	1989	Aug	149.76
139	1989	Sep	149.81
140	1989	Oct	148.96
141	1989	Nov	148.76
142	1989	Dec	149.56

Table (5) Predicted values of CPI.

4. RESULTS AND CONCLUSIONS

This study has examined the behavior of the consumer price index series in Kuwait, from the period starting from January 1979 to April 1988 to identifies and estimates a univariate time series model for the CPI, to be used in forecasting the future values of the series. Different stages of Box-Jenkins approach were used to arrive at a best estimated seasonal autoregressive integrated moving average model.

In the identification stage, four stationary time series models were identified. The first two models were based on the first regular differenced CPI series. The third was based on the first regular and the second seasonal differencing of the CPI series. The last model used the second regular and the first seasonal differences of the CPI series.

All four models were estimated using the non-linear least squares method. The last two models were eliminated in the diagnostic checking stage, based on the overall inadequacy resulting from the Box-Ljung chi-square statistics.

Due to the adequacy of the first two models, validation test were performed on both. In which, part of the CPI series were used to re-estimated the two models. The other part of the series were use to examine the forecasting ability of the re-estimated models.

Based on the validation results a modified version of the first model was chosen as the best estimated model. The model was a seasonal autoregressive of the second regular, and first seasonal order.

The estimated SARIMA model was then used to forecast the values of the CPI series from March 1988 to December 1989.

The same Box-Jenkins technique was used on the inverse transformation of the CPI series [i.e $(CPI)^{-1}$]. But None of the estimated models; based on this transformation, were as good as the above estimated model.

The model performs reliably well compared to the few newly published CPI values since this study was finalized. But since this approach is good for forecasting values on a short term bases, it is recommended to re-check it's performance and update it, if needed, especially when the time series start to shift, or behave in a different form from the historical pattern that the estimated model was based on.

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APPENDIX

Table (A1)
Autocorrelation and partial autocorrelation functions of CPI series.

Lag	ac	pac
1	0.974	0.974
2	0.948	-0.010
3	0.922	-0.015
4	0.895	-0.044
5	0.868	-0.008
6	0.842	0.015
7	0.815	-0.056
8	0.786	-0.033
9	0.757	-0.027
10	0.726	-0.037
11	0.695	-0.041
12	0.665	0.012
13	0.634	-0.026
14	0.606	0.021
15	0.577	-0.023
16	0.547	-0.045
17	0.518	0.013
18	0.491	0.002
19	0.461	-0.056
20	0.430	-0.053
21	0.403	0.046
22	0.376	-0.001
23	0.348	-0.052
24	0.321	0.003

Q-Statistic (24 lags) 1243.927 S.E. of Correlations 0.094

Table (A2)
Autocorrelation and partial autocorrelation functions of D10 series.

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     Autocorrelations      Partial Autocorrelations       ac      pac

|      **| .        |      **| .        |  1 -0.171 -0.171
|      **| .        |     ***| .        |  2 -0.182 -0.218
|      . |****      |      . |***       |  3  0.295  0.237
|      . | .        |      . |*.        |  4  0.020  0.088
|      .*| .        |      . | .        |  5 -0.078  0.037
|      . |***       |      . |**        |  6  0.207  0.167
|      .*| .        |      . | .        |  7 -0.055 -0.035
|      . |*.        |      . |**        |  8  0.082  0.152
|      . |***       |      . |***       |  9  0.250  0.229
|      **| .        |      .*| .        | 10 -0.186 -0.089
|      .*| .        |      **| .        | 11 -0.086 -0.132
|      . |********  |      . |*****     | 12  0.534  0.400
|    ****| .        |    ****| .        | 13 -0.336 -0.279
|      . | .        |      . |*.        | 14 -0.020  0.108
|      . |****      |      . | .        | 15  0.291  0.002
|      .*| .        |      . |*.        | 16 -0.047  0.063
|      .*| .        |      .*| .        | 17 -0.099 -0.065
|      . |***       |      . |*.        | 18  0.211  0.062
|      **| .        |      .*| .        | 19 -0.147 -0.114
|      . | .        |      .*| .        | 20  0.008 -0.046
|      . |**        |      . | .        | 21  0.191  0.018
|      **| .        |      . | .        | 22 -0.186 -0.033
|      . | .        |      . | .        | 23 -0.007  0.006
|      . |****      |      . | .        | 24  0.328  0.003

Q-Statistic (24 lags)  116.871          S.E. of Correlations  0.095
```

Table (A3)
Autocorrelation and partial autocorrelation
functions of D11 series.

Autocorrelations	Partial Autocorrelations		ac	pac
. ¦* .	. ¦* .	1	0.039	0.039
.**¦ .	.**¦ .	2	-0.162	-0.164
. ¦ .	. ¦ .	3	0.017	0.031
. ¦ .	. *¦ .	4	-0.025	-0.055
. ¦ .	. ¦ .	5	0.021	0.034
. ¦ .	. ¦ .	6	-0.003	-0.020
. ¦**.	. ¦**.	7	0.143	0.161
. ¦* .	. ¦* .	8	0.106	0.087
. ¦ .	. ¦ .	9	-0.028	0.020
. ¦* .	. ¦* .	10	0.065	0.094
. ¦ .	. ¦ .	11	0.028	0.029
***¦ .	***¦ .	12	-0.256	-0.251
.**¦ .	.**¦ .	13	-0.159	-0.161
. ¦* .	. ¦ .	14	0.063	-0.037
. ¦* .	. ¦ .	15	0.094	0.020
. ¦ .	. ¦ .	16	0.011	-0.005
.**¦ .	. *¦ .	17	-0.116	-0.112
. ¦ .	. ¦* .	18	0.037	0.050
. *¦ .	. ¦ .	19	-0.045	-0.006
. ¦ .	. ¦**.	20	-0.009	0.120
. ¦ .	. ¦ .	21	0.028	0.037
. *¦ .	. *¦ .	22	-0.077	-0.039
. ¦* .	. ¦* .	23	0.071	0.108
. ¦ .	. *¦ .	24	-0.029	-0.089

Q-Statistic (24 lags) 19.798   S.E. of Correlations 0.101

Table (A4)
Autocorrelation and partial autocorrelation functions of D21 series.

```
        Autocorrelations         Partial Autocorrelations          ac      pac
------------------------------------------------------------------------------
|         *****|  .        |         *****|  .        |  1  -0.399  -0.399
|           ***|  .        |        ******|  .        |  2  -0.200  -0.428
|          .   |**.        |           ***|  .        |  3   0.141  -0.210
|          .  *|  .        |           ***|  .        |  4  -0.057  -0.240
|          .   |  .        |           .**|  .        |  5   0.028  -0.140
|          .  *|  .        |           ***|  .        |  6  -0.062  -0.234
|          .   |* .        |           .**|  .        |  7   0.073  -0.122
|          .   |* .        |           . *|  .        |  8   0.044  -0.039
|          .  *|  .        |           . *|  .        |  9  -0.104  -0.089
|          .   |* .        |           .  |  .        | 10   0.070  -0.004
|          .   |**.        |           .  |***        | 11   0.128   0.241
|           ***|  .        |           .  |* .        | 12  -0.205   0.093
|          .  *|  .        |           . *|  .        | 13  -0.069  -0.075
|          .   |* .        |           .**|  .        | 14   0.105  -0.135
|          .   |* .        |           . *|  .        | 15   0.066  -0.059
|          .   |  .        |           .  |* .        | 16   0.029   0.081
|          .**|   .        |           . *|  .        | 17  -0.154  -0.078
|          .   |* .        |           .  |  .        | 18   0.113  -0.010
|          .  *|  .        |           . *|  .        | 19  -0.043  -0.090
|          .   |  .        |           .  |  .        | 20  -0.005   0.025
|          .   |* .        |           .  |* .        | 21   0.060   0.074
|          .**|   .        |           . *|  .        | 22  -0.136  -0.097
|          .   |**.        |           .  |* .        | 23   0.141   0.099
|          .  *|  .        |           . *|  .        | 24  -0.113  -0.075
------------------------------------------------------------------------------
Q-Statistic (24 lags)   41.421              S.E. of Correlations  0.101
------------------------------------------------------------------------------
```

Table (A5)
Autocorrelation and partial autocorrelation functions of D12 series.

Autocorrelations	Partial Autocorrelations		ac	pac
. ¦* .	. ¦* .	1	0.085	0.085
***¦ .	***¦ .	2	-0.193	-0.202
. ¦ .	. ¦ .	3	-0.008	0.030
. *¦ .	.**¦ .	4	-0.075	-0.122
. ¦ .	. ¦* .	5	0.031	0.060
. ¦ .	. *¦ .	6	-0.022	-0.078
. ¦* .	. ¦**.	7	0.097	0.141
. ¦**.	. ¦* .	8	0.121	0.065
. ¦ .	. ¦ .	9	-0.023	0.020
. ¦**.	. ¦**.	10	0.146	0.188
. *¦ .	. *¦ .	11	-0.047	-0.077
*******¦ .	******¦ .	12	-0.522	-0.484
***¦ .	***¦ .	13	-0.198	-0.198
. ¦* .	.**¦ .	14	0.048	-0.151
. ¦**.	. ¦* .	15	0.136	0.056
. ¦ .	. *¦ .	16	0.036	-0.042
.**¦ .	. *¦ .	17	-0.133	-0.104
. ¦* .	. ¦* .	18	0.053	0.053
. ¦ .	. ¦* .	19	-0.036	0.087
. *¦ .	. ¦* .	20	-0.088	0.097
. *¦ .	. ¦ .	21	-0.049	-0.021
. *¦ .	. ¦ .	22	-0.091	0.034
. ¦* .	. ¦* .	23	0.104	0.101
. ¦* .	***¦ .	24	0.098	-0.223

Q-Statistic (24 lags) 42.983 S.E. of Correlations 0.107

Table (A6)

Non-linear estimation results for model (1).

PAR	LAG	ESTIMATE	STD ERROR	T RATIO
AR	2	-0.16094	0.94977E-01	-1.6945
SAR	12	0.70681	0.68312E-01	10.347

CORRELATION MATRIX OF THE ESTIMATES

PAR	LAG		
AR	2	1.00000	0.10249
SAR	12	0.10249	1.00000

STANDARD DEVIATION OF RESIDUAL SERIES = 0.80980

Table (A7)

Non-linear estimation results for Model (2).

PAR	LAG	ESTIMATE	STD ERROR	T RATIO
SAR	12	0.87500	0.58974E-01	14.837
SMA	12	0.37344	0.11280	3.3107

CORRELATION MATRIX OF THE ESTIMATES

PAR	LAG		
SAR	12	1.00000	0.64021
SMA	12	0.64021	1.00000

STANDARD DEVIATION OF RESIDUAL SERIES = 0.81347

Table (A8)

Non-linear estimation results for model (3).

PAR	LAG	ESTIMATE	STD ERROR	T RATIO
SMA	12	1.0313	0.72836E-01	14.158
SMA	24	-0.17500	0.77424E-01	-2.2603

CORRELATION MATRIX OF THE ESTIMATES

PAR	LAG		
SMA	12	1.00000	-0.87290
SMA	24	-0.87290	1.00000

STANDARD DEVIATION OF RESIDUAL SERIES = 0.91324

Table (A9)

Non-linear estimation results for model (4).

PAR	LAG	ESTIMATE	STD ERROR	T RATIO
MA	1	0.91719	0.42132E-01	21.770
SMA	12	0.58906	0.95095E-01	6.1945
SMA	24	0.26563E-01	0.10302	0.25785
SMA	36	0.78125E-01	0.93537E-01	0.83523

CORRELATION MATRIX OF THE ESTIMATES

PAR	LAG				
MA	1	1.00000	-0.12895	-0.06200	-0.09044
SMA	12	-0.12895	1.00000	-0.52290	0.11637
SMA	24	-0.06200	-0.52290	1.00000	-0.72776
SMA	36	-0.09044	0.11637	-0.72776	1.00000

STANDARD DEVIATION OF RESIDUAL SERIES = 0.82354

Table (A10)
Diagnostic checking - model (1)

BOX-LJUNG CHI-SQUARE STAT. FOR RESIDUAL SERIES OF VARIABLE CPI

LAG	CHI-SQ.	D.F.	PROB.
6	1.29	4	0.8626
12	6.29	10	0.7907
18	16.63	16	0.4098
24	19.30	22	0.6267
30	23.37	28	0.7141
36	25.32	34	0.8591

RESIDUAL AUTOCORRELATION FUNCTION FOR VARIABLE CPI
AUTOCORRELATIONS = *
TWO STANDARD ERROR LIMITS = .

```
     AUTO. STAND.
LAG  CORR.   ERR. -1 -.75 -.5 -.25  0  .25  .5  .75   1
                  :----:----:----:----:----:----:----:----:
  1 -0.009  0.099                 .   *   .
  2 -0.031  0.099                 .  *:   .
  3  0.060  0.098                 .   :*  .
  4 -0.059  0.098                 .  *:   .
  5  0.046  0.097                 .   :*  .
  6  0.031  0.097                 .   :*  .
  7  0.087  0.096                 .   :** .
  8  0.097  0.096                 .   :** .
  9  0.007  0.095                 .   *   .
 10  0.039  0.095                 .   :*  .
 11  0.006  0.094                 .   *   .
 12 -0.148  0.094                 .***:   .
 13 -0.220  0.093                 ****:   .
 14  0.024  0.092                 .   *   .
 15  0.112  0.092                 .   :** .
 16  0.036  0.091                 .   :*  .
 17 -0.113  0.091                 . **:   .
 18  0.071  0.090                 .   :*  .
 19 -0.049  0.090                 .  *:   .
 20 -0.029  0.089                 .  *:   .
 21  0.038  0.088                 .   :*  .
 22 -0.094  0.088                 . **:   .
 23  0.072  0.087                  .  :* .
 24  0.021  0.087                  .  *  .
```

Table (A11)
Diagnostic checking - model (2)

BOX-LJUNG CHI-SQUARE STAT. FOR RESIDUAL SERIES OF VARIABLE CPI

LAG	CHI-SQ.	D.F.	PROB.
6	3.66	4	0.4538
12	8.19	10	0.6100
18	18.25	16	0.3095
24	21.02	22	0.5195
30	24.99	28	0.6285
36	28.82	34	0.7192

```
RESIDUAL AUTOCORRELATION FUNCTION FOR VARIABLE CPI
AUTOCORRELATIONS         = *
TWO STANDARD ERROR LIMITS = .

      AUTO.  STAND.
LAG   CORR.    ERR. -1  -.75  -.5 -.25   0   .25  .5  .75   1
                     :----:----:----:----:----:----:----:----:
  1 -0.019  0.098                    .   *   .
  2 -0.143  0.098                    .***:   .
  3  0.087  0.097                    .   :** .
  4  0.020  0.097                    .   *   .
  5  0.020  0.096                    .   *   .
  6  0.046  0.096                    .   :*  .
  7  0.142  0.095                    .   :***.
  8  0.100  0.095                    .   :** .
  9  0.005  0.094                    .   *   .
 10  0.007  0.094                    .   *   .
 11  0.068  0.093                    .   :*  .
 12  0.044  0.093                    .   :*  .
 13 -0.204  0.092                    ****:   .
 14  0.068  0.092                    .   :*  .
 15  0.131  0.091                    .   :***.
 16 -0.004  0.091                    .   *   .
 17 -0.081  0.090                    . **:   .
 18  0.084  0.089                    .   :** .
 19 -0.032  0.089                    .  *:   .
 20 -0.004  0.088                    .   *   .
 21  0.059  0.088                    .   :*  .
 22 -0.090  0.087                     .**:  .
 23  0.077  0.087                     .  :**.
 24 -0.031  0.086                     . *:  .
```

Table (A12)
Diagnostic checking - model (3)

BOX-LJUNG CHI-SQUARE STAT. FOR RESIDUAL SERIES OF VARIABLE CPI

LAG	CHI-SQ.	D.F.	PROB.
6	3.74	4	0.4427
12	12.05	10	0.2817
18	23.84	16	0.0931
24	30.00	22	0.1184
30	34.50	28	0.1848
36	36.26	34	0.3635

RESIDUAL AUTOCORRELATION FUNCTION FOR VARIABLE CPI
AUTOCORRELATIONS = *
TWO STANDARD ERROR LIMITS = .

```
       AUTO.  STAND.
 LAG   CORR.    ERR. -1   -.75  -.5  -.25   0   .25   .5   .75   1
                      :----:----:----:----:----:----:----:----:
   1   0.011   0.105                  .   *   .
   2  -0.166   0.104                  .***:   .
   3   0.022   0.104                  .   *   .
   4   0.019   0.103                  .   *   .
   5   0.009   0.102                  .   *   .
   6  -0.084   0.102                  . **:   .
   7   0.094   0.101                  .   :** .
   8   0.213   0.100                  .   :****
   9  -0.018   0.100                  .   *   .
  10   0.012   0.099                  .   *   .
  11   0.056   0.098                  .   :*  .
  12  -0.128   0.098                  .***:   .
  13  -0.229   0.097                 *.***:   .
  14   0.029   0.096                  .   :*  .
  15   0.160   0.096                  .   :***.
  16   0.037   0.095                  .   :*  .
  17  -0.136   0.094                  .***:   .
  18  -0.005   0.094                  .   *   .
  19  -0.057   0.093                  .  *:   .
  20  -0.018   0.092                  .   *   .
  21  -0.099   0.092                  . **:   .
  22  -0.053   0.091                  .  *:   .
  23   0.156   0.090                  .   :***.
  24  -0.080   0.089                  . **:   .
```

Table (A13)
Diagnostic checking - model (4)

BOX-LJUNG CHI-SQUARE STAT. FOR RESIDUAL SERIES OF VARIABLE CPI

LAG	CHI-SQ.	D.F.	PROB.
6	7.28	2	0.0263
12	10.25	8	0.2482
18	22.17	14	0.0752
24	26.54	20	0.1486
30	29.23	26	0.3005
36	34.00	32	0.3712

RESIDUAL AUTOCORRELATION FUNCTION FOR VARIABLE CPI
AUTOCORRELATIONS = *
TWO STANDARD ERROR LIMITS = .

```
       AUTO.  STAND.
  LAG  CORR.   ERR.  -1  -.75 -.5 -.25   0   .25  .5   .75   1
                     :----:----:----:----:----:----:----:----:
    1 -0.092  0.099                  . **:   .
    2 -0.223  0.098                  ****:   .
    3  0.062  0.098                  .   :*  .
    4 -0.004  0.097                  .   *   .
    5 -0.028  0.097                  .  *:   .
    6  0.025  0.096                  .   *   .
    7  0.087  0.096                  .   :** .
    8  0.065  0.095                  .   :*  .
    9  0.008  0.095                  .   *   .
   10 -0.052  0.094                  .  *:   .
   11  0.047  0.094                  .   :*  .
   12  0.085  0.093                  .   :** .
   13 -0.237  0.093                 *.***:   .
   14  0.009  0.092                  .   *   .
   15  0.150  0.091                  .   :***.
   16  0.000  0.091                  .   *   .
   17 -0.090  0.090                  . **:   .
   18  0.074  0.090                  .   :*  .
   19 -0.048  0.089                  .  *:   .
   20  0.005  0.089                  .   *   .
   21  0.070  0.088                  .   :*  .
   22 -0.123  0.087                   .**:  .
   23  0.089  0.087                   .  :**.
   24 -0.027  0.086                   . *:  .
```

Table (A14)
Forecasted CPI values and 95 % confidence limits form
Jan. 1987 to Dec. 1987 for model (1) and (2)
using first part of the CPI series.

MODEL (1)

OBS NUM	REAL CPI	LOW CONF LIM	FORECAST	UPP CONF LIM	S.E.
97	145.3	142.75	144.40	146.05	0.8399
98	144.1	142.22	144.55	146.88	1.1879
99	144.3	141.67	144.39	147.11	1.3880
100	144.6	141.33	144.40	147.46	1.5626
101	144.8	140.51	143.89	147.28	1.7271
102	145.6	141.08	144.76	148.44	1.8773
103	145.0	140.30	144.25	148.20	2.0153
104	145.4	140.05	144.25	148.46	2.1445
105	145.5	140.75	145.19	149.63	2.2665
106	143.8	139.87	144.54	149.21	2.3822
107	143.4	138.79	143.68	148.56	2.4925
108	145.0	139.45	144.54	149.64	2.5982

MODEL (2)

OBS NUM	REAL CPI	LOW CONF LIM	FORECAST	UPP CONF LIM	S.E.
97	145.3	142.55	144.21	145.88	0.8509
98	144.1	142.04	144.40	146.76	1.2035
99	144.3	141.56	144.45	147.34	1.4740
100	144.6	141.29	144.63	147.97	1.7020
101	144.8	140.14	143.87	147.60	1.9029
102	145.6	140.62	144.70	148.79	2.0845
103	145.0	139.78	144.19	148.60	2.2515
104	145.4	139.50	144.22	148.93	2.4070
105	145.5	140.28	145.29	150.29	2.5530
106	143.8	139.36	144.64	149.92	2.6911
107	143.4	138.37	143.91	149.44	2.8224
108	145.0	139.07	144.85	150.63	2.9479

Table (A15) Forecasted values for Jan. 1987 to Dec. 1987 for model (1) and (2) using first part of the CPI series.

DEFINITIONS OF SYMBOL :	
DATA	= *
FORECASTS AT LEAD 1	= +
ESTIMATED 95% CONFIDENCE LIMITS	= .
FORECAST FUNCTION	= 0
OVERLAP	= X

MODEL (1)

```
  OBS.   DATA        132       137       142       147
                   :---:---------:---------:---------:-----:
  94   144.000     :                     . *+  .
  95   142.800     :                   . *+  .
  96   144.000     :                   .  +* .
  97   144.400   F :                     .  0  .
  98   144.547   F :                   .    0    .
  99   144.387   F :                  .     0    .
 100   144.397   F -                  .     0     .
 101   143.894   F :                .      0      .
 102   144.759   F :                 .       0      .
 103   144.253   F :                .       0      .
 104   144.253   F :               .        0       .
 105   145.192   F :                .        0        .
 106   144.542   F :               .        0        .
 107   143.676   F :             .        0         .
 108   144.542   F :              .         0         .
```

MODEL(2)

```
  OBS.   DATA              137       142       147       152
                   :---------:---------:---------:---------:
  94   144.000     :                    .  *+  .
  95   142.800     :                   . * +  .
  96   144.000     :                   .  +*  .
  97   144.214   F :                    .  0   .
  98   144.401   F :                   .    0    .
  99   144.451   F :                  .     0     .
 100   144.629   F -                  .     0      .
 101   143.871   F :               .       0      .
 102   144.704   F :                .       0        .
 103   144.189   F :               .       0        .
 104   144.216   F :              .        0         .
 105   145.285   F :                .         0         .
 106   144.640   F :              .         0          .
 107   143.907   F :            .          0          .
 108   144.852   F :             .           0          .
```