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Reliable Analytical Approach for Multi-taper Spectrum Sensing in Cognitive Radio Networks

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ABSTRACT

Multi-taper detection method (MTM) is a powerful technique in spectrum sensing for Cognitive radio networks. In this paper, reliable and simple analytical expressions for the mean and variance of the Probability Density Function (PDF) of the MTM spectrum detector are derived. Then, closed-form expressions for detection and false alarm probabilities for the MTM spectrum detector have been obtained. Intensive simulation based work is conducted under AWGN channel conditions using MATALB to confirm and evaluate the proposed theoretical study. The confirmation and the evaluation processes are designated to verify many perspectives such as: the receiver operating characteristics (ROCs), the detection rate with respect to SNR, and minimum required sample points $(N)_{min}$ to achieve a certain performance. All these perspectives are simulated under setting of multiple Slepian tapers (K), sample points (N), and false-alarm probability (P_f). Also, a comparison with energy detection method is presented. The simulation results confirm that the proposed model is reliable and robust under all settings of the simulation parameters.

1. INTRODUCTION

The use of the electromagnetic radio frequency RF spectrum is licensed by governments since it is a scarce resource. In case of static RF access, fixed channels are assigned to licensed primary users. These fixed channels cannot be assigned to unlicensed secondary users even if they are unoccupied.

Cognitive radio (CR) appeared as a suitable solution to solve the problem of inefficient use of frequency resource [1]. A cognitive radio system detects the available spectrum, gains information about, and then captures the spectrum holes. These unoccupied holes are assigned to the unlicensed secondary users [2]. A monitoring of these holes is very important to check the reappearance of the licensed primary users [3-8]. Spectrum sensing can detect spectrum holes in different techniques.

Matched filtering [9] and Cyclo-stationary detector [10] are of the spectrum detection techniques. They have high performance compared with other techniques. Their problem is that they require prior knowledge about the primary users' signaling. Energy detection [11] is a non-coherent detection technique. The main advantages of energy detection are short time of sensing and simplicity. These advantages come at the expense of moderate performance due to the use of single rectangular window tapering [12-13].

The multi-taper spectrum estimation method [14] is another spectrum detection technique. It overcomes some of the limitations of conventional Fourier analysis.

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When we apply the Fourier transform to get spectral information from a signal, each Fourier coefficient is assumed to be a reliable representation of the amplitude and relative phase of the corresponding component frequency. This assumption is not always true [15-18].

For instance, a single trial represents only one noisy realization of the process considered. The same situation happens in statistics when estimating measures of central tendency, it is not accurate to estimate qualities of a population using small samples. Likewise, a single sample of a process does not provide a reliable estimate of its spectral properties. These problems can be overcome by averaging over many realizations of the same event. Instead of ensemble averaging, the multi-taper method reduces estimation bias by getting multiple independent estimates from the same sample [19-24].

Each taper is multiplied by the signal to provide a windowed trial to estimate the power at each component frequency. Since each taper is orthogonal to all other tapers, the windowed signals give statistically independent estimates of the spectrum. The final spectrum is obtained by averaging all the tapered spectra [25]. In [26] the Discrete Prolate Slepian Sequences (DPSS), which are developed by David J. Thomson [27], have been chose as tapers since they are mutually orthogonal. In practice, a weighted average is often used to overcome the increased energy loss at higher order tapers [28]. MTM is considered as a less complex approximation of the Maximum Likelihood (ML) optimal spectrum estimate method [29].

Although, there are many published papers on the Multi-taper spectral detector, there is a missing of analytical closed-form equations suitable for numerical evaluations.

Few papers such as [18-21] work on reaching analytical closed-form equations for detection performance of the Multi-taper spectral detector in CR networks.

In [18], simple closed-form expressions for the detection, and false alarm probabilities are presented for spectrum sensing detection based on MTM. However, the derived closed-form for both the mean and variance of both hypotheses don't verify the nature concept of MTM, which is a reduction in the variance at the cost of decreasing resolution.

In [19], an optimal detector solution for investigating detection performance of the Multi-taper spectral detector in CR networks is proposed. The detector is robust for various multiple data tapers and the detection performance is reliable. However, it is difficult to implement the mean and variance values blocks for their complexity. In addition, the system building block need natural logarithmic calculation block.

In [20], Multi-taper spectral detector is formulated as a quadratic function of Gaussian vector, thereby facilitating the determination of detection and false-alarm probabilities. However, the false-alarm probability(P_f) is not a simple function of threshold (γ). Therefore, the Newton-Raphson method is used to determine γ for a given P_f. Also, its characteristic function (CHF) has inherent singular values which exclude a simple expression for detection probability.

In [21], Multi-taper spectral detector is formulated as a quadratic function of Gaussian vector as illustrated in [20]. Also, the calculation of the false-alarm probability(P_f) is not straight forward as a function of threshold (γ).

In this paper, the energy detection and multi- taper spectrum sensing methods are discussed. Closed-form analytical expressions for the mean and the variance of the Probability Density Function (PDF) of the MTM detector are formulated, where the PDF of the MTM detector is approximated to be Gaussian. Then, simple and reliable closed-form expressions for the probability of detection and probability of false alarm are derived.

The remaining parts of the paper are organized as follows: Section II describes the model of the energy detection spectrum sensing. Section III gives a description of the MTM spectrum sensing method with a complete derivation of the probability of detection and the probability of false alarm of the MTM detector. Section IV shows and discusses the simulation results. Finally, Section V is devoted for the main conclusions.

2. ENERGY DETECTION

Energy detection is a non- coherent non- cooperative detection technique. It detects the primary signal based on the energy sensed. Existence or absence of the primary user can be decided by comparing the received energy with a predefined threshold.

The signal detection at the secondary user can be expressed by the following hypothesis testing problem; H_0 for absent signal and H_1 for present signal. As a result, the received signal can be expressed as:

$$Y(t) = n(t), \qquad 0 < t \leq T : H_0 \qquad (1)$$

$$Y(t) = S(t) + n(t)\,, \qquad 0 < t \leq T : H_1 \qquad (2)$$

where *Y(t)* is the received signal, *S(t)* is the transmitted signal, and *n(t)* is white noise which is assumed to be Gaussian random variable with mean zero and variance σ_w^2 and *Y(n)*, *S(n)* and *n(n)* are their time sampled form. The decision rule for the previous hypothesis problem is

$$\varepsilon < \gamma \qquad for\ H_\circ \qquad (3)$$

$$\varepsilon \geq \gamma \qquad for\ H_1 \qquad (4)$$

where ε is the test statistic and γ is the threshold voltage. H_0 indicates that primary user is absent while it is actually present. H_1 indicates that primary user is present. It is very important to choose a suitable value for the threshold γ. Accordingly, the probability of false alarm P_f and the probability of detectionP_d can be defined as

$$P_f = P_r(\varepsilon \geq \gamma)H_\circ \qquad (5)$$

$$P_d = P_r(\varepsilon \geq \gamma)H_1 \qquad (6)$$

After comparing the test statistic with the threshold, the final decision on existence or absence of the primary user is taken. The test statistic can be given as

$$\varepsilon = \frac{1}{N} \sum_{n=1}^{N} |Y(n)|^2 \qquad (7)$$

where N is the sample number such that $N \approx TW$, where TW is the time-bandwidth product. In our model, the power spectrum density PSD of the received signal is approximated at higher values of N to normal distribution. The mean of this process for both hypotheses are μ/H_1 and μ/H_0, and variances are σ^2/H_1 and σ^2/H_0. The probability of detection and false alarm are given by:

$$P_f = Q\left(\frac{\gamma - \mu/H_0}{\sqrt{\sigma^2/H_0}}\right) \qquad (8)$$

$$\gamma = (Q^{-1}(P_f))\sqrt{\sigma^2/H_0} + \mu/H_0 \qquad (9)$$

$$P_d = Q\left(\frac{\gamma - \mu/H_1}{\sqrt{\sigma^2/H_1}}\right) \qquad (10)$$

The mean and variance for energy detection has been derived as follows [11-13].

For hypothesis H_0, the mean of energy is σ_w^2 and the variance is $\left(\frac{2\sigma_w^4}{N}\right)$.

For hypothesis H_1, the mean is $(E_s + \sigma_w^2)$ and the variance is $\left(\frac{2\sigma_w^4(SNR+1)^2}{N}\right)$.

The probability of detection and false alarm can be written as:

$$P_d^{En} = Q\left(\frac{\gamma - (E_s + \sigma_w^2)}{\sqrt{\left(\frac{2\sigma_w^4(SNR+1)^2}{N}\right)}}\right) \qquad (11)$$

$$P_f^{En} = Q\left(\frac{\gamma - \sigma_w^2}{\sqrt{\left(\frac{2\sigma_w^4}{N}\right)}}\right) \quad (12)$$

3. MULTI-TAPER SYSTEM MODEL

A non- stationary signal generated from a random statistical process is considered. The signal is sampled to get the finite discrete sample sequence X_t ; t = 0; 1; :N-1, where t is time index. As shown in Fig. 1, X_t is then multiplied with a number of discrete Slepian sequences h^K_t *(N,W)*. The associated Eigen values of k[th] taper are λ_k. Then, the products are applied to Fast Fourier Transform (*FFT*) to get the power concentrated in a chosen bandwidth W. The half time bandwidth product is NW and the total number of generated tapers is 2NW. The received data samples have been assumed to be scaled, so that the noise variance is unity, i.e., $\sigma_w^2 = 1$.

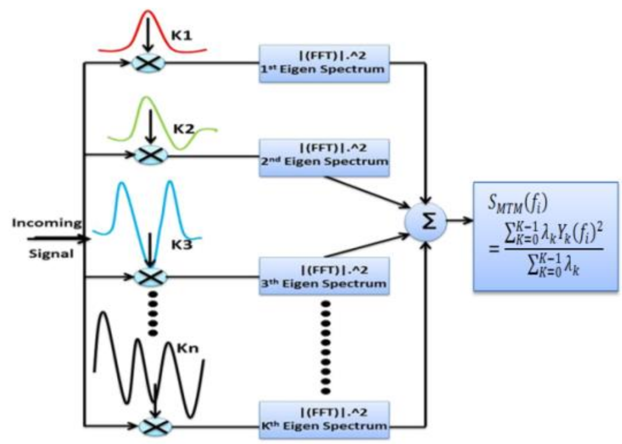

Fig. 1: Multi-taper system model.

The K different Eigen spectrums produced are defined as

$$Y_k(f_i) = \sum_{t=0}^{N-1} h_t^k(N,W) X_t e^{-2j\pi f t} \quad (13)$$

where f_i are normalized frequency bins. Moreover, we got the total estimated power, according to Thomson equation [18, 26].

$$S_{MTM}(f_i) = \frac{\sum_{K=0}^{K-1}\lambda_k(N,W)Y_k(f_i)^2}{\sum_{K=0}^{K-1}\lambda_k(N,W)} \quad (14)$$

On the other side, the energy detection method gives the power spectrum density estimation as follows.

$$S_{ED}(f_i) = \frac{1}{N}\sum_{t=0}^{N-1}|X_t e^{-2j\pi f t}|\text{^}2 \quad (15)$$

In order to compare the MTM detector with other systems, we follow the hypothesis model stated in equations (1) and (2).

In the proposed model, the power spectrum density (PSD) of the received signal is approximated at higher values of N to normal distribution. The mean of this process for both hypotheses are μ/H_1 and μ/H_0, and variances are σ^2/H_1 and σ^2/H_0.

The probability of detection and false alarm are given by equations (8) and (9).

Here, we follow the method that reported in [18] with recalculation of the mean and variance considering the MTM nature concepts.

Since we have a number of K independent random variables *g(x)*, the expectation of K tapers random process *G(x)* with λ_k as the weights, can be calculated as:

$$E[G(x)] = \sum_{i=0}^{K-1} a_i E[g(x)_i] \quad (16)$$

where $g(x)_i = \sum_{i=0}^{K-1}(|FFT(Signal \times Taper_i)|^2)$ and $a_i = \frac{\lambda_i}{\sum k\,\lambda_k}$, and the variance is calculated as

$$Variance = \sum_{i=0}^{K-1} a_i^2\, Var[g(x)_i] + 2\sum_{i\&j=0}^{k} a_i a_j Cov[g(x)_i g(x)_j] \quad (17)$$

where covariance (*Cov)* for i ≠ j is calculated by

$$Cov[g(x)_i g(x)_j] = \rho\sigma_i\sigma_j \quad (18)$$

where ρ is correlation coefficient . For H_0 hypothesis where noise only exists, the MTM mean μ_{MTM} and the variance σ_{MTM}^2 can be calculated as ,

$$\mu_{MTM}\backslash H_0 = \left(\frac{\lambda_1}{\sum k\,\lambda_k}\right)E[g(x)_1] + \left(\frac{\lambda_2}{\sum k\,\lambda_k}\right)E[g(x)_2] \ldots\ldots\ldots + \left(\frac{\lambda_K}{\sum k\,\lambda_k}\right)E[g(x)_K]$$

$$= \left(\frac{\lambda_1}{\sum k\,\lambda_k}\right)\sigma_w^2 + \left(\frac{\lambda_2}{\sum k\,\lambda_k}\right)\sigma_w^2 \ldots\ldots\ldots + \left(\frac{\lambda_K}{\sum k\,\lambda_k}\right)\sigma_w^2$$

$$= \left(\frac{\sigma_w^2}{\sum k\,\lambda_k}\right)(\lambda_1 + \lambda_2 \ldots\ldots\ldots + \lambda_k) = \sigma_w^2 \quad (19)$$

$$\sigma_{MTM}^2/H_\circ = \left(\frac{\lambda_1^2}{(\sum k\,\lambda_k)^2}\right)\times\left(\frac{2\sigma_w^4}{N}\right) + \left(\frac{\lambda_2^2}{(\sum k\,\lambda_k)^2}\right)\times\left(\frac{2\sigma_w^4}{N}\right)\ldots\ldots\ldots + \left(\frac{\lambda_k^2}{(\sum k\,\lambda_k)^2}\right)\times\left(\frac{2\sigma_w^4}{N}\right) + 2\rho\left\{\left[\left(\frac{\lambda_1\lambda_2}{(\sum k\,\lambda_k)^2}\right)\times\left(\frac{2\sigma_w^4}{N}\right)\right] + \left[\left(\frac{\lambda_1\lambda_3}{(\sum k\,\lambda_k)^2}\right)\times\left(\frac{2\sigma_w^4}{N}\right)\right] + \cdots + \left[\left(\frac{\lambda_1\lambda_k}{(\sum k\,\lambda_k)^2}\right)\times\left(\frac{2\sigma_w^4}{N}\right)\right] + \left[\left(\frac{\lambda_2\lambda_3}{(\sum k\,\lambda_k)^2}\right)\times\left(\frac{2\sigma_w^4}{N}\right)\right] + \cdots + \left[\left(\frac{\lambda_2\lambda_k}{(\sum k\,\lambda_k)^2}\right)\times\left(\frac{2\sigma_w^4}{N}\right)\right] + \cdots + \left[\left(\frac{\lambda_k\lambda_{k-1}}{(\sum k\,\lambda_k)^2}\right)\times\left(\frac{2\sigma_w^4}{N}\right)\right]\right\}$$

$$= \left(\frac{2\sigma_w^4}{(\sum k\,\lambda_k)^2 N}\right)\times(\lambda_1^2 + \lambda_2^2 \ldots\ldots + \lambda_K^2) + \left[\left(\frac{2\rho\times 2\sigma_w^4}{(\sum k\,\lambda_k)^2 N}\right)\times(\lambda_1\lambda_2 + \lambda_1\lambda_3 \ldots\ldots + \lambda_1\lambda_k \ldots\ldots + \lambda_{k-1}\lambda_k)\right] \quad (20)$$

For independent uncorrelated Tapers with ρ =0, and for first higher order tapers where power concentration ≃1.

$$\frac{\lambda_1^2 + \lambda_2^2 + \cdots + \lambda_K^2}{(\sum k\,\lambda_k)^2} \simeq \frac{1}{K} \quad (21)$$

$$and\ \sigma_{MTM}^2\backslash H_\circ = \left(\frac{2\sigma_w^4}{NK}\right) \quad (22)$$

For hypothesis H_1 where signal and noise exist,

$\mu_{MTM}\backslash H_1 =$

$$\left(\frac{\lambda_1}{\sum k\,\lambda_k}\right) E[g(x)_1]$$

$$+ \left(\frac{\lambda_2}{\sum k\,\lambda_k}\right) E[g(x)_2] \ldots\ldots\ldots + \left(\frac{\lambda_K}{\sum k\,\lambda_k}\right) E[g(x)_K]$$

$$= \left(\frac{\lambda_1}{\sum k\,\lambda_k}\right)(E_s + \sigma_w^2) + \left(\frac{\lambda_2}{\sum k\,\lambda_k}\right)(E_s + \sigma_w^2) \ldots + \left(\frac{\lambda_K}{\sum k\,\lambda_k}\right)(E_s + \sigma_w^2)$$

$$= \left(\frac{(E_s+\sigma_w^2)}{\sum k\,\lambda_k}\right)(\lambda_1 + \lambda_2 \ldots\ldots\ldots + \lambda_k) = \; = (E_s +^2_w) \qquad (23)$$

And variance,

$$\sigma_{MTM}^2 \backslash H_1 = \left(\frac{2\sigma_w^4(SNR+1)^2}{(\sum k\,\lambda_k)^2 N}\right) \times (\lambda_1^2 + \lambda_2^2 \ldots\ldots\ldots + \lambda_K^2) + \left[\left(\frac{2\rho \times 2\sigma_w^4(SNR+1)^2}{(\sum k\,\lambda_k)^2 N}\right) \times (\lambda_1\lambda_2 + \lambda_1\lambda_3 \ldots\ldots + \lambda_1\lambda_k \ldots\ldots\ldots + \lambda_{k-1}\lambda_k)\right] \qquad (24)$$

For independent uncorrelated Tapers, ρ =0, and for first higher order tapers where power concentration ≃1.

$$\sigma_{MTM}^2 \backslash H1 = \left(\frac{2\sigma_w^4(SNR+1)^2}{NK}\right) \qquad (25)$$

So, the probability of detection and false alarm become

$$P_d^{MTM} = Q\left(\frac{\gamma - \sigma_w^2\,(SNR+1)}{\sqrt{\left(\frac{2\sigma_w^4(SNR+1)^2}{NK}\right)}}\right) \qquad (26)$$

$$P_f^{MTM} = Q\left(\frac{\gamma - \sigma_w^2}{\sqrt{\left(\frac{2\sigma_w^4}{NK}\right)}}\right) \qquad (27)$$

$$\gamma = Q^{-1}(P_f^{MTM}) \times \sqrt{\left(\frac{2\sigma_w^4}{NK}\right)} + \sigma_w^2 \qquad (28)$$

4. SIMULATION RESULTS AND DISSCUSION

In this section, the accuracy of the proposed theoretical formulas for the mean and variance of the Probability Density Function (PDF) of the MTM spectrum detector is evaluated. Consequently, the verification of the closed-form expressions for the probability of detection (P_d^{MTM}), the probability of false alarm P_f^{MTM}, and the threshold γ are verified. This is done by comparing the theoretical values of P_d^{MTM}, P_f^{MTM}, and γ determined by equations (26), (27), and (28), respectively, with their values computed directly from computer-simulated data using MATLAB software.

The computer-simulated data is computed under two hypotheses H_0 and H_1 and used in the verification process through two approaches.

1. The first approach computes the mean and variance of the primary users' received (PDF), then, P_f, γ, and P_d are obtained by equations (8), (9), and (10), respectively, under different simulation conditions. PDF for the energy detector is $(Y(t))^2$and for the MTM spectrum detector can be obtained by equation (14), or using the MATLAB's function ($pmtm$) which generate the power spectrum density ($pi \times pxx$).

2. The second approach computes the theoretical threshold (γ) by equation (9). Then, the decision rule given by equations (3) and (4) is used to compute P_d for a given P_f under different simulation conditions using Monte Carlo simulation model. The decision rule depends on the test statistic (ε) which can be obtained by calculating the mean value of the primary users' received (PDF). Monte Carlo simulation model is used; where the primary user's signal is assumed to be normally random distributed signal. The simulation runs 10000 times for realization.

Some simulation results are given using randomly generated signals to illustrate the performance of the proposed analytical detection approach. The system model is simulated under different conditions, such as, different number of tapers K, AWGN channel with different values of Signal-to-Noise Ratio (SNR), and different number of samples (N). Also, the performance of the MTM system is compared with Energy detection under the same conditions. We exploit our results under two hypotheses H_0 and H_1

First, the accuracy of the relationship between probability of detection and probability of false alarm has been verified by comparing theoretical one determined by equations (26) and (28) with the one derived directly from computer-generated data using equations (9) and (10), i.e., using the first approach. The comparison results are shown in Fig. 2.

In Fig. 2, the proposed system model was simulated with N=512, K=5 & 3 respectively, and SNR = -10 dB.

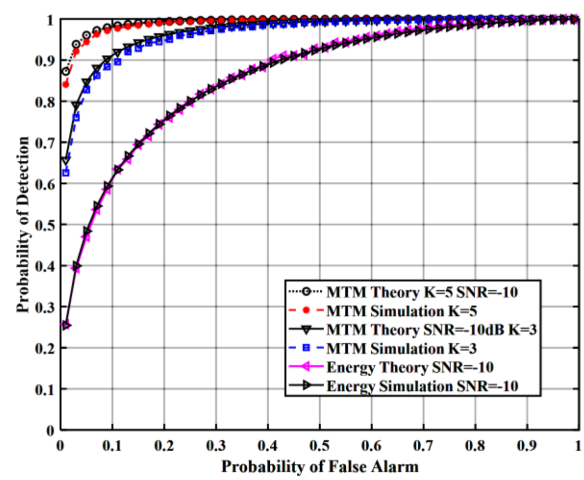

Fig. 2: Probability of detection versus probability of false alarm for MTM and Energy detection at SNR=-10 dB.

The Figure shows that the receiver operating characteristics (ROC) which were generated from the proposed theoretical formulas for the probability of detection (P_d^{MTM}) and the probability of false alarm P_f^{MTM} is matched well with that generated by simulation under all settings of false-alarm rate. This confirms that the accuracy of the proposed theoretical formulas for the mean and variance of the Probability Density Function (PDF) of the MTM spectrum detector matches well under all settings of false-alarm rate and other system parameters.

From this Figure, it is obviously noted that, probability of detection for MTM is significantly increased to reach 90% at probability of false alarm less than 10%. In the same Figure, the detection performance of MTM is compared with Energy detection. We notice that MTM outperforms Energy detection by about 40% and 30% at $p_f = 10\%$ for $K = 5$ and $K = 3$, respectively, under the same conditions.

Second, the accuracy of the proposed formula to determine the threshold, for MTM detector, has been verified by comparing theoretical one determined by equations (28) with one derived directly from computer-generated data using equations (9) with the first approach under H0 hypothesis. The comparison result is given in Fig. 3. The Figure also shows this comparison for Energy detector.

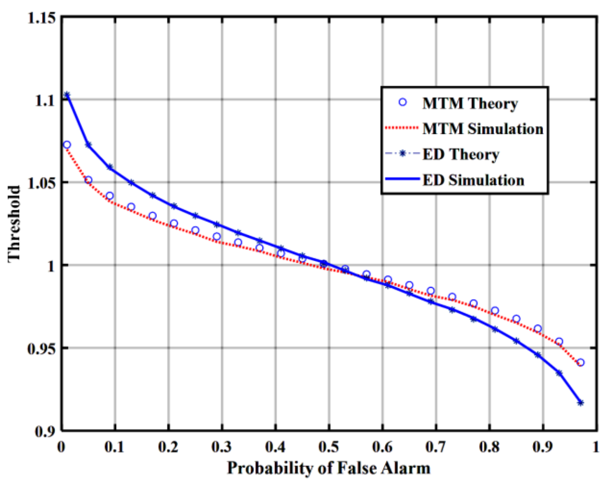

Fig. 3: Comparison between theoretical and simulated thresholds for MTM and Energy detectors for N=1024, K=2 and SNR=-15dB

From Fig. 3, it is observed that the proposed theoretical threshold matches well with the threshold generated by simulation under all settings of probability of false-alarm for both MTM and ED detectors.

Also, the verification of the accuracy of the proposed closed-forms formulas for both the mean and variance of the Probability Density Function (PDF) of the MTM spectrum detector is done through the simulation using the second approach. In this simulation, to enhance the matching between theoretical results with simulation results, we multiply test statistic (ε) by an empirical correction factor Cf which adapts according to the simulation parameters as given in equation (29).

$$Cf = 1 + 0.025 \times K \times (p_f - 0.2) + 0.3 \times SNR \qquad (29)$$

The suggested empirical correction factor Cf is tested for different values of K, p_f, N and SNR. It is found that, Cf enhances the accuracy of the simulation results effectively, as indicated in the following Figure.

In Fig. 4, for different values of K ($K = 5$ and $K = 2$) with $SNR = -15$dB, and $N = 512$ there is a good fitting for ROC curves that are generated from the Monte Carlo simulation (MTM Simulation), using adapted test statistic ε, with that generated from the proposed analytical formulas (MTM Theory). Also, the Figure shows that for increase in K, the detection performance is enhanced and the detector becomes more reliable under the same SNR and N. For example, at $p_f = 20\%$,the P_d equals about 60 % at $K = 5$ while it equals about 45% at $K = 2$.

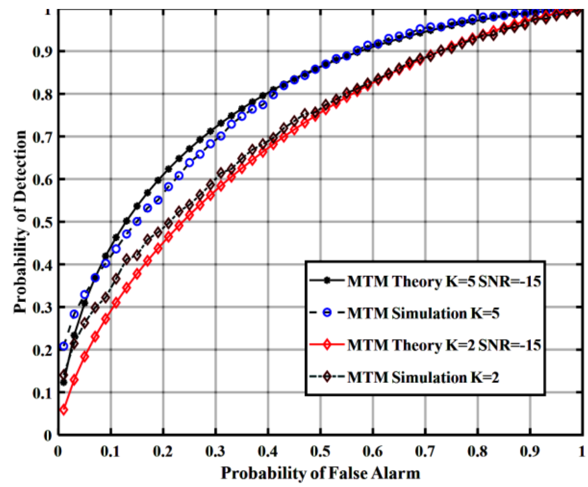

Fig. 4: MTM probability of detection versus probability of false alarm for different number of tapers at SNR=-15 dB.

Also, the proposed model verification and behaviour is tested for a wide range of SNRs with different p_f as shown in Fig. 5.

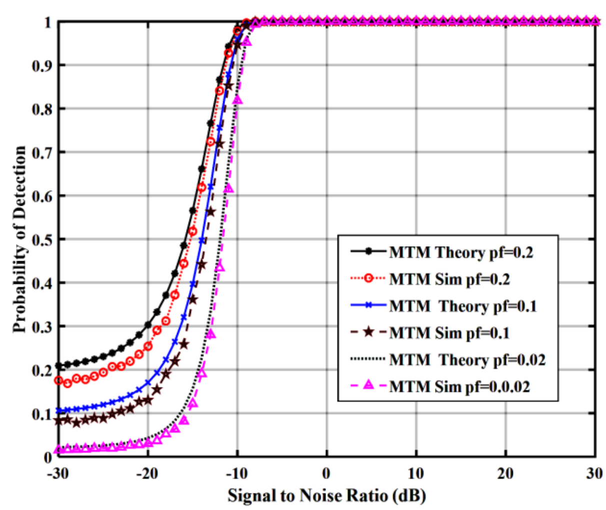

Fig. 5: Probability of detection versus SNR at K=4 and N=512 for different values of pf.

The probability of detection values start to increase with the increase in SNR with noticeable performance enhancement with the increase in p_f .

In Fig. 6, the values of P_d for both theoretical and simulation results versus different SNR values with different number of tapers K are shown. The Figure shows that the probability of detection becomes higher with the increase in SNR and number of tapers K.

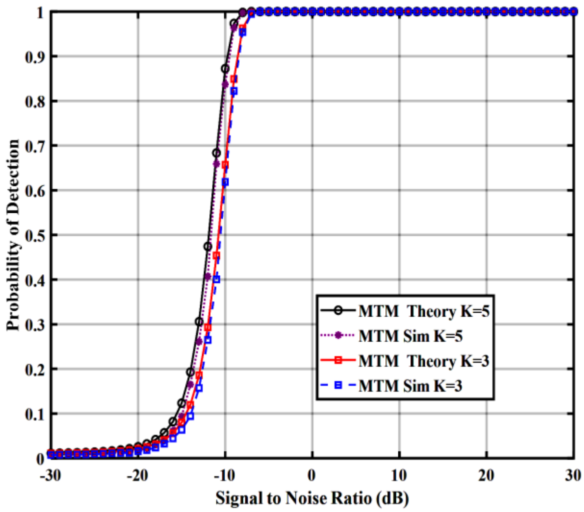

Fig. 6: Probability of detection versus SNR at Pf=0. 1and N=512 for different number of tapers K.

The effect of sample size (N) on the relation between P_d and SNR is shown in Fig. 7. It is clear that the detection performance is enhanced and the detector becomes more reliable with increasing N under the same SNR, K and P_f.

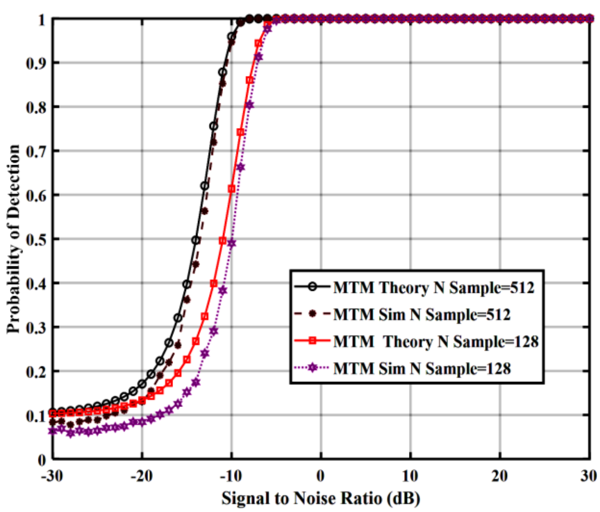

Fig. 7: Probability of detection versus SNR at pf=0.1 and K=4 for different values of sample size.

Moreover, the the proposed MTM model is well confirmed by comparison with the model reported in [19], as shown in the following Figures.

From Fig. 8, it is clear that the ROC curves generated using the proposed model matched well with that generated with the model reported in [23] under all settings of false-alarm rate, especially, in the low and moderate SNR ranges, which are the important ranges in the detection process.

The performance comparison of the proposed model with that reported in [23] is illustrated in Fig. 9. The Figure shows reasonable matching under all settings of N, especially, at low SNR. Also, the probability of detection increases as N increases.

Also, the proposed MTM model is tested for the minimum required number of samples $(N)_{min}$ to achieve required p_d and p_f. $(N)_{min}$ is a function which monotonically decreases with increasing K, as given by equation (30).

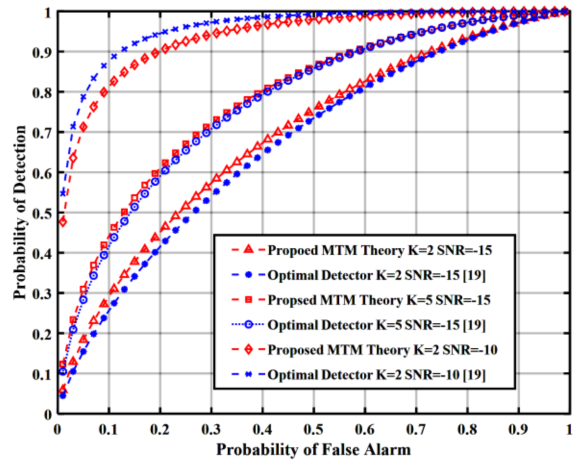

Fig. 8: Comparison between ROC curves generated using the proposed model and the model reported in [23] with N=512.

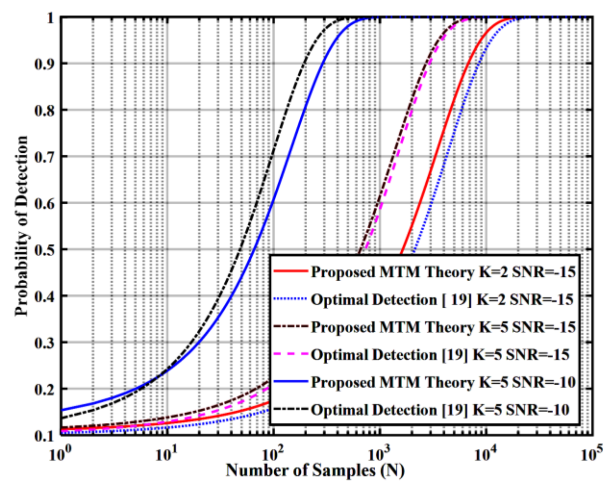

Fig. 9: Performance comparison of the proposed model with the model reported in [19] with respect to the number of samples with Pf=0.1.

The number of samples required for the proposed MTM model and the traditional energy detection algorithms can be computed using equation (30) and equation (31), respectively, and are given by

$$(N)_{min} = \frac{2}{K} \times \left(\frac{(Q^{-1}(p_f) - Q^{-1}(p_d)(SNR+1)\,)}{SNR} \right)^2 \qquad (30)$$

$$(N)_{min} = 2 \times \left(\frac{(Q^{-1}(p_f) - Q^{-1}(p_d)(SNR+1)\,)}{SNR} \right)^2 \qquad (31)$$

The equations show that the required number of samples for a target performance varies as order of

($1/SNR^2$) and can be considered as an important parameter in calculating the computational complexity of the system.

Figure 10 shows the required sample size to achieve the required probability of detection for K=2, 5 and ED as a function of SNR, with Pd = 0.99 and Pf = 0.001. The Figure shows that the proposed MTM model requires smaller sample size to achieve the same performance of the energy detection algorithm. Also, the sample size decreases and N increases and could be decreased more by increasing number of tapers K according to equation (30).

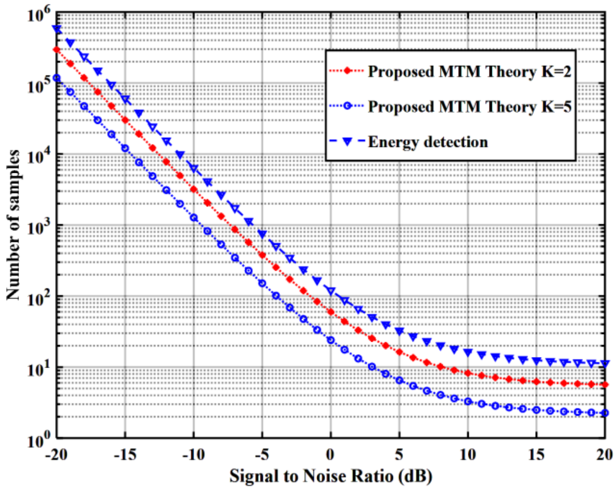

Fig. 10: The sample size to achieve the required probability of detection for K=2, 5 as a function of SNR with Pd=0.99 and Pf=0.001.

Also, from Fig. 10, for $K = 2$, if $N_{min} < 6$, the desirable performance, i.e. $Pf = 0.001$ and $Pd = 0.99$, cannot be achieved at any SNR. However, N_{min} can be decreased by increasing K, e.g., for $K = 5$, N_{min} is decreased from 7 to 2. However, in "pmtm" MATLAB function, which is used to estimate multi-taper power spectral density, N_{min} must be greater than the time-bandwidth product, i.e., $(N_{min} > (K + 1))$.

5. CONCLUSION

In this paper, simple and reliable analytical closed-form approach to analyse and evaluate the detection performance of multi-taper detection based technique in CR networks is proposed. Starting from formulating closed-form expressions for the the mean and variance of the two hypotheses H_0 and H_1 of multi-taper detection technique, closed-form expressions for the detection and false alarm probabilities for the MTM spectrum detector have been derived. The validity of the proposed theoretical formulas is examined intensively through computer simulations. The accuracy of validity of the computer simulation using the decision rule which depends on the test statistic (ε) is enhanced by multiplying test statistic (ε) by a proposed empirical correction factor which is adapted according to the simulation parameters. The simulation results confirm the effectiveness of the proposed method and it is reliable and robust under all settings of simulation parameters. A comparison of the proposed model with one of the reliable existing models but with more implementation complexity indicates well matched results. Also, a comparison between the proposed model and the energy detection method is presented. The results of this comparison have been compared with those concluded and reported in the well-known literature. Similar results and highly concordant conclusion are obtained.

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العنوان باللغة العربية:

منهج تحليلي موثوق لاستشعار الطيف متعدد النوافذ في شبكات الراديو الإدراكية

الملخص باللغة العربية:

تعد طريقة الكشف متعدد النوافذ (MTM) تقنية قوية في استشعار الطيف في شبكات الراديو الإدراكية. في هذه الورقة البحثية، تم استنتاج تعبيرات رياضية (معادلات) تحليلية موثوقة وبسيطة لمتوسط وتباين دالة الكثافة الاحتمالية (PDF) لكاشف الطيف متعدد النوافذ. ومن ثم تم استنتاج تعبيرات رياضية تحليلية لاحتمالية الكشف والإنذار الكاذب. و قد تم التأكد وتقييم الدراسة النظرية المقترحة بإجراء عمليات محاكاة مكثفة باستخدام برمجية الماتلاب بفرضية وجود قناة اتصال ذات الضوضاء المضافة البيضاء من نوع جاوسين (AWGN). وقد تم تصميم هذه العمليات بحيث تشمل العديد من الجوانب الهامه لإستشعار الطيف في شبكات الراديو الإدراكية مثل: خصائص تشغيل المستقبل ومعدل الكشف مقابل نسبة الإشارة إلى الضوضاء (SNR)، وكذلك الحد الأدنى المطلوب لنقاط العينة لتحقيق أداء محدد. و قد تمت جميع عمليات المحاكاة لتشمل معظم العوامل التي تؤثر على كفاءة منهجية الاستشعار المقترحة، مثل: عدد النوافذ المستخدمة (K)، عدد نقاط العينة (N)، واحتمالية الإنذار الكاذب (P_f). وقد تم أيضاً المقارنة مع طريقة كشف الطاقة. وتؤكد جميع نتائج المحاكاة والمقارنات أن النموذج المقترح موثوق به وقوي تحت جميع العوامل المستخدمة في المحاكاة.