The Generalized Weibull Family of Distribution

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The Generalized Weibull Family of Distribution

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Abstract

In this article the T-X family is introduced by givening the cumulative distribution function (CDF) R{W(F(x))}, where R is the CDF of a random variable T, F is the CDF of X and W is an increasing function defined on [0, 1] having the support of T as its range. This family provides a new method of generating univariate distributions. Different choices of the R, F and W functions give different families of distributions. We used the quintile functions to define the W function. Some general properties of this T-X system of distributions are studied. A new distribution of the T-X family is derived, namely, the Generalized Weibull Uniform Log Logistic Distribution (GRU {LL}) with four parameter. Different methods of estimation are used to estimate the unknown parameters in complete and censored random samples.

Key wards

T-X families, Weibull Distribution, Log logistic Distribution, Generalized Weibull Uniform Log Logistic Distribution, quintile function.

1. Introduction

Many existing well known distributions have been extensively used for modeling several areas of applications, such as medical, biological, economics studies and lifetime analysis. Despite the large number of distributions, it is not enough to meet the various real applications. The recent literature has suggested many ways of extending well known distributions for generating new generalized families. The generalized families are : the Pearson system [16],Burr system [3], Johnson system [9][10], Marshal Olkin [14], Lai [13], Lambda distribution (LD) Tukey [23], Generalized Lambda distribution (G L D) [11], Ramberg and

Schiser [17] [18], Fremier et al.[8], Karian and Dudewicz [11], Eugene, N., Lee, C., Famoye, F, Eygene et al.(2002) Beta-G [7], Jones[2;(2004)], The Kumaraswamy Weibull distribution [4], The Kumaraswamy generalized half-normal distribution [5], The Kumaraswamy generalized gamma distribution - Castro et al.(2011)[6], McDonald-G (5;Mc-G, Generalized beta-generated distributions - Alexander et al.[1], Alzaatreh et al.(2014) [2], gamma-G(type1) Zografos and Balakrishanan (2009) [24], gamma-G (type2) Ristic and Balakrishanan (2012) [19], gamma-G (type3) Tarabi Montazari (2012) [21], Logistic-G (Tarabi and Montazari,2014), transformed-T-X, Alzatreh et al .(2013) [2] , Logistic-X family of Distributions, Taher et al (2015b) [20].

2. Generating Families Of Continuous Probability Distributions:

Let T be a continuous probability random variable with PDF r(t) defined on $[m, n]$, i.e, the support of T is closed interval $[m, n]$, for $-\infty < m \leq n < \infty$, X be a continuous random variable with PDF f(x; δ) and CDF F(x; δ) depending on a vector parameter δ, and W[F(x; δ)] be a continuous function of F(x; δ).

let W[F(x)] Satisfying the following conditions:

1- $W[F(X;\delta)] \in [m, n]$, i.e W(.)is defined on the support of T.
2- $[FW(X;\delta)]$ is differentiable and monotonically increasing.
3- $W([FW(X;\delta)]) \to m$ as $x \to -\infty$ and $\to n$ as $x + \infty$

Alzaatreh et al.(2013) [2], defined the CDF of the T-X family of distributions by:

$$G(X;\delta) = \int_{m}^{W[F(X;\delta)]} r(t)dt \qquad (1)$$

If both Functions $W(.)$ and $F(.)$ are absolutely continuous, then $G(X;\delta)$ is absolutely continuous and has the PDF as:

$$g(X;\delta) = \frac{d\, G(X;\delta)}{dx}$$

Let y be a random variable with PDF $v(y)$ and CDF $V(y)$, its quintile function is $Q_y(\theta), \theta \in (0,1)$. If $V(y)$is continuous and strictly increasing, then $Q_y(\theta) = V^{-1}(\theta)$ is a continuous and strictly increasing. Taking $W(.)$ in (1) to be the quintile function of y, then the CDF $G(X;\delta)$ of (1) is defined by :

$$G(X;\delta) = \int_m^{Q_y[F(X;\delta)]} r(t)dt \qquad (2)$$

$$= R\{Q_y[F(X;\delta)]\}, \text{ where:}$$

$x \in [-\infty, \infty]$, R is the CDF of T,

and the corresponding PDF of (2) is:

$$g(X;\delta) = r\{W[F(X;\delta)]\}\{\frac{d}{dx}W[F(X;\delta)]\}$$

$$= \frac{f(X;\delta)}{v\{Q_y[F(X;\delta)]\}} r\{Q_y[F(X;\delta)]\} \qquad (3)$$

2.1 The Generalized Weibull Uniform $\{Log - Logistic\}$ Distribution GWU $\{LL\}$.

Let the random variable Y has the log – logistic {LL} distribution with two Parameters $(s^\backslash , c^\backslash)$, then the PDF $v(y)$ and the quintile function $Q_y(\theta)$ of y are:

$$v(y) = \frac{\left(\frac{s^\backslash}{c^\backslash}\right)\left(\frac{y}{c^\backslash}\right)^{s^\backslash - 1}}{\left[1 + \left(\frac{y}{c^\backslash}\right)^{s^\backslash}\right]^2} \quad , s^\backslash, c^\backslash > 0 \; ; \; y \geq 0 \; , \qquad (4)$$

$$Q_y(\theta) = c^{\backslash} \left(\frac{\theta}{1-\theta}\right)^{1/s^{\backslash}} \quad (5)$$

Substituting equations (4) and (5) into (3) we get the PDF $g(X;\delta)$ of the T-X {LL} family as:

$$g(X;\delta) = \frac{\left(\frac{c^{\backslash}}{s^{\backslash}}\right) f(X;\delta).r\left\{c^{\backslash}\left(\frac{F(X;\delta)}{1-F(X;\delta)}\right)^{\frac{1}{s^{\backslash}}}\right\}}{F(X;\delta)^{(s^{\backslash}-1)/s^{\backslash}}\left(1-F(X;\delta)\right)^{\frac{\left(s^{\backslash}+1\right)}{s^{\backslash}}}} \quad (6)$$

From (2) and (5) the CDF $G(X;\delta)$ is defined as :

$$G(X;\delta) = R\left\{c^{\backslash}\left(\frac{F(X;\delta)}{1-F(X;\delta)}\right)^{\frac{1}{s^{\backslash}}}\right\} \quad , \quad (7)$$

where R and F are the CDF`s of T and X respectively.

When $s^{\backslash} = c^{\backslash} = 1$ the PDF $g(X;\delta)$ and the CDF $G(X;\delta)$ of the T-X {LL} family reduced to:

$$g(X;\delta) = \frac{f(X;\delta)}{\left(1-F(X;\delta)\right)^2}.r\left\{\frac{F(X;\delta)}{1-F(X;\delta)}\right\} \quad \text{and} \quad (8)$$

$$G(X;\delta) = R\left\{\frac{F(X;\delta)}{1-F(X;\delta)}\right\} \quad (9)$$

Suppose that a random variable T has the standard Weibull distribution with the PDF $r(t)$ and CDF $R(t)$ with two Parameters c and s, and $f(X;\delta)$ and $F(X;\delta)$ are the PDF and the CDF of a uniform distribution with 2 parameters (a,b) , $-\infty \le a \le b \le \infty$, then:

$$r(t) = c\,s\,t^{c-1}\,e^{-st^{c}} \qquad (10)$$

where $s > 0$ is a scale parameter and $c > 0$ is a shape parameter, $t > 0$,

$$R(t) = 1 - e^{-st^{c}} \;, \qquad (11)$$

$$f(x) = \frac{1}{b-a} \;, \qquad a \leq x \leq b \;, \qquad (12)$$

$$F(x) = \frac{x-a}{b-a} \quad a \leq x \leq b \;. \qquad (13)$$

Then substituting in (10), (11) , (12) and (13) ,into (8),and (9) we obtain the PDF $g(x)$ and the CDF $G(x)$ of the Generalized Weibull Uniform {Log-Logistic} (GWU{LL}) distribution as :

$$g(x) = \frac{b-a}{(b-x)^2}\, c\, s \left(\frac{x-a}{b-x}\right)^{c-1} e^{\left\{-s\left(\frac{x-a}{b-x}\right)^{c}\right\}} \;, \qquad (14)$$

$$G(x) = 1 - e^{\left\{-s\left(\frac{x-a}{b-x}\right)^{c}\right\}} \;, \qquad (15)$$

where $s, c > 0\,, -\infty \leq a \leq x \leq b \leq \infty$.

The Survival function $S(x)$ and the hazard function $h(x)$ of the GWU{LL} distribution are defined as:

$$S(x) = 1 - G(x) \;, \qquad (16)$$

$$h(x) = \frac{g(x)}{1-G(x)} = \frac{g(x)}{S(x)} \qquad (17)$$

where $g(x)$ and $G(x)$ as given in (14) and (15).

Plots of the GWU{LL} density g(x) and hazard h(x) functions are given in Figures (1) and (2). The graphs in Figure (1) show that the GWU{LL} distribution can be wright skewed, bathtub shape, or unimodal. The graphs in Figure (2) show increasing failure which can be useful in analyzing various data set.

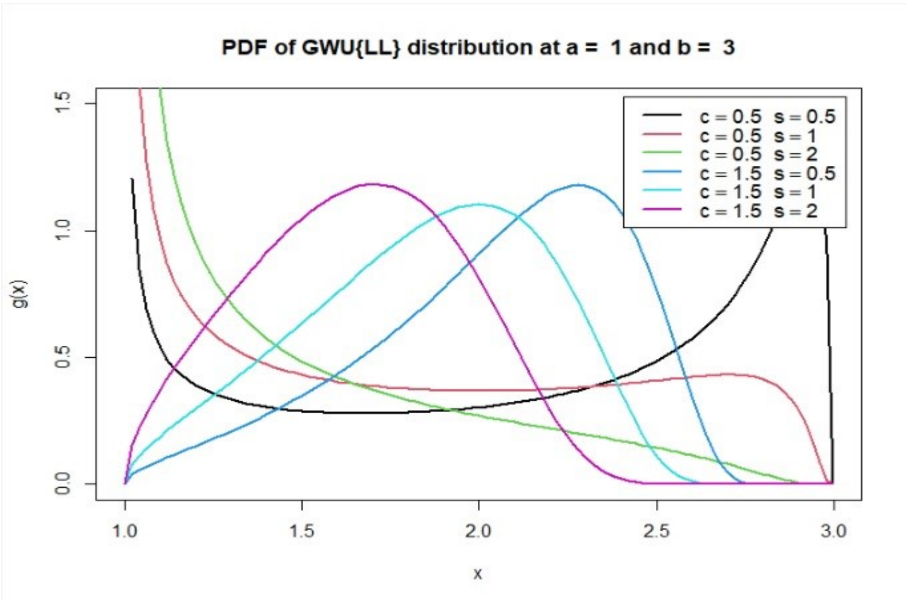

Figure (1): The GWU{LL} density function g(x)

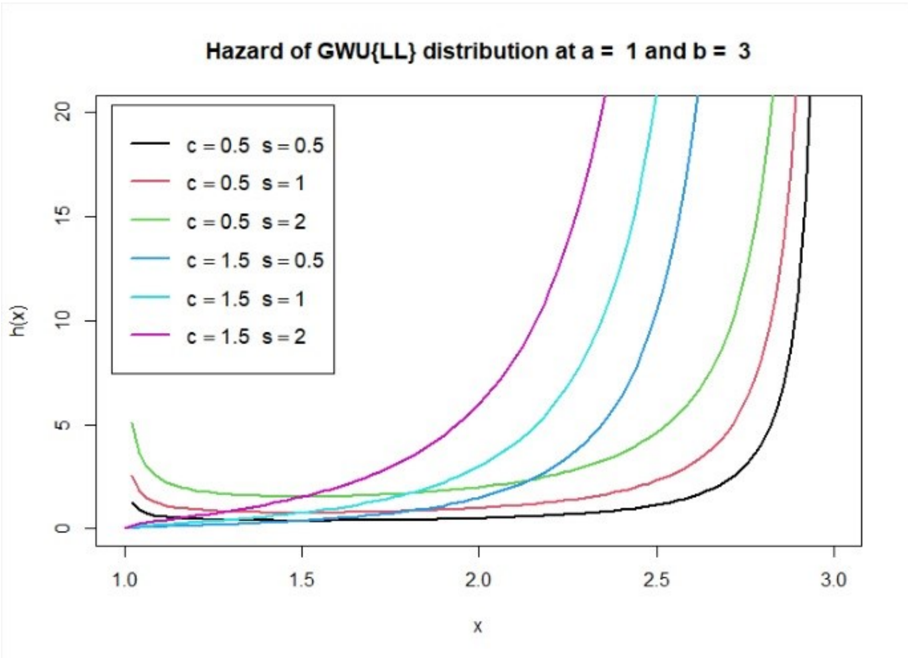

Figure (2): The GWU{LL} hazard function h(x)

2.2 Quantile and Generating Functions of GWU{LL} Distribution.

The CDF G(.) of the GWU{LL} distribution is given in (15), and hence the quantile function of the GWU{LL} can be written as:

$$G(Q(p)) = p\ ,$$

$$Q(p) = \frac{bz + a}{1 + z} \quad , \tag{18}$$

where

$$Z = \left[ln \left(\frac{1}{1-p} \right)^{\frac{1}{s}} \right]^{\frac{1}{c}}$$

and p is the vector of percentiles.

If we replaced p by a vector of Uniform (0,1) random variables we get, $Q(u)$, then we can use $Q(u)$ to generate random samples of GWU{LL} distribution.

The skewness and the kurtosis of the GWU{LL} distribution can be calculated using the Bowley skewness measure Bsk [12] and the Moor's kurtosis measure Mkur [15] and defined by :

$$\mathrm{Bsk} = \frac{Q_{0.75} - 2Q_{0.5} + Q_{0.25}}{Q_{0.75} - Q_{0.25}}$$

$$\mathrm{Mkur} = \frac{Q_{0.875} - Q_{0.625} - Q_{0.375} + Q_{0.125}}{Q_{0.75} - Q_{0.25}}$$

The above measures are less Sensitive to outliers. Table (1) given below gives the Bsk and Mkur of the GWU{LL}distribution for the different values of the parameters.

Table (1): The skewness and the kurtosis of the GWU{LL} distribution

Q.00	Q.25	Q.5	Q.75	Q.12	Q.375	Q.625	Q.875	Bskewness	Mkurtosis
parm1	1.14	1.3	1.43	1.01	1.23	1.37	1.51	-0.1049631	-0.2771996
parm2	0.94	1.14	1.31	0.78	1.05	1.22	1.41	-0.0863416	-0.2213147
parm3	1.02	1.18	1.3	0.9	0.99	1.24	1.38	-0.0968515	-0.254507
parm4	0.88	1.07	1.24	0.73	0.99	1.15	1.34	-0.0806048	-0.2057707
parm5	0.99	1.14	1.26	0.86	1.07	1.2	1.34	-0.0942984	-0.2474117
parm6	1.08	1.21	1.31	0.98	1.15	1.26	1.37	-0.1028613	-0.2735059
parm7	1.15	1.26	1.34	1.06	1.21	1.3	1.39	-0.1075713	-0.2880878

2.3 The Raw Moments.

The r^{th} raw moment of a random variable X having The GWU{LL} distribution is given as :

$$\mu_r^{\backslash} = E(x^r) = cs(b-a)\int_a^b \frac{x^r}{(b-x)^2}\left(\frac{x-a}{b-x}\right)^{c-1} \exp^{\left\{-s\left(\frac{x-a}{b-x}\right)^c\right\}} dx$$

The above integration can be easily calculated using the integer (.) function of the R software. Table (2) given below presents the first four raw moments (m_1 , m_2 ,m_3 , m_4), the variance (M_2) , the coefficient of variation (cv), the skewness (sk) and the kurtosis (kur) of the GWU{LL} distribution for different parameter values.

Table (2): The first four raw moments

	moments	**parm1**	**parm2**	**parm3**	**parm4**
1	mean	1.058981	1.12063726	0.68095403	0.8613142
2	m2	1.3316846	1.32124777	0.79661147	0.9492416
3	m3	1.7086185	1.59727421	0.97797871	1.1111625
4	m4	2.2274972	1.9704627	1.23991413	1.3547455
5	M2	0.2102438	0.06541992	0.33291307	0.2073795
6	CV	0.4329856	0.22823904	0.84732032	0.5287146
7	sk	-1.5238166	-1.7922586	-0.0930523	-0.6742021
8	kur	1.0147765	5.14308102	-1.6704448	-0.6592806

2.4 Mean Residual life (MRL)

For a random lifetime X, the mean residual life (MRL) or the life expectancy at age t is the expected additional life length for a unit which is a life at age t. The MRL has many important applications in fuzzy set engineering modeling, insurance assessment of human life expectancy, demography, and economic etc.. The MRL is the conditional expectation $E(x - t|x > t)$ where $t > 0$. The MRL function can be simply represented with the survival function $S(x)$. For a random lifetime X, the MRL is :

$$\mathrm{MRL} = \frac{1}{S(x)}\int_{t}^{\infty} S(x)dx \quad , \qquad S(x) > 0$$

When $S(0) = 1$ at $= 0$, the MRL equal the average lifetime. When the MRL is represented with $S(x)$ it is denoted by the theoretical MRL (TMRL), and when we calculate it from a random sample $x_1, x_2, .., x_n$ of size n of a GWU$\{LL\}|$ distribution using the following expression, it is called the empirical MRL (EMRL) which can be calculated as :

$$\mathrm{EMRL} = \frac{1}{(n-k)}\sum_{k=1}^{n-1}(x_{(k+1)} - x_{(k)})$$

Where $x_{(k)}$ is the k^{th} order statistic of the sample . Table(3) given below present the first 10 values (in table MRLa) and the last 10 values (in table MRLb) of the EMRL and TMRL when the values of the age t is the order statistics of a random sample of size 50 from a GWU$\{LL\}$distribution with parameters $a = 0$, $b = 3$, $c = 4$, and $s = 2$.

Table (3): The mean residual life (MRL)

MRLa				**MRLb**			
	Death.Time	EMRL	TMRL		Death.Time	EMRL	TMRL
1	0.7513064	0.5508368	0.5360701	41	1.513187	0.05284205	0.06399281
2	0.8305732	0.4813943	0.4657889	42	1.518133	0.05388263	0.06267947
3	0.842894	0.4790538	0.4551519	43	1.520368	0.05902647	0.06209298
4	0.976722	0.3527308	0.3456578	44	1.526271	0.06197743	0.06056379
5	1.0299766	0.3061312	0.3056489	45	1.528297	0.07194141	0.06004559
6	1.0351625	0.3077849	0.301875	46	1.564465	0.04471682	0.05135845
7	1.0406299	0.3093482	0.2979206	47	1.581494	0.03691699	0.04762468
8	1.0472063	0.3099806	0.2931973	48	1.584647	0.05064613	0.04695749
9	1.1143018	0.2488092	0.2471678	49	1.593631	0.08332443	0.0450969
10	1.1278793	0.2411125	0.2383499	50	1.676955	0.00000000	0.00000000

2.5 The Renyi Entropy

Entropy is a thermodynamic quantity used as a measure of uncertainty variation of systems . If X is a random variable has the GWU{LL} distribution then the Renyi entropy of X is defined as:

$$R(p) = \frac{1}{1-p} \ln\left[\int_0^\infty g^p(x)dx\right],$$

where $g(x)$ is given by (14).The value of $R(p)$can be evaluated using the integrate(.) function of the R software. The values of R(P) for different values of the parameters c, s, a and b and for $p = 2,3$ and 4 are presented in Table (4) given below:

Table (4): The value of $R(p)$for different values of the parameters

Column1	**par**	**p1**	**p2**	**p3**
1	par1	1.2446781	3.5727358	11.5969903
2	par2	1.4024262	4.0228459	13.3895997
3	par3	2.3603439	4.163993	14.1605476
4	par4	2.0580426	12.863749	93.8015588
5	par5	0.6780948	0.5239334	0.4501118
6	par6	0.422858	0.2730861	0.2087424
7	par7	0.9539423	1.5673198	2.9007317

3. Methods of Estimation and Simulation Study:

In this section we estimate the parameters a, b, c and s of the GWU{LL} distribution by five different methods of estimation using complete sampling technique. These methods are: maximum likelihood estimation (MLE), least – squares (LS) and weighted least – squares (WLS) estimation, percentile based estimation , and product spacing (MPS) estimation [GWU{LL}_3Parameter].

The performance of all methods are studied through Monte Carlo simulations using Mathcad 14 software.

The aim of this section is to compare the performance of the methods of estimation, namely: MLE, MPS, LS, WLS, and PE for the GWU_LL distribution which discussed in the previous section. A Monte Carlo study is employed to check the behavior of the proposed methods of estimation. The Monte Carlo process is carried by generating 5000 random data from the GWU{LL} distribution with the following assumptions:

1. Sample sizes are $n = 20, 50, 100$.
2. Assuming the following selected cases of parameters a and b of the GWU_LL distribution:

   $\alpha = 0.2, \quad b = 1$

   $a = 0.5, \quad b = 1.5$
3. Assuming the following selected cases of parameters c and s of the GWU_LL distribution:
   a. $c = 0.5,\ s = 0.5$
   b. $c = 0.5,\ s = 1.5$
   c. $c = 1.5,\ s = 0.5$
   d. $c = 1.5,\ s = 1.5$
   e. $c = 2.5,\ s = 2.5$

Based on the generated data and applying different methods of estimation, all the means square error (MSE) and relative biases (RB) are represented in Tables (3.1), (3.2), (3.3), (3.5) and (3.6) for six different methods of estimation.

Table (3.1): The MSE and RB for different estimates of the GWU_LL distribution with parameters ($a = 0.2, b = 1$) and different values for $c,\ s$ at sample size $n = 25$.

Parameters	MLE		MPS		LS		WLS		PE	
	MSE	RB	MSE	RB	MSE	RB	MSE	RB	MSE	RB
Case I:										
$a = 0.2$	0.0005	0.0487	0.0005	0.0482	8.5817	0.7606	0.2876	0.1927	0.0075	0.0243
$b = 1$	0.0002	0.0069	0.0004	0.0093	0.1692	0.0644	0.0368	0.0257	0.0052	0.0076
$c = 0.5$	0.0075	0.0489	0.0126	0.0518	0.2110	0.2135	0.0663	0.1156	0.0542	0.0109
$s = 0.5$	0.0176	0.0453	0.0185	0.0214	0.1880	0.0969	0.0539	0.0526	0.0373	0.0630
Case II:										
$a = 0.2$	0.0000	0.0057	0.0000	0.0057	0.0542	0.0392	0.0001	0.0096	0.0003	0.0032
$b = 1$	0.0076	0.0270	0.1488	0.0501	4.8805	0.6775	3.3851	0.4671	1.8279	0.2816
$c = 0.5$	0.0074	0.0304	0.0125	0.0750	0.0434	0.1102	0.0331	0.0834	0.0410	0.0216
$s = 1.5$	0.0471	0.0354	0.5510	0.0260	16.2229	0.8932	11.3440	0.6229	9.3180	0.4738
Case III:										
$a = 0.2$	8.0566	0.8209	9.1521	0.1836	33.7605	5.7962	15.7247	39.5068	27.1965	8.0615
$b = 1$	0.2417	0.0409	0.1677	0.0637	2.6270	0.4277	3.2632	0.5064	1.8212	0.3036
$c = 1.5$	1.7211	0.2746	0.7850	0.2033	12.9526	0.6830	14.5746	0.7902	8.1506	0.4505
$s = 0.5$	0.8035	0.4782	0.5757	0.3490	1.3434	0.7193	1.9640	0.9089	1.8374	0.7849
Case IV:										
$a = 0.2$	0.0066	0.2518	0.0187	0.2666	6.6777	4.7650	11.7866	12.7757	13.2769	1.1274
$b = 1$	0.0806	0.1087	0.0659	0.1441	2.0367	0.3364	1.6064	0.3186	0.8069	0.2098
$c = 1.5$	0.5080	0.3302	0.2668	0.2296	7.9589	0.4487	7.0065	0.3927	3.2787	0.1417
$s = 1.5$	9.9330	0.1537	4.1250	0.0386	7.8725	0.5899	13.7248	1.0131	18.0730	1.1372
Case V:										
$a = 0.2$	6.7189	0.5043	0.9413	0.0946	22.0078	5.8652	191.8819	5.5905	63.8579	5.7428
$b = 1$	0.6671	0.1225	0.4592	0.1772	3.4620	0.5359	3.5192	0.5382	4.4431	0.5365
$c = 2.5$	8.0814	0.2956	4.9700	0.3097	49.0921	0.9225	49.7105	0.8942	38.9887	0.6422
$s = 2.5$	33.0628	1.0291	13.8895	0.2112	16.2477	1.3181	23.3900	0.4637	31.0362	0.5224

Table (3.2): The MSE and RB for different estimates of the GWU_LL distribution with parameters ($a = 0.2, b = 1$) and different values for c, s at sample size $n = 50$.

Parameters	MLE		MPS		LS		WLS		PE	
	MSE	RB	MSE	RB	MSE	RB	MSE	RB	MSE	RB
Case I:										
$a = 0.2$	0.0000	0.0126	0.0000	0.0125	0.0144	0.1048	0.0002	0.0161	0.0006	0.0039
$b = 1$	0.0000	0.0027	0.0001	0.0038	0.0105	0.0142	0.0004	0.0050	0.0011	0.0015
$c = 0.5$	0.0033	0.0190	0.0048	0.0421	0.0390	0.0791	0.0108	0.0383	0.0149	0.0009
$s = 0.5$	0.0087	0.0282	0.0093	0.0196	0.0186	0.0099	0.0112	0.0115	0.0129	0.0226
Case II:										
$a = 0.2$	0.0000	0.0014	0.0000	0.0014	0.0000	0.0074	0.0000	0.0017	0.0001	0.0012
$b = 1$	0.0020	0.0121	0.0048	0.0327	1.3421	0.2711	0.4171	0.1188	0.1341	0.0696
$c = 0.5$	0.0028	0.0080	0.0048	0.0402	0.0157	0.0560	0.0103	0.0337	0.0181	0.0131
$s = 1.5$	0.0281	0.0242	0.0750	0.0090	5.2282	0.3929	1.9997	0.2013	1.4409	0.1477
Case III:										
$a = 0.2$	0.1372	0.2028	0.0716	0.3072	26.5802	9.6227	15.0095	22.4117	20.2250	5.4408
$b = 1$	0.1460	0.0342	0.0738	0.0532	2.2004	0.3715	2.4308	0.3801	1.3863	0.2391
$c = 1.5$	0.8795	0.1894	0.3323	0.1644	11.4274	0.6434	10.7999	0.6077	5.4679	0.3550
$s = 0.5$	0.4552	0.2749	0.2690	0.1809	0.7909	0.5297	0.8892	0.5756	1.1211	0.6389
Case IV:										
$a = 0.2$	0.0019	0.1542	0.0020	0.1790	4.6254	1.3140	13.2265	2.5300	0.0478	0.0961
$b = 1$	0.0675	0.0845	0.0409	0.1122	0.8472	0.1811	0.3226	0.1210	0.1407	0.1102
$c = 1.5$	0.2845	0.2221	0.1780	0.1792	5.2829	0.3105	1.5319	0.1445	0.4969	0.0557
$s = 1.5$	10.5160	0.1937	4.6273	0.0288	4.1970	0.4154	8.0444	0.7306	13.2795	1.0038
Case V:										
$a = 0.2$	2.9855	0.1156	0.5483	0.2034	14.6947	4.3224	18.6796	2.7263	12.6554	3.8113
$b = 1$	0.2935	0.1157	0.2449	0.1782	2.7444	0.4467	2.1252	0.3447	2.4909	0.3179
$c = 2.5$	4.4937	0.2468	1.9155	0.3139	41.0502	0.8243	34.0735	0.6259	23.0253	0.4101
$s = 2.5$	22.4126	0.3184	9.4647	0.2076	10.9168	0.2518	15.2256	0.3666	22.0103	0.5483

Table (3.3): The MSE and RB for different estimates of the GWU_LL distribution with parameters $(a = 0.2, b = 1)$ and different values for c, s at sample size $n = 100$.

Parameters	**MLE**		**MPS**		**LS**		**WLS**		**PE**	
	MSE	**RB**	**MSE**	**RB**	**MSE**	**RB**	**MSE**	**RB**	**MSE**	**RB**
Case I:										
$\boldsymbol{a = 0.2}$	0.0000	0.0032	0.0000	0.0032	0.0003	0.0261	0.0000	0.0031	0.0002	0.0021
$\boldsymbol{b = 1}$	0.0000	0.0009	0.0000	0.0012	0.0003	0.0053	0.0001	0.0020	0.0004	0.0007
$\boldsymbol{c = 0.5}$	0.0012	0.0055	0.0019	0.0304	0.0060	0.0330	0.0036	0.0166	0.0067	0.0004
$\boldsymbol{s = 0.5}$	0.0039	0.0162	0.0045	0.0106	0.0060	0.0053	0.0051	0.0063	0.0064	0.0121
Case II:										
$\boldsymbol{a = 0.2}$	0.0000	0.0004	0.0000	0.0004	0.0000	0.0025	0.0000	0.0003	0.0001	0.0010
$\boldsymbol{b = 1}$	0.0007	0.0065	0.0016	0.0194	0.3195	0.0912	0.0147	0.0262	0.0075	0.0158
$\boldsymbol{c = 0.5}$	0.0011	0.0004	0.0021	0.0251	0.0055	0.0258	0.0035	0.0138	0.0072	0.0020
$\boldsymbol{s = 1.5}$	0.0170	0.0133	0.0293	0.0026	1.2708	0.1294	0.1422	0.0471	0.0972	0.0292
Case III:										
$\boldsymbol{a = 0.2}$	0.0261	0.1501	0.0032	0.2173	3.9953	4.4925	7.9119	1.5201	0.7230	0.3610
$\boldsymbol{b = 1}$	0.0321	0.0283	0.0075	0.0450	1.4681	0.2879	0.9999	0.1903	0.1423	0.0718
$\boldsymbol{c = 1.5}$	0.3226	0.1174	0.1143	0.1237	9.0694	0.5675	4.8807	0.3314	1.2228	0.1355
$\boldsymbol{s = 0.5}$	0.1465	0.1264	0.0709	0.0614	0.4241	0.3377	0.3459	0.3055	0.4556	0.3659
Case IV:										
$\boldsymbol{a = 0.2}$	0.0007	0.0877	0.0008	0.1121	2.6741	0.5438	0.0045	0.0640	0.0016	0.0389
$\boldsymbol{b = 1}$	0.0385	0.0475	0.0240	0.0806	0.4606	0.1139	0.0509	0.0686	0.0626	0.0887
$\boldsymbol{c = 1.5}$	0.1370	0.1257	0.0960	0.1264	2.4079	0.1787	0.2463	0.0613	0.1733	0.0476
$\boldsymbol{s = 1.5}$	5.8627	0.1530	2.3973	0.0591	2.8512	0.3653	4.8329	0.5533	7.7960	0.7680
Case V:										
$\boldsymbol{a = 0.2}$	0.1472	0.1392	0.1585	0.2463	2.0425	2.3634	4.3049	2.4851	0.6575	0.3710
$\boldsymbol{b = 1}$	0.0789	0.0801	0.0582	0.1516	1.5282	0.2956	0.5910	0.1299	0.2500	0.0802
$\boldsymbol{c = 2.5}$	0.7922	0.1820	0.8462	0.2580	3.6395	0.6065	1.6096	0.2841	3.1768	0.1222
$\boldsymbol{s = 2.5}$	1.0986	0.5043	1.6932	0.0834	7.3349	0.2407	8.7977	0.3339	1.6735	0.6021

Table (3.4): The MSE and RB for different estimates of the GWU_LL distribution with parameters ($a = 0.5, b = 1.5$) and different values for c, s at sample size $n = 25$.

Parameters	MLE		MPS		LS		WLS		PE	
	MSE	RB	MSE	RB	MSE	RB	MSE	RB	MSE	RB
Case I:										
$a = 0.5$	0.0009	0.0254	0.0010	0.0250	4.3257	0.2925	1.6109	0.1314	0.3181	0.0279
$b = 1.5$	0.0003	0.0054	0.0008	0.0070	0.5795	0.0608	0.0306	0.0208	0.0089	0.0068
$c = 0.5$	0.0080	0.0513	0.0154	0.0614	0.1351	0.1921	0.1013	0.1303	0.0577	0.0154
$s = 0.5$	0.0203	0.0547	0.0194	0.0167	0.1898	0.0972	0.0705	0.0566	0.0486	0.0639
Case II:										
$a = 0.5$	0.0000	0.0028	0.0000	0.0028	2.4025	0.0603	0.0001	0.0047	0.0005	0.0012
$b = 1.5$	0.0111	0.0224	0.1059	0.0474	9.3167	0.5572	6.6423	0.4185	3.6390	0.2542
$c = 0.5$	0.0071	0.0343	0.0125	0.0748	0.0504	0.1081	0.0321	0.0773	0.0413	0.0232
$s = 1.5$	0.0503	0.0374	0.4018	0.0238	13.9045	0.7779	12.3367	0.6292	9.1616	0.4737
Case III:										
$a = 0.5$	25.2952	0.1502	34.3412	0.0643	61.1782	16.5225	22.9209	9.4561	76.6203	13.2825
$b = 1.5$	0.3525	0.0320	0.1648	0.0566	5.6066	0.3940	5.2613	0.4140	4.2529	0.2887
$c = 1.5$	1.5079	0.2883	0.6418	0.2021	13.0187	0.6831	14.2228	0.7684	7.9837	0.4318
$s = 0.5$	0.7581	0.4946	0.6871	0.3920	1.3579	0.7481	2.0231	0.9231	2.0170	0.8097
Case IV:										
$a = 0.5$	0.0078	0.1270	0.0563	0.1269	71.0813	3.7432	90.3400	1.9386	3.3862	0.3891
$b = 1.5$	0.1254	0.0922	0.1347	0.1191	4.9781	0.3358	2.7488	0.2820	1.3908	0.1801
$c = 1.5$	0.4526	0.3353	0.3028	0.2278	9.7875	0.5352	7.9186	0.4318	3.4450	0.1673
$s = 1.5$	9.2111	0.1401	4.0315	0.0549	7.3622	0.5639	14.8884	1.0504	18.1780	1.1349
Case V:										
$a = 0.5$	9.5434	0.2150	13.3181	0.2100	20.4341	17.1167	16.5544	2.1983	11.1771	5.3925
$b = 1.5$	1.9863	0.0901	1.4107	0.1234	5.1027	0.4359	4.9454	0.4149	10.2774	0.5321
$c = 2.5$	6.2303	0.3431	5.6775	0.3134	53.3550	0.9415	46.2900	0.8415	38.8795	0.6113
$s = 2.5$	32.6554	0.0027	16.0837	0.1908	14.9920	0.2881	23.8359	0.5039	35.0043	0.5995

Table (3.5): The MSE and RB for different estimates of the GWU_LL distribution with parameters $(a = 0.5, b = 1.5)$ and different values for c, s at sample size $n = 50$.

Parameters	MLE		MPS		LS		WLS		PE	
	MSE	RB	MSE	RB	MSE	RB	MSE	RB	MSE	R
Case I:										
$a = 0.5$	0.0001	0.0064	0.0001	0.0064	0.0780	0.0578	0.0006	0.0087	0.0009	0.00
$b = 1.5$	0.0000	0.0020	0.0001	0.0028	0.0790	0.0151	0.0008	0.0045	0.0017	0.00
$c = 0.5$	0.0030	0.0167	0.0050	0.0453	0.0424	0.0801	0.0113	0.0397	0.0153	0.00
$s = 0.5$	0.0087	0.0344	0.0091	0.0180	0.0179	0.0119	0.0112	0.0123	0.0131	0.0
Case II:										
$a = 0.5$	0.0000	0.0007	0.0000	0.0007	0.0000	0.0036	0.0000	0.0008	0.0002	0.00
$b = 1.5$	0.0034	0.0093	0.0071	0.0283	2.5472	0.2337	0.5946	0.0914	0.1667	0.0
$c = 0.5$	0.0027	0.0064	0.0050	0.0425	0.0155	0.0546	0.0101	0.0329	0.0180	0.0
$s = 1.5$	0.0279	0.0270	0.0676	0.0080	5.2498	0.3832	1.8981	0.1864	1.2636	0.1
Case III:										
$a = 0.5$	0.2714	0.1000	0.9654	0.1398	28.5381	5.5296	13.8863	91.7671	46.2798	2.04
$b = 1.5$	0.2849	0.0274	0.0336	0.0469	4.1548	0.3251	3.9529	0.3197	2.1483	0.2
$c = 1.5$	0.7700	0.1923	0.3801	0.1651	10.7842	0.6381	9.1593	0.5747	7.0997	0.3
$s = 0.5$	0.4208	0.2805	0.3360	0.2108	0.7459	0.5140	0.9495	0.6062	1.2397	0.6
Case IV:										
$a = 0.5$	0.0030	0.0749	0.0031	0.0877	13.9461	0.6743	1.3216	0.2043	0.0941	0.0
$b = 1.5$	0.1009	0.0726	0.0637	0.0946	1.4836	0.1575	0.6803	0.1086	0.2369	0.0
$c = 1.5$	0.2902	0.2241	0.1795	0.1791	5.2556	0.3244	1.2701	0.1384	0.4174	0.0
$s = 1.5$	8.9689	0.1459	4.4971	0.0333	4.3222	0.4269	8.6966	0.7567	13.1996	0.9
Case V:										
$a = 0.5$	1.3265	0.0173	2.0772	0.0664	31.7059	7.2172	33.0424	1.3550	15.4927	1.2
$b = 1.5$	0.6525	0.0913	0.4676	0.1451	4.9395	0.3720	3.4586	0.2708	4.1280	0.2
$c = 2.5$	3.1312	0.2644	2.0404	0.3120	38.4116	0.7787	28.7006	0.5765	19.7706	0.3
$s = 2.5$	41.3766	0.3188	16.5263	0.1777	11.1977	0.2788	14.6505	0.4032	24.5421	0.6

Table (3.6): The MSE and RB for different estimates of the GWU_LL distribution with parameters ($a = 0.5, b = 1.5$) and different values for c, s at sample size $n = 100$.

Parameters	MLE		MPS		LS		WLS		PE	
	MSE	RB	MSE	RB	MSE	RB	MSE	RB	MSE	RB
Case I:										
$a = 0.5$	0.0000	0.0016	0.0000	0.0016	0.0005	0.0129	0.0000	0.0015	0.0004	0.0016
$b = 1.5$	0.0000	0.0007	0.0000	0.0010	0.0004	0.0043	0.0001	0.0016	0.0007	0.0006
$c = 0.5$	0.0010	0.0038	0.0019	0.0316	0.0059	0.0327	0.0035	0.0165	0.0066	0.0010
$s = 0.5$	0.0037	0.0165	0.0044	0.0114	0.0059	0.0051	0.0050	0.0061	0.0063	0.0123
Case II:										
$a = 0.5$	0.0000	0.0002	0.0000	0.0002	0.0000	0.0013	0.0000	0.0001	0.0001	0.0004
$b = 1.5$	0.0011	0.0050	0.0025	0.0159	0.4694	0.0734	0.0173	0.0207	0.0123	0.0138
$c = 0.5$	0.0011	0.0018	0.0020	0.0242	0.0055	0.0237	0.0036	0.0118	0.0073	0.0015
$s = 1.5$	0.0162	0.0156	0.0306	0.0006	0.9412	0.1209	0.1228	0.0460	0.1036	0.0317
Case III:										
$a = 0.5$	0.0051	0.0830	0.0050	0.1094	28.4671	8.8170	10.2986	3.6626	13.0675	0.4972
$b = 1.5$	0.0161	0.0256	0.0115	0.0367	2.7863	0.2347	1.8207	0.1614	0.2752	0.0619
$c = 1.5$	0.1698	0.1197	0.1113	0.1215	7.1659	0.5138	4.8417	0.3379	1.1492	0.1359
$s = 0.5$	0.1551	0.1297	0.0816	0.0668	0.4177	0.3298	0.3877	0.3226	0.4176	0.3596
Case IV:										
$a = 0.5$	0.0011	0.0439	0.0013	0.0559	1.7980	0.2278	0.0044	0.0317	0.0025	0.0193
$b = 1.5$	0.0584	0.0415	0.0387	0.0684	0.5259	0.0855	0.0766	0.0551	0.0977	0.0720
$c = 1.5$	0.1373	0.1277	0.0979	0.1283	2.2875	0.1751	0.2032	0.0592	0.1714	0.0455
$s = 1.5$	6.0727	0.1437	2.2664	0.0717	2.7448	0.3622	4.5786	0.5377	7.8700	0.7570
Case V:										
$a = 0.5$	0.0286	0.0818	5.5537	0.0691	52.2586	1.2591	9.8337	0.4694	2.4230	0.2312
$b = 1.5$	0.1079	0.0659	0.0906	0.1248	2.8865	0.2684	1.3381	0.1231	0.5459	0.0732
$c = 2.5$	1.0996	0.1781	1.3777	0.2507	32.6458	0.6322	11.6240	0.2619	3.7650	0.1290
$s = 2.5$	42.6185	0.5216	18.1413	0.0657	8.3051	0.2860	8.5949	0.3238	16.3701	0.6172

From the above tabulated results, one can indicate that the ordering of performance of estimators in terms of MSEs (from best to worst) is (MLE, MPS, WLS, LS, PE).

4. Estimation under Type-I and Type-II censoring

We use censoring types I and II to estimate WULL parameters using the MLE method.

For a type-I censoring, a sample of n units is followed for a fixed time xc. The number of units for which the event is occurred is random, but the total duration of the study is fixed.

Under censoring of type-II, a sample of n units is followed until "r" units have failed. The number of failures "r" determines the precision of the study and is fixed in advance. The total duration of the study is random and cannot be known with certainty in advance.

Suppose that we have n units with lifetimes have a survival function S(x), with associated density function $g(x)$ and hazard function $h(x)$ where:

$$S(x) = 1 - G(x) \,,$$

$$h(x) = \frac{g(x)}{S(x)} \,,$$

where $G(x)$ and $g(x)$ are as given by (15) and (14).

Suppose also that the observation time of unit "i" is a fixed time "xc", for $i = 1,2,\ldots,n$. If the ith unit died at time xc, where $x_i \leq xc$, its contribution to the likelihood function is the density $g_{(x_i)}$ which can be written as $L_i = g_{(x_i)} = S(x_i)\, h(x_i)$،

If the unit is still alive at xc , all we know is that its lifetime exceeds xc, and its contribution to the likelihood function is the probability of this event as:

$$L_i = P_r\,(x > xc) = S\,(xc),$$

Both types of contributions share the survival function S(.), so we can write the two contributions in a single expression. Let d_i be a death indicator, taking the value 1 if unit "i" died $(x_i \leq xc)$ and the value 0 otherwise, then the likelihood function can be written as follows:

$L = \prod_{i=1}^{n} h(x_i)^{d_i} S(x_i).$

4.1 The likelihood function for censoring of Type-I

In censoring of type-I, unit "i" is followed for a fixed time xc_i, i = 1, 2, …, n, where n is the sample size and the number of failures in the sample, say s units, is random, where, $s = \sum_{i=1}^{n} di$. $S(x)$ and $h(x)$ are as given by (16) and (17).

The log likelihood function is:

$$LL1 = \sum_{i=1}^{n} di\, \ell n\big(g(x_i)\big) + (n - s)\, \ell n(S(xc)) \,,$$

$S(x)$ and $h(x)$ are as given by (16) and (17).

$d_i = 1$ for $x_i \leq xc$,

$d_i = 0$ for $x_i > xc$,

$s = \sum_{i=1}^{n} di$.

The fixed time xc can be determined as follows:

$$G(xc) = q \quad , \quad 0 < q < 1$$

where q is called a censoring type-I parameter which determines the precision of the study and the above equation can be solve using equation (18).

Taking the partial derivatives of the log likelihood function "LL1" with respect to the parameters, setting these derivatives to zero and solving the resulting equations simultaneously yields the maximum likelihood estimates of the parameters. But the equations cannot be solved analytically so we used the R statistical package to solve them numerically.

4.2 The likelihood function for censoring of Type-II

In censoring of type-II, the sample is followed until "r" units have failed. The number of failures "r" determines the precision of the study and is fixed in advance, and the likelihood function is:

$$L = \prod_{i=1}^{n} g_{(x_i)}^{d_i} \, S_{(x_i)}^{1-d_i}$$

Let $x_{(1)}, x_{(2)}, \dots, x_{(n)}$ be the sample order statistics, then the log likelihood function can be written from the above equation as:

$$LL2 = \sum_{i=1}^{n} d_i \, \ell n \, g\left(x_{(i)}\right) + (n-r) \, \ell n \, S(x_{(r)}) \, ,$$

where;

$d_i = 1$ for $i \le r$

$d_i = 0$ for $i > r$

Since r determines the precision of the study, then "r" can be taken as the greatest integer less than pn where P , $0 < P < 1$, is the precision of the study and can be fixed in advance and n is the sample size. The maximum likelihood estimates of the parameters can be obtained exactly as the censoring type-I case.

4.3 Data Analysis and Simulation Study

In this section, we employ a Monte Carlo simulation study to generating data from GWU_LL(α, b, c, s) distribution under Type-I and Type-II censoring schemes for different choice of n and various censroing schemes. We suppose the following information about simulation:

Number of simulation 10000

Sample size: $n = 25, 50, 100, 200$

Initial parameters of GUW_LL:

$(\alpha, b, c, s) = (0.25, 1.75, 0.5, 0.5)$

$(\alpha, b, c, s) = (0.25, 1.75, 0.5, 1.5)$

$(\alpha, b, c, s) = (0.25, 1.75, 1.5, 0.5)$

$(\alpha, b, c, s) = (0.25, 1.75, 2.5, 2.5)$

Supposed schemes for Type-II censoring: $q_r = 30\%, 60\%, 90\%$

Supposed schemes for Type-I censoring: $q_t = 25\%, 50\%, 75\%$

where q_r represent a fraction (in %) of time censoring and q_t represent a fraction of number of failure items.

The average estimate, interval estimate, means square error (MSE) and average interval length (AIL) with coverage percentage (CP) are reported for both type censoring schemes (Type-II and Type-I) are in two parts:

4.3.1 Part I: Results of Type-II censoring schemes

Table (4.1.a): Avg. and MSEs estimated values of the GWU_LL distribution with parameters: $(a = 0.25, b = 1.75, \mathrm{c} = 0.5, \mathrm{s} = 0.5)$ under Type-II censoring with different cases of q_r and sample sizes n

n	Param	$q_r = 30\%$		$q_r = 60\%$		$q_r = 90\%$	
		Avg.	**MSE**	**Avg.**	**MSE**	**Avg.**	**MSE**
50	a	0.2547	1.29E-04	0.2545	1.12E-04	0.2548	1.37E-04
	b	1.8130	0.1584	1.7625	0.0126	1.7464	6.17E-04
	c	0.4683	0.0166	0.4934	0.0063	0.4961	0.0035
	s	0.5440	0.0209	0.5341	0.0115	0.5216	0.0090
100	a	0.2512	9.25E-06	0.2512	7.90E-06	0.2512	8.99E-06
	b	1.8110	0.0819	1.7662	0.0069	1.7494	1.97E-04
	c	0.4894	0.0071	0.5012	0.0029	0.4998	0.0013
	s	0.5366	0.0107	0.5234	0.0056	0.5127	0.0038
200	a	0.2503	5.98E-07	0.2503	4.29E-07	0.2503	4.32E-07
	b	1.8160	0.0694	1.7611	0.0034	1.7504	6.96E-05
	c	0.4982	0.0031	0.5025	0.0012	0.5005	4.17E-04
	s	0.5279	0.0062	0.5130	0.0021	0.5081	0.0015

Table (4.1.b): Avg. and MSEs estimated values of the GWU_LL distribution with parameters: $(a = 0.25, b = 1.75, \mathrm{c} = 0.5, \mathrm{s} = 1.5)$ under Type-II censoring with different cases of q_r and sample sizes n

n	Param	$q_r = 30\%$		$q_r = 60\%$		$q_r = 90\%$	
		Avg.	**MSE**	**Avg.**	**MSE**	**Avg.**	**MSE**
50	a	0.2505	1.58E-06	0.2506	1.82E-06	0.2505	1.43E-06
	b	1.8478	0.3667	1.7945	0.1056	1.7540	0.0487
	c	0.4707	0.0073	0.4890	0.0053	0.4986	0.0031
	s	1.5675	0.1010	1.5691	0.0541	1.5600	0.0400
100	a	0.2501	9.26E-08	0.2501	9.84E-08	0.2501	1.09E-07
	b	1.8299	0.2956	1.8089	0.3065	1.7610	0.0120
	c	0.4904	0.0028	0.4972	0.0021	0.5030	0.0013
	s	1.5606	0.0589	1.5530	0.0442	1.5452	0.0186
200	a	0.2500	5.71E-09	0.2500	6.91E-09	0.2500	6.95E-09
	b	1.8152	0.1726	1.7991	0.0652	1.7666	6.11E-03
	c	0.4987	9.69E-04	0.5009	7.75E-04	0.5035	4.66E-04
	s	1.5484	0.0345	1.5422	0.0231	1.5375	0.0111

Table (4.1.c): Avg. and MSEs estimated values of the GWU_LL distribution with parameters: $(a = 0.25, b = 1.75, \mathrm{c} = 1.5, \mathrm{s} = 0.5)$ under Type-II censoring with different cases of q_r and sample sizes n

n	Param	$q_r = 30\%$		$q_r = 60\%$		$q_r = 90\%$	
		Avg.	**MSE**	**Avg.**	**MSE**	**Avg.**	**MSE**
50	a	0.3849	0.0257	0.3797	0.0260	0.3700	0.0248
	b	1.6120	1.1145	1.5614	0.3640	1.6390	0.2149
	c	0.8670	0.9600	0.9897	0.6395	1.1266	0.5162
	s	0.4705	0.2779	0.6170	1.0246	0.6680	0.9095
100	a	0.3348	0.0115	0.3242	0.0108	0.3182	0.0106
	b	1.5291	0.5730	1.6210	0.3419	1.6815	0.1180
	c	0.9741	0.4655	1.1326	0.3645	1.2779	0.3951
	s	0.5437	1.1875	0.7295	1.8852	0.6248	0.5696
200	a	0.3012	0.0047	0.2930	0.0043	0.2888	0.0041
	b	1.5347	0.6852	1.6717	0.2381	1.7089	0.0454
	c	1.0971	0.4337	1.2684	0.2068	1.3734	0.1041
	s	0.6409	1.8601	0.6665	1.1531	0.5620	0.2276

Table (4.1.d): Avg. and MSEs estimated values of the GWU_LL distribution with parameters: $(a = 0.25, b = 1.75, c = 2.5, s = 2.5)$ under Type-II censoring with different cases of q_r and sample sizes n

n	Param	$q_r = 30\%$		$q_r = 60\%$		$q_r = 90\%$	
		Avg.	MSE	Avg.	MSE	Avg.	MSE
50	a	0.3930	0.0300	0.3892	0.0292	0.3832	0.0290
	b	1.9794	7.1708	1.4516	2.6035	1.4703	1.3074
	c	2.2090	16.5640	1.8554	9.8940	1.7204	3.6398
	s	1.5996	15.2089	2.0638	22.1536	3.0528	38.1347
100	a	0.3432	0.0171	0.3427	0.0166	0.3317	0.0153
	b	1.4736	1.6143	1.4309	1.2081	1.5418	0.5370
	c	2.0555	8.0518	1.7806	3.0733	1.9262	1.1219
	s	1.8353	17.2999	2.5903	27.5992	3.4166	35.8636
200	a	0.3086	0.0093	0.3101	0.0088	0.3002	0.0079
	b	1.4403	0.6374	1.4990	0.2990	1.6197	0.1874
	c	1.9925	2.1418	1.9411	0.8496	2.1284	0.5162
	s	2.2694	17.6488	3.0053	26.4246	3.7453	35.2647

4.3.2 Part II: Results of Type-I censoring schemes

Table (4.2.a): Avg. and MSEs estimated values of the GWU_LL distribution with parameters: $(a = 0.25, b = 1.75, c = 0.5, s = 0.5)$ under Type-I censoring with different cases of q_t and sample sizes n

n	Param	$q_t = 30\%$		$q_t = 60\%$		$q_t = 90\%$	
		Avg.	MSE	Avg.	MSE	Avg.	MSE
50	a	0.2518	0.0068	0.2519	0.0066	0.2515	0.0041
	b	1.8396	0.3254	1.8238	0.1846	1.7798	0.0724
	c	0.4360	0.1295	0.4761	0.0832	0.4902	0.0453
	s	0.5657	0.1405	0.5521	0.1109	0.5388	0.0970
100	a	0.2504	0.0009	0.2504	0.0011	0.2506	0.0017
	b	1.8606	0.3158	1.8168	0.1564	1.7676	0.0475
	c	0.4478	0.0936	0.4834	0.0481	0.4943	0.0331
	s	0.5333	0.0976	0.5390	0.0806	0.5253	0.0630
200	a	0.2502	0.0005	0.2502	0.0005	0.2502	0.0005
	b	1.8934	0.4526	1.8008	0.1144	1.7573	0.0307
	c	0.4760	0.0526	0.4928	0.0340	0.4979	0.0257
	s	0.5325	0.0813	0.5249	0.0534	0.5156	0.0431

Table (4.2.b): Avg. and MSEs estimated values of the GWU_LL distribution with parameters: $(a = 0.25, b = 1.75, \mathrm{c} = 0.5, \mathrm{s} = 1.5)$ under Type-I censoring with different cases of q_t and sample sizes n

n	Param	$q_t = 30\%$		$q_t = 60\%$		$q_t = 90\%$	
		Avg.	**MSE**	**Avg.**	**MSE**	**Avg.**	**MSE**
50	a	0.2501	0.0004	0.2501	0.0003	0.2502	0.0005
	b	1.7865	0.4524	1.8196	0.6463	1.7997	0.2562
	c	0.4571	0.0814	0.4670	0.0668	0.4781	0.0571
	s	1.6089	0.3780	1.6026	0.3005	1.5891	0.2117
100	a	0.2501	0.0001	0.2501	0.0001	0.2501	0.0002
	b	1.8129	0.3359	1.8208	0.3831	1.8110	0.2702
	c	0.4768	0.0517	0.4796	0.0454	0.4903	0.0359
	s	1.5578	0.2705	1.5584	0.2072	1.5733	0.1709
200	a	0.2500	0.0001	0.2500	0.0001	0.2500	0.0001
	b	1.8235	0.2776	1.8126	0.2590	1.7977	0.1553
	c	0.4922	0.0302	0.4929	0.0265	0.4977	0.0222
	s	1.5492	0.1833	1.5369	0.1273	1.5508	0.1231

Table (4.2.c): Avg. and MSEs estimated values of the GWU_LL distribution with parameters: $(a = 0.25, b = 1.75, \mathrm{c} = 1.5, \mathrm{s} = 0.5)$ under Type-I censoring with different cases of q_t and sample sizes n

n	Param	$q_t = 30\%$		$q_t = 60\%$		$q_t = 90\%$	
		Avg.	**MSE**	**Avg.**	**MSE**	**Avg.**	**MSE**
50	a	0.2901	0.1115	0.3151	0.1220	0.3145	0.1221
	b	2.0951	1.3550	1.8740	0.3859	1.8355	0.3180
	c	2.5119	4.6265	1.5746	0.6985	1.5649	0.5562
	s	0.5777	0.3759	0.8168	0.6382	0.7972	0.6473
100	a	0.3097	0.0945	0.2970	0.0800	0.2938	0.0816
	b	1.8974	0.5076	1.7810	0.3412	1.7624	0.2676
	c	1.4205	0.3624	1.4348	0.2728	1.4347	0.3062
	s	0.6847	0.4510	0.6982	0.6202	0.6476	0.4949
200	a	0.2839	0.0539	0.2861	0.0559	0.2863	0.0586
	b	1.7625	0.4147	1.7140	0.3148	1.7360	0.2417
	c	1.4051	0.2534	1.3753	0.2777	1.4057	0.2807
	s	0.6058	0.4983	0.5815	0.4620	0.5968	0.4255

Table (4.2.d): Avg. and MSEs estimated values of the GWU_LL distribution with parameters: $(a = 0.25, b = 1.75, \mathrm{c} = 2.5, \mathrm{s} = 2.5)$ under Type-I censoring with different cases of q_t and sample sizes n

n	Param	$q_t = 30\%$		$q_t = 60\%$		$q_t = 90\%$	
		Avg.	**MSE**	**Avg.**	**MSE**	**Avg.**	**MSE**
50	a	0.3385	0.1426	0.3484	0.1629	0.3582	0.1518
	b	2.2318	2.8703	2.0780	1.3198	1.6462	0.6665
	c	2.4610	1.6058	3.0616	3.9647	2.4624	3.0224
	s	3.3126	3.4973	3.1417	3.3833	2.0119	1.5998
100	a	0.3230	0.1285	0.3336	0.1232	0.3449	0.1181
	b	2.3063	2.5776	1.7966	1.2457	1.4412	0.4277
	c	2.4886	1.7127	2.1623	1.0911	1.7833	0.9277
	s	3.4087	4.8111	2.6661	1.8836	1.5790	1.6012
200	a	0.2858	0.0945	0.3024	0.0925	0.3189	0.0905
	b	1.9032	0.4121	1.5857	0.3590	1.4685	0.3779
	c	2.5419	1.3890	2.0721	0.7129	1.9234	0.7334
	s	2.8148	1.3914	2.0336	1.4305	1.5554	1.5248

Refernces

[1] Alexander, C., Cordeiro, G.M., Ortega, E.M.M., Sarabia, J.M. :Generalized beta-generated distributions. Computational Statistics and Data Analysis 56:1880–1897. (2012)

[2] Alzaatreh, A, Famoye, F. and Lee, C. ;A new method for generating families of continuous distributions. Metron 71(1), pp. 63–79. (2013b)

[3] Burr, I.W.: Cumulative frequency functions. Ann. Math. Stat. 13, 215–232 (1942)

[4] Cordeiro, G.M., Ortega, E.M.M., Nadarajah, S.: The Kumaraswamy Weibull distribution with application to failure data. J. Frankl. Inst. 347, 1399–1429 (2010)

[5] Cordeiro, G.M., Pescim, R.R., Ortega, E.M.M.: The Kumaraswamy generalized half-normal distribution for skewed positive data. J. Data Sci. 10, 195–224 (2012)

[6] De Castro, M.A.R., Ortega, E.M.M., Cordeiro, G.M.: The Kumaraswamy generalized gamma distribution with application in survival analysis. Stat. Methodol. 8(5), 411–433 (2011)

[7] Eugene, N., Lee, C., Famoye, F.: The beta-normal distribution and its applications. Commun. Stat. Theory Methods 31(4), 497–512 (2002)

[8] Freimer, M., Kollia, G., Mudholkar, G.S., Lin, C.T.: A study of the generalized Tukey lambda family. Commun. Stat. Theory Methods 17, 3547–3567 (1988)

[9] Johnson, N.L.: Systems of frequency curves generated by methods of translation. Biometrika 36, 149–176 (1949)

[10] Johnson, N.L., Kotz, S., Balakrishnan, N.: Continuous Univariate Distributions, vol. 1, 2nd edn. Wiley, New York (1994)

[11] Karian, Z.A., Dudewicz, E.: Fitting Statistical Distributions—The Generalized Lambda Distribution and Generalized Bootstrap Methods. Chapman & Hall/CRC Press, Boca Raton (2000)

[12] M-19 Kenney, J. F. and Keeping, E. S. :Mathematics of statistics. 3rd ed. Princeton, NJ: Chapman and Hall. pp. 101-102. (1962)

[13] _Lai, CD: Constructions and applications of lifetime distributions. Appl Stoch Model Bus Ind 29, 127–140 (2013)

[14] Marshall-Olkin. Marshall, AW, Olkin, I: A new method for adding a parameter to a family of distributions with applications to the exponential and Weibull families. Biometrika 84, 641–652 (1997)

[15] M_22 Moors, J. J. :A quantile alternative for kurtosis. J. Royal Statist. Soc. D, vol. 37, pp. 25-32. (1988)

[16] Pearson, K.: Contributions to the mathematical theory of evolution. II. Skew variation in homogeneous material. Philos. Trans. Royal Soc. Lond. A 186, 343–414 (1895)

[17] Ramberg, J.S., Schmeiser, B.W.: An approximate method for generating symmetric random variables. Commun. Assoc. Comput. Mach. 15, 987–990 (1972)

[18] Ramberg, J.S., Schmeiser, B.W.: An approximate method for generating asymmetric random variables. Commun. Assoc. Comput. Mach. 17, 78–82 (1974)

[19] Ristic, M.M., Balakrishnan, N. :The gamma-exponentiated exponential distribution. ´ Journal of Statistical Computation and Simulation 82:1191–1206. (2012)

[20] Tahir, M. H., Cordeiro, G. M., Alzaatreh, A., Mansoor, M., Zubair, M. :The Logistic-X Family of Distributions and its Applications, Communications in Statistics- Theory and Methods, DOI:10.1080/03610926.2014.980516. (2015b)

[21] Torabi, H., Montazari, N.H. :The gamma-uniform distribution and its application. Kybernetika 48:16–30. (2012)

[22] Torabi, H., Montazari, N.H. :The logistic-uniform distribution and its application. Communications in Statistics–Simulation and Computation 43:2551–2569. (2014)

[23] Tukey, J.W.: The practical relationship between the common transformations of percentages of counts and amounts. Technical Report 36, Statistical Techniques Research Group, Princeton University, Princeton, NJ (1960)

[24] Zografos, K., Balakrishnan, N. :On families of beta- and generalized gamma-generated distributions and associated inference. Statistical Methodology 6:344–362. (2009)

توزيع تعميم عائلة ويبل

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الملخص:

في هذه المقالة تم تقديم عائلة T-X من خلال دالة التوزيع التراكمي {(R{W(F(x (CDF) حيث R هو CDF لمتغير عشوائيT ، Fهو CDF لـ X و Wهو دالة متزايدة محددة على المدى [٠، ١]. توفر هذه العائلة طريقة جديدة لتوليد توزيعات أحادية المتغير. توفر الاختيارات المختلفة لوظائف R و Fو Wعائلات مختلفة من التوزيعات. استخدمنا دوال الربيع لتحديد الدالة .W تمت دراسة بعض الخصائص العامة لنظام التوزيعات .T-X تم اشتقاق توزيعات جديدة لعائلة T-X ، وهو توزيع وايبل المنتظم اللوغاريتمى اللوجيستى المعمم ({GRU {LL) بأربعة معلمات. تم استخدام طرق تقدير مختلفة لتقدير المعلمات غير المعروفة في العينات العشوائية المراقبة الكاملة

الكلمات الافتتاحية:

عائلاتT-X ، توزيع وايبل، التوزيع اللوجستي اللوغاريتمي، توزيع وايبل المنتظم اللوغاريتمى اللوجيستى المعمم، دالة الربيع.