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SOFT S_b-METRIC SPACES AND SOME OF ITS PROPERTIES

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Abstract. Firstly, we have founded a generalized concept of soft S-metric spaces, named soft S_b-metric space, based on soft points of soft sets, and some basic properties regarding soft S_b-metric spaces are studied with examples. After that, we have established a fixed point theorem on soft S_b-metric spaces with an application.

1. Introduction and Preliminaries

Soft set theory was first initiated by Molodtsov [4] in 1999, which is an extension of fuzzy set theory [7]. After that, Maji et al. [10] studied this theory in detail. The concepts of soft real set, soft real number, soft point, and soft metric spaces were introduced, and some of their important properties were studied by Das and Samanta in [13, 15].

In 2012, S-metric space was introduced by Sedghi et al. [12] and obtained some fixed point results on S-metric spaces. Thereafter, some more fixed-point results are discussed by many researchers in [8, 9, 16, 17, 19].

As a continuation, Aras et al. [2] have extended the concept of S-metric spaces to soft S-metric spaces, and some important fixed point results were established in 2018 [3] with the help of soft mapping [1, 5, 11, 18]. Recently, soft S_b-metric space using soft elements was discussed in [6].

In the present study, using soft points, we have introduced a generalized notion of soft S-metric spaces called soft S_b- metric spaces, and some of their fundamental properties are established with proper examples. An important soft fixed point result on soft S_b-metric spaces is also discussed with an application.

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Choosing $X(\neq \emptyset)$, universal set; E^*, set of parameters, and $P(X)$, power set of X.

Definition 1.1. [4] *Taking $F^* : Q^* \to P(X)$, a function; $Q^* \subseteq E^*$, then $(F^*,\ Q^*)$ is soft set over X.*

Definition 1.2. [10] *If $F^*(q^*) = X,\ \forall\ q^* \in Q^*$, then $(F^*,\ Q^*)$, a soft set over X, is designated absolute soft set and is glossed by $\widetilde{X}$.*

Definition 1.3. [13] *A function $H^* : E^* \to B(\mathbb{R})$ is a soft real set and is glossed by $(H^*,\ E^*)$.*
If $(H^*,\ E^*)$ is singleton, then it is designated a soft real number. It is prevailed as $\tilde{t^*}$, whereas $\overline{t^*}$ prevailed a especial form of soft real numbers, where $\overline{t^*}(a^*) = t^*,\ \forall\ a^* \in E^*$.

Definition 1.4. [15] *$(F^*,\ Q^*)$ is mentioned as a soft point of X if there is a specific one $q^* \in Q^*$ for which $F^*(q^*) = \{z\}$, for few $z \in X$ and $F^*(b^*) = \emptyset,\ \forall\ b^* \in Q^* \backslash \{q^*\}$. It is prevailed by $P^z_{q^*}$.*

Definition 1.5. [14] *$P^{z_1}_{q_1^*} = P^{z_2}_{q_2^*} \ \Rightarrow\ z_1 = z_2$ and $q_1^* = q_2^*$.*
Again, $P^{z_1}_{q_1^*} \neq P^{z_2}_{q_2^*} \Rightarrow$ either $z_1 \neq z_2$ or, $q_1^* \neq q_2^*$.

2. Soft S_b-Metric Spaces

In this part, we have initiated soft S_b-metric spaces, and some of their properties are discussed with application.

Definition 2.6. *Let $SP(\widetilde{X})$ be the collection of all soft points of $\widetilde{X}$ and $\mathbb{R}(E^*)^*$ be the collection of all non-negative soft real numbers.*
A mapping $\widetilde{S_b} : SP(\widetilde{X}) \times SP(\widetilde{X}) \times SP(\widetilde{X}) \to \mathbb{R}(E^*)^*$, is entitled a soft S_b- metric on the soft set $\widetilde{X}$ with constant soft real number $\overline{s} \widetilde{\geq} \overline{1}$, if $\widetilde{S_b}$ content the following conditions $\forall\ P^{x_1}_{a_1},\ P^{x_2}_{a_2},\ P^{x_3}_{a_3},\ P^{x_4}_{a_4} \widetilde{\in} SP(\widetilde{X})$,

(S_1) $\widetilde{S_b}(P^{x_1}_{a_1},\ P^{x_2}_{a_2},\ P^{x_3}_{a_3}) \widetilde{\geq} \overline{0}$, *equality holds if and only if* $P^{x_1}_{a_1} = P^{x_2}_{a_2} = P^{x_3}_{a_3}$

(S_2) $\widetilde{S_b}(P^{x_1}_{a_1},\ P^{x_2}_{a_2},\ P^{x_3}_{a_3}) \widetilde{\leq} \overline{s}\ \{\ \widetilde{S_b}(P^{x_1}_{a_1},\ P^{x_1}_{a_1},\ P^{x_4}_{a_4})\ +\ \widetilde{S_b}(P^{x_2}_{a_2},\ P^{x_2}_{a_2},\ P^{x_4}_{a_4})$
$\quad +\ \widetilde{S_b}(P^{x_3}_{a_3},\ P^{x_3}_{a_3},\ P^{x_4}_{a_4})\}$

and the soft set $\widetilde{X}$ with a soft S_b-metric on $\widetilde{X}$ is called a soft S_b-metric space and is denoted by $(\widetilde{X},\ S_b,\ E^*)$.

Example 1. *Choose $X = \mathbb{R} = E^*$ and $\forall\ P^{x_1}_{a_1},\ P^{x_2}_{a_2},\ P^{x_3}_{a_3} \widetilde{\in} SP(\widetilde{X})$, define the function $\widetilde{S_b}$ by,*
$\widetilde{S_b}(P^{x_1}_{a_1},\ P^{x_2}_{a_2},\ P^{x_3}_{a_3}) = \Big[\ |\ \overline{x_1} - \overline{x_2}\ | + |\ \overline{x_2} - \overline{x_3}\ | + |\ \overline{x_3} - \overline{x_1}\ |\ \Big]^2$
$+\ \Big[\ |\ \overline{a_1} - \overline{a_2}\ | + |\ \overline{a_2} - \overline{a_3}\ | + |\ \overline{a_3} - \overline{a_1}\ |\ \Big]^2$, *where $\overline{x_1}(\lambda^*)\ =\ x_1,\ \forall\ \lambda^* \in E^*$.*
Then, definitely the condition (S_1) is satisfied. Now, for condition (S_2),

$$
\begin{aligned}
\widetilde{S_b}(P_{a_1}^{x_1},\ P_{a_2}^{x_2},\ P_{a_3}^{x_3}) &= \Big[\,|\,\overline{x_1}-\overline{x_2}\,|\ +\ |\,\overline{x_2}-\overline{x_3}\,|\ +\ |\,\overline{x_3}-\overline{x_1}\,|\,\Big]^2\ +\ \Big[\,|\,\overline{a_1}-\overline{a_2}\,| \\
&\quad +\ |\,\overline{a_2}-\overline{a_3}\,|\ +\ |\,\overline{a_3}-\overline{a_1}\,|\,\Big]^2 \\
&\widetilde{\leq} \Big[\,|\,\overline{x_1}-\overline{x_4}\,|\ +\ |\,\overline{x_4}-\overline{x_2}\,|\ +\ |\,\overline{x_2}-\overline{x_4}\,|\ +\ |\,\overline{x_4}-\overline{x_3}\,| \\
&\quad +\ |\,\overline{x_3}-\overline{x_4}\,|\ +\ |\,\overline{x_4}-\overline{x_1}\,|\,\Big]^2 \\
&\quad +\Big[\,|\,\overline{a_1}-\overline{a_4}\,|\ +\ |\,\overline{a_4}-\overline{a_2}\,|\ +\ |\,\overline{a_2}-\overline{a_4}\,|\ +\ |\,\overline{a_4}-\overline{a_3}\,| \\
&\quad +\ |\,\overline{a_3}-\overline{a_4}\,|\ +\ |\,\overline{a_4}-\overline{a_1}\,|\,\Big]^2 \\
&= [\ \overline{2}\ \ |\,\overline{x_1}-\overline{x_4}\,|\,]^2\ +\ [\ \overline{2}\ \ |\,\overline{x_2}-\overline{x_4}\,|\,]^2\ +\ [\ \overline{2}\ \ |\,\overline{x_3}-\overline{x_4}\,|\,]^2 \\
&\quad +\overline{8}\ \ |\,\overline{x_1}-\overline{x_4}\,||\,\overline{x_2}-\overline{x_4}\,|\ \ +\overline{8}\ \ |\,\overline{x_2}-\overline{x_4}\,||\,\overline{x_3}-\overline{x_4}\,| \\
&\quad +\overline{8}\ \ |\,\overline{x_3}-\overline{x_4}\,||\,\overline{x_1}-\overline{x_4}\,|\ \ +[\ \overline{2}\ \ |\,\overline{a_1}-\overline{a_4}\,|\,]^2 \\
&\quad +[\ \overline{2}\ \ |\,\overline{a_2}-\overline{a_4}\,|\,]^2\ +\ [\ \overline{2}\ \ |\,\overline{a_3}-\overline{a_4}\,|\,]^2\ +\ \overline{8}\ \ |\,\overline{a_1}-\overline{a_4}\,||\,\overline{a_2}-\overline{a_4}\,| \\
&\quad +\overline{8}\ \ |\,\overline{a_2}-\overline{a_4}\,||\,\overline{a_3}-\overline{a_4}\,|\ \ +\overline{8}\ \ |\,\overline{a_3}-\overline{a_4}\,||\,\overline{a_1}-\overline{a_4}\,| \\
&\widetilde{\leq} \overline{3}\left[\ \Big[\ \overline{2}\ \ |\,\overline{x_1}-\overline{x_4}\,|\,\Big]^2\ +\ \Big[\ \overline{2}\ \ |\,\overline{x_2}-\overline{x_4}\,|\,\Big]^2\ +\ \Big[\ \overline{2}\ \ |\,\overline{x_3}-\overline{x_4}\,|\,\Big]^2\right. \\
&\quad \left. +\ \Big[\ \overline{2}\ \ |\,\overline{a_1}-\overline{a_4}\,|\,\Big]^2\ +\ \Big[\ \overline{2}\ \ |\,\overline{a_2}-\overline{a_4}\,|\,\Big]^2\ +\ \Big[\ \overline{2}\ \ |\,\overline{a_3}-\overline{a_4}\,|\,\Big]^2\ \right] \\
&= \overline{3}\ \Big[\ \widetilde{S_b}(P_{a_1}^{x_1},\ P_{a_1}^{x_1},\ P_{a_4}^{x_4})+\widetilde{S_b}(P_{a_2}^{x_2},\ P_{a_2}^{x_2},\ P_{a_4}^{x_4})+\widetilde{S_b}(P_{a_3}^{x_3},\ P_{a_3}^{x_3},\ P_{a_4}^{x_4})\ \Big]
\end{aligned}
$$

Thus, $(\widetilde{X},\ S_b,\ E^*)$ *is a soft* S_b*-metric space with constant* $\overline{s}=\overline{3}$.

Note: Every soft S-metric space is a soft S_b-metric space with $\overline{s}=1$, but the function $\widetilde{S_b}$ may not be a soft S-metric, if we pick $\overline{x_1}=\overline{4},\ \overline{x_2}=\overline{6},\ \overline{x_3}=\overline{8},\ \overline{x_4}=\overline{5}$; and $\overline{a_1}=\overline{2},\ \overline{a_2}=\overline{4},\ \overline{a_3}=\overline{6},\ \overline{a_4}=\overline{3}$, then from Example 1 we have $\forall\ \lambda^*\ \in\ E^*$,

$$
\begin{aligned}
\widetilde{S_b}(P_{a_1}^{x_1},\ P_{a_2}^{x_2},\ P_{a_3}^{x_3})(\lambda^*) &= 128 \\
&\widetilde{\leq} \Big[\ \widetilde{S_b}(P_{a_1}^{x_1},\ P_{a_1}^{x_1},\ P_{a_4}^{x_4})\ +\ \widetilde{S_b}(P_{a_2}^{x_2},\ P_{a_2}^{x_2},\ P_{a_4}^{x_4}) \\
&\quad +\widetilde{S_b}(P_{a_3}^{x_3},\ P_{a_3}^{x_3},\ P_{a_4}^{x_4})\ \Big](\lambda^*) \\
&= 88,\ \text{which is a contradiction.}
\end{aligned}
$$

Lemma 2.1. *In a soft* S_b*-metric space* $(\widetilde{X},\ S_b,\ E^*)$ *with* $\overline{s}\ \widetilde{\geq}\ \overline{1}$,

$$\widetilde{S_b}(P_{a_1}^{x_1},P_{a_1}^{x_1},P_{a_2}^{x_2})\ \widetilde{\leq}\ \overline{s}\ \widetilde{S_b}(P_{a_2}^{x_2},P_{a_2}^{x_2},P_{a_1}^{x_1}).$$

Proof. Since, $(\widetilde{X},\ S_b,\ E^*)$ is a soft S_b-metric space with $\overline{s}\ \widetilde{\geq}\ \overline{1}$, we have

$$
\begin{aligned}
\widetilde{S_b}(P_{a_1}^{x_1},\ P_{a_1}^{x_1},\ P_{a_2}^{x_2}) &\ \widetilde{\leq}\ \overline{s}\ \Big\{\ \widetilde{S_b}(P_{a_1}^{x_1},\ P_{a_1}^{x_1},\ P_{a_1}^{x_1})+\widetilde{S_b}(P_{a_1}^{x_1},\ P_{a_1}^{x_1},\ P_{a_1}^{x_1}) \\
&\quad +\widetilde{S_b}(P_{a_2}^{x_2},\ P_{a_2}^{x_2},\ P_{a_1}^{x_1})\ \Big\} \\
\Rightarrow \widetilde{S_b}(P_{a_1}^{x_1},\ P_{a_1}^{x_1},\ P_{a_2}^{x_2}) &\ \widetilde{\leq}\ \overline{s}\ \widetilde{S_b}(P_{a_2}^{x_2},\ P_{a_2}^{x_2},\ P_{a_1}^{x_1})
\end{aligned}
$$

□

Definition 2.7. *A soft S_b-metric space $(\widetilde{X},\ S_b,\ E^*)$ with $\overline{s} \mathrel{\widetilde{\geq}} \overline{1}$ is called symmetric if*

$$\widetilde{S_b}(P^{x_1}_{a_1},\ P^{x_1}_{a_1},\ P^{x_2}_{a_2}) = \widetilde{S_b}(P^{x_2}_{a_2},\ P^{x_2}_{a_2},\ P^{x_1}_{a_1}).$$

Example 2. *In Example 1, the function $\widetilde{S_b}$ is a symmetric soft S_b-metric on $\widetilde{X}$.*

Example 3. *Take $X = \mathbb{R} = E^*$ and $\forall\ P^{x_1}_{a_1},\ P^{x_2}_{a_2},\ P^{x_3}_{a_3} \mathrel{\widetilde{\in}} SP(\widetilde{X})$ pick $\widetilde{S_b}$ as,*

$$\begin{aligned}
\widetilde{S_b}(P^0_0,\ P^0_0,\ P^1_1) &= \overline{3}, \\
\widetilde{S_b}(P^1_1,\ P^1_1,\ P^0_0) &= \overline{6}, \\
\widetilde{S_b}(P^{x_1}_{a_1},\ P^{x_2}_{a_2},\ P^{x_3}_{a_3}) &= \overline{0},\ \text{if } P^{x_1}_{a_1} = P^{x_2}_{a_2} = P^{x_3}_{a_3}, \\
\widetilde{S_b}(P^{x_1}_{a_1},\ P^{x_2}_{a_2},\ P^{x_3}_{a_3}) &= \overline{1},\ \text{otherwise},
\end{aligned}$$

Then $\widetilde{S_b}$ is soft S_b-metric, but not symmetric.

Definition 2.8. *A sequence $\{P^{x_1}_{d,n}\}$ in a soft S_b-metric space $(\widetilde{X},\ S_b,\ E^*)$ is converges to P^y_a if and only if $\widetilde{S_b}(P^{x_1}_{d,n},\ P^{x_1}_{d,n},\ P^y_a) \to \overline{0}$ as $n \to \infty$.*
i.e, for each $\widetilde{\varepsilon} \mathrel{\widetilde{>}} \overline{0},\ \exists\ k \in \mathbb{N}$ such that $\widetilde{S_b}(P^{x_1}_{d,n},\ P^{x_1}_{d,n},\ P^y_a) \mathrel{\widetilde{<}} \widetilde{\varepsilon},\ \forall\ n \geq k$.
It is denoted by $\lim\limits_{n\to\infty} P^{x_1}_{d,n} = P^y_a$.

Example 4. *Take $E^* = \{d_1,\ d_2\}$ and $X = \mathbb{R}$.*
Pick S_b from Example 1.
Define $\{P^{x_1}_{d,n}\}$ by $P^{x_1}_{d,n}(d_i) = \frac{i}{n},\ \forall\ n \in \mathbb{N};\ i = 1, 2$.
Then $\forall\ d_i \in E^*;\ i = 1, 2$,

$$\begin{aligned}
\widetilde{S_b}(P^{x_1}_{d,n},\ P^{x_1}_{d,n},\ P^0_d)(d_i) &= \widetilde{S_b}(P^{\frac{i}{n}}_d,\ P^{\frac{i}{n}}_d,\ P^0_d) \\
&= \left[\ \left|\ \overline{\frac{i}{n}} - \overline{\frac{i}{n}}\ \right| + \left|\ \overline{\frac{i}{n}} - \overline{0}\ \right| + \left|\ \overline{0} - \overline{\frac{i}{n}}\ \right|\ \right]^2 \\
&= \left[\ \left| \frac{i}{n} - \frac{i}{n} \right| + \left| \frac{i}{n} - 0 \right| + \left| 0 - \frac{i}{n} \right|\ \right]^2 \\
&= 4\left[\frac{i}{n}\right]^2 \\
&\to 0,\ \text{as } n \to \infty
\end{aligned}$$

Therefore, $\lim\limits_{n\to\infty} P^{x_1}_{d,n} = P^0_d$

Theorem 2.1. *If a sequence $\{P^{x_1}_{d,n}\}$ in a symmetric soft S_b- metric space $(\widetilde{X},\ S_b,\ E^*)$ converges to P^y_a, then P^y_a is unique.*

Proof. Let $\{P^{x_1}_{d,n}\} \to P^y_a$, as $n \to \infty$ and $\{P^{x_1}_{d,n}\} \to P^z_b$, as $n \to \infty$, where $P^y_a \neq P^z_b$.
So, for each $\widetilde{\varepsilon} \mathrel{\widetilde{>}} \overline{0},\ \exists\ k_1, k_2 \in \mathbb{N}$ such that,

$$\widetilde{S_b}(P^{x_1}_{d,n},\ P^{x_1}_{d,n},\ P^y_a) \mathrel{\widetilde{<}} \frac{\widetilde{\varepsilon}}{4\ \overline{s}},\ \forall\ n \geq k_1$$

and

$$\widetilde{S_b}(P^{x_1}_{d,n},\ P^{x_1}_{d,n},\ P^z_b) \mathrel{\widetilde{<}} \frac{\widetilde{\varepsilon}}{2\ \overline{s}},\ \forall\ n \geq k_2$$

If we take $k^* = \max\{k_1,\ k_2\}$, then

$$\widetilde{S_b}(P^{x_1}_{d,n},\ P^{x_1}_{d,n},\ P^{y}_{a}) \,\widetilde{<}\, \frac{\widetilde{\varepsilon}}{4\ \overline{s}},\ \forall\ n \geq k^*$$

and

$$\widetilde{S_b}(P^{x_1}_{d,n},\ P^{x_1}_{d,n},\ P^{z}_{b}) \,\widetilde{<}\, \frac{\widetilde{\varepsilon}}{2\ \overline{s}},\ \forall\ n \geq k^*$$

Now,

$$\begin{aligned}\widetilde{S_b}(P^{y}_{a},\ P^{y}_{a},\ P^{z}_{b}) &= \overline{s}\ \left\{2\ \widetilde{S_b}(P^{y}_{a},\ P^{y}_{a},\ P^{x_1}_{d,n}) + \widetilde{S_b}(P^{z}_{b},\ P^{z}_{b},\ P^{x_1}_{d,n})\right\} \\ &= \overline{s}\ \left\{2\ \widetilde{S_b}(P^{x_1}_{d,n},\ P^{x_1}_{d,n},\ P^{y}_{a}) + \widetilde{S_b}(P^{x_1}_{d,n},\ P^{x_1}_{d,n},\ P^{z}_{b})\right\}, \\ &\quad \text{since } \widetilde{S_b} \text{ is symmetric} \\ &\widetilde{<}\ \widetilde{\varepsilon},\ \forall\ n \geq k^*\end{aligned}$$

Since $\widetilde{\varepsilon} > \overline{0}$ is arbitrary, so $\widetilde{S_b}(P^{y}_{a},\ P^{y}_{a},\ P^{z}_{b}) = \overline{0},\ i.e.,\ P^{y}_{a} = P^{z}_{b}$.

□

Note: If the function S_b is not symmetric, then P^{y}_{a} in Theorem 2.1 may not be unique.
In Example 3, P^{y}_{a} is not unique.

Definition 2.9. *A sequence $\{P^{x_1}_{d,n}\}$ in a soft S_b-metric space $(\widetilde{X},\ S_b,\ E^*)$ is Cauchy if $\widetilde{S_b}(P^{x_1}_{d,n},\ P^{x_1}_{d,n},\ P^{x_1}_{d,m}) \to \overline{0}$, as $n, m \to \infty$.*
i.e, for each $\widetilde{\varepsilon} \,\widetilde{>}\, \overline{0}$, $\exists\ k \in \mathbb{N}$ such that $\widetilde{S_b}(P^{x_1}_{d,n},\ P^{x_1}_{d,n},\ P^{x_1}_{d,m}) \,\widetilde{<}\, \widetilde{\varepsilon},\ \forall\ n, m \geq k$.

Example 5. *In Example 4, the sequence $\{P^{x_1}_{d,n}\}$, where $P^{x_1}_{d,n}(d_i) = \frac{i}{n},\ \forall\ n \in \mathbb{N};\ i = 1, 2$ is a Cauchy sequence, as for all $d_i \in E^*;\ i = 1, 2,$*

$$\begin{aligned}\widetilde{S_b}(P^{x_1}_{d,n},\ P^{x_1}_{d,n},\ P^{x_1}_{d,m})(d_i) &= \widetilde{S_b}(P^{\frac{i}{n}}_{d},\ P^{\frac{i}{n}}_{d},\ P^{\frac{i}{m}}_{d}) \\ &= \left[\ \left|\ \overline{\frac{i}{n}} - \overline{\frac{i}{n}}\ \right| + \left|\ \overline{\frac{i}{n}} - \overline{\frac{i}{m}}\ \right| + \left|\ \overline{\frac{i}{m}} - \overline{\frac{i}{n}}\ \right|\ \right]^2 \\ &= \left[\ \left|\ \frac{i}{n} - \frac{i}{n}\ \right| + \left|\ \frac{i}{n} - \frac{i}{m}\ \right| + \left|\ \frac{i}{m} - \frac{i}{n}\ \right|\ \right]^2 \\ &= 4\left[\frac{i}{m} - \frac{i}{n}\right]^2 \\ &\to 0,\ \text{as } n, m \to \infty\end{aligned}$$

Theorem 2.2. *If a sequence $\{P^{x_1}_{d,n}\}$ in a soft S_b- metric space $(\widetilde{X},\ S_b,\ E^*)$ is converges to P^{y}_{a}, then $\{P^{x_1}_{d,n}\}$ is a Cauchy sequence.*

Proof. As $\lim\limits_{n\to\infty} P^{x_1}_{d,n} = P^{y}_{a}$, so for any $\widetilde{\varepsilon} \,\widetilde{>}\, \overline{0}$, $\exists\ k_1, k_2 \in \mathbb{N}$ such that,

$$\widetilde{S_b}(P^{x_1}_{d,n},\ P^{x_1}_{d,n},\ P^{y}_{a}) \,\widetilde{<}\, \frac{\widetilde{\varepsilon}}{4\ \overline{s}},\ \forall\ n \geq k_1,$$

and

$$\widetilde{S_b}(P^{x_1}_{d,m},\ P^{x_1}_{d,m},\ P^{y}_{a}) \,\widetilde{<}\, \frac{\widetilde{\varepsilon}}{2\ \overline{s}},\ \forall\ n \geq k_2.$$

Set $k^* = \max\{k_1,\ k_2\}$
Now,

$$\begin{aligned}
\widetilde{S_b}(P^{x_1}_{d,n},\ P^{x_1}_{d,n},\ P^{x_1}_{d,m}) &\widetilde{\leq} \overline{s}\,\{2\,\widetilde{S_b}(P^{x_1}_{d,n},\ P^{x_1}_{d,n},\ P^{y}_{a}) + \widetilde{S_b}(P^{x_1}_{d,m},\ P^{x_1}_{d,m},\ P^{y}_{a})\} \\
&\widetilde{<} \frac{\widetilde{\varepsilon}}{2} + \frac{\widetilde{\varepsilon}}{2},\ \forall\ n, m \geq k^* \\
\Rightarrow \widetilde{S_b}(P^{x_1}_{d,n},\ P^{x_1}_{d,n},\ P^{x_1}_{d,m}) &\widetilde{<} \widetilde{\varepsilon}
\end{aligned}$$

Therefore, $\{P^{x_1}_{d,n}\}$ is a Cauchy sequence. □

Definition 2.10. *A soft S_b-metric space $(\widetilde{X},\ S_b,\ E^*)$ is complete if every Cauchy sequence in $\widetilde{X}$ is converges to some soft point in $\widetilde{X}$.*

Example 6. *In Example 4, if we take $(Y,\ E^*)\ \widetilde{\subset}\ \widetilde{X}$, where $Y(d) = [0, 1],\ \forall\ d \in E^*$, then $(\widetilde{Y},\ S_b,\ E^*)$ is a complete soft S_b-metric spaces.*

Theorem 2.3. *Let $(\widetilde{X},\ S_b,\ E^*)$ be a complete soft S_b- metric space with $\overline{s}\ \widetilde{\geq}\ \overline{1}$. If f_φ and T_ψ are two soft mappings on $(\widetilde{X},\ S_b,\ E^*)$, content the following condition,*

$$\widetilde{S_b}(f_\varphi(P^x_{\lambda^*}),\ f_\varphi(P^x_{\lambda^*}),\ T_\psi(P^y_{\mu^*}))\ \widetilde{\leq}\ \overline{a}\ \left[\ \widetilde{S_b}(P^x_{\lambda^*},\ P^x_{\lambda^*},\ P^y_{\mu^*})\right],$$
$$\forall\ P^x_{\lambda^*}, P^y_{\mu^*}\ \widetilde{\in}\ SP(\widetilde{X}),\ \text{where}\ \overline{a}\ \widetilde{\in}\ \left[\ \overline{0},\ \overline{\frac{1}{s^2}}\ \right), \quad (1)$$

then f_φ and T_ψ have a unique common fixed soft point in $(\widetilde{X},\ S_b,\ E^*)$.

Proof. Let $P^x_{\lambda^*,0}\ \widetilde{\in}\ SP(\widetilde{X})$.
Let us consider a sequence of soft points $\{P^x_{\lambda^*,n}\}$ in $(\widetilde{X},\ S_b,\ E^*)$ defined as,
$P^x_{\lambda^*,2k+1} = f_\varphi(P^x_{\lambda^*,2k})$, $P^x_{\lambda^*,2k+2} = T_\psi(P^x_{\lambda^*,2k+1})$; $k = 0, 1, 2, ...$
Now,

$$\begin{aligned}
\widetilde{S_b}(P^x_{\lambda^*,2k+1},\ P^x_{\lambda^*,2k+1},\ P^x_{\lambda^*,2k+2}) &= \widetilde{S_b}(f_\varphi(P^x_{\lambda^*,2k}),\ f_\varphi(P^x_{\lambda^*,2k}),\ T_\psi(P^x_{\lambda^*,2k+1})) \\
&\widetilde{\leq}\ \overline{a}\ \widetilde{S_b}(P^x_{\lambda^*,2k},\ P^x_{\lambda^*,2k},\ P^x_{\lambda^*,2k+1})
\end{aligned}$$

Again,

$$\begin{aligned}
\widetilde{S_b}(P^x_{\lambda^*,2k+2},\ P^x_{\lambda^*,2k+2},\ P^x_{\lambda^*,2k+3}) &= \widetilde{S_b}(T_\psi(P^x_{\lambda^*,2k+1}),\ T_\psi(P^x_{\lambda^*,2k+1}),\ f_\varphi(P^x_{\lambda^*,2k+2})) \\
&= \overline{s}\ \widetilde{S_b}(f_\varphi(P^x_{\lambda^*,2k+2}),\ f_\varphi(P^x_{\lambda^*,2k+2}),\ T_\psi(P^x_{\lambda^*,2k+1})), \\
&\quad \text{from Lemma 2.1} \\
&\widetilde{\leq}\ \overline{a}\ \overline{s}\ \widetilde{S_b}(P^x_{\lambda^*,2k+2},\ P^x_{\lambda^*,2k+2},\ P^x_{\lambda^*,2k+1}) \\
&= \overline{a}\ \overline{s}^2\ \widetilde{S_b}(P^x_{\lambda^*,2k+1},\ P^x_{\lambda^*,2k+1},\ P^x_{\lambda^*,2k+2}), \\
&\quad \text{from Lemma 2.1}
\end{aligned}$$

Therefore,

$$\begin{aligned}
\widetilde{S_b}(P^x_{\lambda^*,2k+2},\ P^x_{\lambda^*,2k+2},\ P^x_{\lambda^*,2k+3}) &\widetilde{\leq}\ \overline{a}\ \overline{s}^2\ \widetilde{S_b}(P^x_{\lambda^*,2k+1},\ P^x_{\lambda^*,2k+1},\ P^x_{\lambda^*,2k+2}) \\
&\widetilde{\leq}\ \overline{a}^2\ \overline{s}^2\ \widetilde{S_b}(P^x_{\lambda^*,2k},\ P^x_{\lambda^*,2k},\ P^x_{\lambda^*,2k+1});\ k = 0, 1, 2, ...
\end{aligned}$$

Now, $\forall\ n \in \mathbb{N}$,

$$\begin{aligned}
\widetilde{S_b}(P^x_{\lambda^*,n+1},\ P^x_{\lambda^*,n+1},\ P^x_{\lambda^*,n+2}) &\ \widetilde{\leq}\ \overline{a}\ \widetilde{S_b}(P^x_{\lambda^*,n},\ P^x_{\lambda^*,n},\ P^x_{\lambda^*,n+1}) \\
&\ \widetilde{\leq}\ \overline{a}^2\ \overline{s}^2\ \widetilde{S_b}(P^x_{\lambda^*,n-1},\ P^x_{\lambda^*,n-1},\ P^x_{\lambda^*,n}) \\
&\ \vdots \\
&\ \widetilde{\leq}\ \overline{a}^{n+1}\ \overline{s}^{n+1}\ \widetilde{S_b}(P^x_{\lambda^*,0},\ P^x_{\lambda^*,0},\ P^x_{\lambda^*,1})
\end{aligned}$$

Using Lemma 2.1, for $m > n$,

$$\begin{aligned}
\widetilde{S_b}(P^x_{\lambda^*,n},\ P^x_{\lambda^*,n},\ P^x_{\lambda^*,m}) &\ \widetilde{\leq}\ \overline{s}\ \big[2\ \widetilde{S_b}(P^x_{\lambda^*,n},\ P^x_{\lambda^*,n},\ P^x_{\lambda^*,n+1}) + \\
&\quad \overline{s}\ \widetilde{S_b}(P^x_{\lambda^*,n+1},\ P^x_{\lambda^*,n+1},\ P^x_{\lambda^*,m})\big] \\
&\ \widetilde{\leq}\ \overline{s}\ \Big[2\ \widetilde{S_b}(P^x_{\lambda^*,n},\ P^x_{\lambda^*,n},\ P^x_{\lambda^*,n+1}) \\
&\quad + \overline{s}\ \big[2\ \widetilde{S_b}(P^x_{\lambda^*,n+1},\ P^x_{\lambda^*,n+1},\ P^x_{\lambda^*,n+2}) \\
&\quad + \overline{s}\ \widetilde{S_b}(P^x_{\lambda^*,n+2},\ P^x_{\lambda^*,n+2},\ P^x_{\lambda^*,m})\big]\Big] \\
&\ \vdots \\
&\ \widetilde{\leq}\ 2\ \overline{s}\ \widetilde{S_b}(P^x_{\lambda^*,n},\ P^x_{\lambda^*,n},\ P^x_{\lambda^*,n+1}) \\
&\quad + 2\ \overline{s}^2\ \widetilde{S_b}(P^x_{\lambda^*,n+1},\ P^x_{\lambda^*,n+1},\ P^x_{\lambda^*,n+2}) \\
&\quad + \cdots +\ 2\ \overline{s}^{(m-n-1)}\ \ \widetilde{S_b}(P^x_{\lambda^*,m-2},\ P^x_{\lambda^*,m-2},\ P^x_{\lambda^*,m-1}) \\
&\quad + 2\ \overline{s}^{(m-n)}\ \ \widetilde{S_b}(P^x_{\lambda^*,m-1},\ P^x_{\lambda^*,m-1},\ P^x_{\lambda^*,m}) \\
&\ \widetilde{\leq}\ 2\ \overline{s}\ \widetilde{S_b}(P^x_{\lambda^*,n},\ P^x_{\lambda^*,n},\ P^x_{\lambda^*,n+1}) \\
&\quad + 2\ \overline{s}^2\ \widetilde{S_b}(P^x_{\lambda^*,n+1},\ P^x_{\lambda^*,n+1},\ P^x_{\lambda^*,n+2}) \\
&\quad + 2\ \overline{s}^3\ \widetilde{S_b}(P^x_{\lambda^*,n+2},\ P^x_{\lambda^*,n+2},\ P^x_{\lambda^*,n+3}) \\
&\quad + 2\ \overline{s}^4\ \widetilde{S_b}(P^x_{\lambda^*,n+3},\ P^x_{\lambda^*,n+3},\ P^x_{\lambda^*,n+4}) + \cdots + \\
&\quad 2\ \overline{s}^{(m-n-1)}\ \ \widetilde{S_b}(P^x_{\lambda^*,m-2},\ P^x_{\lambda^*,m-2},\ P^x_{\lambda^*,m-1}) \\
&\quad + 2\ \overline{s}^{(m-n)}\ \ \widetilde{S_b}(P^x_{\lambda^*,m-1},\ P^x_{\lambda^*,m-1},\ P^x_{\lambda^*,m}) + \cdots \\
&\ \widetilde{\leq}\ 2\ \big[\ \overline{a}^n\ \overline{s}^{n+1} + \overline{a}^{n+1}\ \overline{s}^{n+3} + \overline{a}^{n+2}\ \overline{s}^{n+5} \\
&\quad + \overline{a}^{n+3}\ \overline{s}^{n+7} + \cdots\big]\ \widetilde{S_b}(P^x_{\lambda^*,0},\ P^x_{\lambda^*,0},\ P^x_{\lambda^*,1}) \\
&\ \widetilde{\leq}\ 2\ \overline{a}^n\ \overline{s}^{n+1}\ \big[1 + (\overline{a}\ \overline{s}^2) + (\overline{a}\ \overline{s}^2)^2 \\
&\quad + (\overline{a_1}\ \overline{s}^2)^3 + \cdots\big]\ \widetilde{S_b}(P^x_{\lambda^*,0},\ P^x_{\lambda^*,0},\ P^x_{\lambda^*,1}) \\
&\ \widetilde{\leq}\ 2\ \frac{\overline{a}^n\ \overline{s}^{n+1}}{1-(\overline{a}\ \overline{s}^2)}\ \widetilde{S_b}(P^x_{\lambda^*,0},\ P^x_{\lambda^*,0},\ P^x_{\lambda^*,1}) \\
&\ \to\ \overline{0},\ \text{as } n \to \infty\ \left[\text{since, } \overline{a_1}\ \widetilde{\in}\ \left[\ \overline{0}, \overline{\frac{1}{s^2}}\ \right)\right].
\end{aligned}$$

Therefore, $\{P^x_{\lambda^*,n}\}$ is a Cauchy sequence.

Since, $(\widetilde{X},\ S_b,\ E^*)$ is a complete soft S_b-metric space, so $\exists\ P^t_\alpha\ \widetilde{\in}\ SP(\widetilde{X})$ such that $P^x_{\lambda^*,n} \to P^t_\alpha,$ as $n \to \infty$.

Now,

$$\begin{aligned}\widetilde{S_b}(f_\varphi(P^t_\alpha),\ f_\varphi(P^t_\alpha),\ P^t_\alpha) &\widetilde{\leq} 2\ \widetilde{S_b}(f_\varphi(P^t_\alpha),\ f_\varphi(P^t_\alpha),\ T_\psi(P^x_{\lambda^*,2k+1}))\\ &\quad + \widetilde{S_b}(P^t_\alpha,\ P^t_\alpha,\ T_\psi(P^x_{\lambda^*,2k+1}))\\ &\widetilde{\leq} 2\ \overline{a}\ \widetilde{S_b}(P^t_\alpha,\ P^t_\alpha,\ P^x_{\lambda^*,2k+1}) + \widetilde{S_b}(P^t_\alpha,\ P^t_\alpha,\ P^x_{\lambda^*,2k+2})\end{aligned}$$

Taking $k \to \infty$,

$$\begin{aligned}\widetilde{S_b}(f_\varphi(P^t_\alpha),\ f_\varphi(P^t_\alpha),\ P^t_\alpha) &\widetilde{\leq} 2\ \overline{a}\ \widetilde{S_b}(P^t_\alpha,\ P^t_\alpha,\ P^t_\alpha) + \widetilde{S_b}(P^t_\alpha,\ P^t_\alpha,\ P^t_\alpha)\\ \Rightarrow \widetilde{S_b}(f_\varphi(P^t_\alpha),\ f_\varphi(P^t_\alpha),\ P^t_\alpha) &\widetilde{\leq} \overline{0}\\ \Rightarrow f_\varphi(P^t_\alpha) &= P^t_\alpha.\end{aligned}$$

Again,

$$\begin{aligned}\widetilde{S_b}(P^t_\alpha,\ P^t_\alpha,\ T_\psi(P^t_\alpha)) &= \widetilde{S_b}(f_\varphi(P^t_\alpha),\ f_\varphi(P^t_\alpha),\ T_\psi(P^t_\alpha))\\ &\widetilde{\leq} \overline{a}\ \widetilde{S_b}(P^t_\alpha,\ P^t_\alpha,\ P^t_\alpha)\\ &= \overline{0}\\ \Rightarrow T_\psi(P^t_\alpha) &= P^t_\alpha.\end{aligned}$$

Thus, $f_\varphi(P^t_\alpha) = T_\psi(P^t_\alpha) = P^t_\alpha$.
i.e., f_φ and T_ψ have common fixed soft point.
To assert uniqueness, let $P^{t^*}_\beta (\neq\ P^t_\alpha)\ \widetilde{\in}\ SP(\widetilde{X})$ be another fixed soft point of f_φ and T_ψ.
Now,

$$\begin{aligned}\widetilde{S_b}(P^t_\alpha,\ P^t_\alpha,\ P^{t^*}_\beta) &= \widetilde{S_b}(f_\varphi(P^t_\alpha),\ f_\varphi(P^t_\alpha),\ T_\psi(P^{t^*}_\beta))\\ &= \overline{a}\ \widetilde{S_b}(P^t_\alpha,\ P^t_\alpha,\ P^{t^*}_\beta)\\ \Rightarrow \widetilde{S_b}(P^t_\alpha,\ P^t_\alpha,\ P^{t^*}_\beta) &= \overline{0}\\ \Rightarrow P^t_\alpha &= P^{t^*}_\beta\end{aligned}$$

Therefore, f_φ and T_ψ have unique common fixed soft point in $(\widetilde{X},\ S_b,\ E^*)$. □

Corollary 2.0. *Let $(\widetilde{X},\ S_b,\ E^*)$ be a complete soft S_b- metric space with $\overline{s}\ \widetilde{\geq}\ \overline{1}$. If h_γ is a soft mappings on $(\widetilde{X},\ S_b,\ E^*)$, content the following condition,*

$$\widetilde{S_b}(h_\gamma(P^x_{\lambda^*}), h_\gamma(P^x_{\lambda^*}), h_\gamma(P^y_{\mu^*}))\ \widetilde{\leq}\ \overline{b}\ \left[\ \widetilde{S_b}(P^x_{\lambda^*}, P^x_{\lambda^*}, P^y_{\mu^*})\right],$$
$$\forall\ P^x_{\lambda^*}, P^y_{\mu^*}\ \widetilde{\in}\ SP(\widetilde{X}),\ \text{where}\ \overline{b}\ \widetilde{\in}\ \left[\ \overline{0}, \overline{\frac{1}{s^2}}\ \right), \quad (2)$$

then h_γ has a unique fixed soft point in $(\widetilde{X},\ S_b,\ E^*)$.

Proof. Choose $f_\varphi = h_\gamma = T_\psi$ and $\overline{a} = \overline{b}$. Then from Theorem 2.3, we get the result. □

2.1. **Application.** Let $E^* = [-2,\ \infty)$ and $\widetilde{X}(\lambda^*) = [-\frac{1}{4},\ \frac{1}{4}],\ \forall\ \lambda^* \in\ E^*$.
For all $P^{x_1}_{a_1},\ P^{x_2}_{a_2},\ P^{x_3}_{a_3} \widetilde{\in} SP(\widetilde{X})$, define $\widetilde{S_b}$ as,
$\widetilde{S_b}(P^{x_1}_{a_1},\ P^{x_2}_{a_2},\ P^{x_3}_{a_3}) = \big[\ |\ \overline{x_1} - \overline{x_2}\ |\ +|\ \overline{x_2} - \overline{x_3}\ |\ +\ |\ \overline{x_3} - \overline{x_1}\ |\ \big]^2$
$+\ \big[\ |\ \overline{a_1} - \overline{a_2}\ |\ +|\ \overline{a_2} - \overline{a_3}\ |\ +|\ \overline{a_3} - \overline{a_1}\ |\ \big]^2$, where $\overline{x_1}(\lambda^*)\ =\ x_1,\ \forall\ \lambda^* \in E^*$.

Then $(\widetilde{X},\ S_b,\ E^*)$ is a soft S_b-metric space.

Now, we define $\varphi : [-2,\ \infty) \to [-2,\ \infty)$ and $f : [-\frac{1}{4},\ \frac{1}{4}] \to [-\frac{1}{4},\ \frac{1}{4}]$ by $\varphi(x) = \frac{x}{2} - 1,\ \forall\ x \in\ [-2,\ \infty]$ and $f(x) = x^2,\ \forall\ x \in\ [-\frac{1}{4},\ \frac{1}{4}]$ respectively.

Let $f_\varphi : (\widetilde{X},\ S_b,\ E^*) \to (\widetilde{X},\ S_b,\ E^*)$ be such that $f_\varphi(P^x_{\lambda^*}) = P^{f(x)}_{\varphi(\lambda^*)}$.
Now,

$$\begin{aligned}
&\widetilde{S_b}\big(f_\varphi(P^{x_1}_{a_1}),\ f_\varphi(P^{x_2}_{a_2}),\ f_\varphi(P^{x_3}_{a_3})\big)(\lambda^*)\\
&= \Big[\ |\ \overline{f(x_1)}(\lambda^*) - \overline{f(x_2)}(\lambda^*)\ | + |\ \overline{f(x_2)}(\lambda^*) - \overline{f(x_3)}(\lambda^*)\ |\\
&+ |\ \overline{f(x_3)}(\lambda^*) - \overline{f(x_1)}(\lambda^*)\ |\ \Big]^2 + \Big[\ |\ \overline{\varphi(a_1)}(\lambda^*) - \overline{\varphi(a_2)}(\lambda^*)\ |\\
&+ |\ \overline{\varphi(a_2)}(\lambda^*) - \overline{\varphi(a_3)}(\lambda^*)\ | + |\ \overline{\varphi(a_3)}(\lambda^*) - \overline{\varphi(a_1)}(\lambda^*)\ |\ \Big]^2\\
&= \Big[\ |\ f(x_1)\ - f(x_2)\ | + |\ f(x_2) - f(x_3)\ | + |\ f(x_3) - f(x_1)\ |\ \Big]^2\\
&+ \Big[\ |\ \varphi(a_1) - \varphi(a_2)\ | + |\ \varphi(a_2) - \varphi(a_3)\ | + |\ \varphi(a_3) - \varphi(a_1)\ |\ \Big]^2,\ (\text{since } \overline{x_1}(\lambda^*) = x_1)\\
&= \big[\ |\ x_1^2 - x_2^2\ | + |\ x_2^2 - x_3^2\ | + |\ x_3^2 - x_1^2\ |\ \big]^2 + \big[\ |\ \tfrac{a_1}{2} - \tfrac{a_2}{2}\ | + |\ \tfrac{a_2}{2} - \tfrac{a_3}{2}\ | + |\ \tfrac{a_3}{2} - \tfrac{a_1}{2}\ |\ \big]^2\\
&\widetilde{\leq} \tfrac{1}{4}\ \Big[\big[\ |\ x_1 - x_2\ | + |\ x_2 - x_3\ | + |\ x_3 - x_1\ |\ \big]^2 + \big[\ |\ a_1 - a_2\ | + |\ a_2 - a_3\ |\\
&+ |\ a_3 - a_1\ |\ \big]^2\Big]\\
&= \bar{\tfrac{1}{4}}\ \Big[\big[\ |\ \overline{x_1}(\lambda^*) - \overline{x_2}(\lambda^*)\ | + |\ \overline{x_2}(\lambda^*) - \overline{x_3}(\lambda^*)\ | + |\ \overline{x_3}(\lambda^*) - \overline{x_1}(\lambda^*)\ |\ \big]^2\\
&+ \big[\ |\ \overline{a_1}(\lambda^*) - \overline{a_2}(\lambda^*)\ | + |\ \overline{a_2}(\lambda^*) - \overline{a_3}(\lambda^*)\ | + |\ \overline{a_3}(\lambda^*) - \overline{a_1}(\lambda^*)\ |\ \big]^2\Big],
\end{aligned}$$

(since $\overline{x_1}(\lambda^*) = x_1$)

$\Rightarrow\ \widetilde{S_b}\big(f_\varphi(P^{x_1}_{a_1}),\ f_\varphi(P^{x_2}_{a_2}),\ f_\varphi(P^{x_3}_{a_3})\big)(\lambda^*)\ \widetilde{\leq}\ \bar{\frac{1}{4}}\ \widetilde{S_b}(P^{x_1}_{a_1},\ P^{x_2}_{a_2},\ P^{x_3}_{a_3})(\lambda^*)$.

Since this is true for all $\lambda^* \in\ E^*$, so

$$\widetilde{S_b}\big(f_\varphi(P^{x_1}_{a_1}),\ f_\varphi(P^{x_2}_{a_2}),\ f_\varphi(P^{x_3}_{a_3})\big)\ \widetilde{\leq}\ \frac{\bar{1}}{4}\ \widetilde{S_b}(P^{x_1}_{a_1},\ P^{x_2}_{a_2},\ P^{x_3}_{a_3})$$

.
Therefore, $(\widetilde{X},\ S_b,\ E^*)$ is a complete soft S_b-metric space.
Also the condition,

$$\widetilde{S_b}(f_\varphi(P^{x_1}_{a_1}),\ f_\varphi(P^{x_2}_{a_2}),\ f_\varphi(P^{x_3}_{a_3}))\quad \widetilde{\leq}\quad \bar{b}\ \widetilde{S_b}(P^{x_1}_{a_1},\ P^{x_2}_{a_2},\ P^{x_3}_{a_3}),\ \forall\ P^{x_1}_{a_1},\ P^{x_2}_{a_2},\ P^{x_3}_{a_3} \widetilde{\in} SP(\widetilde{X}),$$

is satisfied for $\bar{b}\ (= \bar{\frac{1}{4}}) \widetilde{\in} \mathbb{R}(E^*)$.

Thus, all the conditions of Corollary 2.0 are contented. So, from Corollary 2.0, we can say that f_φ has a unique fixed soft point.
Now, $f_\varphi(P^0_{-2}) = P^{0^2}_{\left(\frac{-2}{2}-1\right)} = P^0_{-2}$.
Hence, P^0_{-2} is a fixed soft point.

3. Conclusions

In this study, we have initiated soft S_b-metric space, and some elementary behaviours are investigated. A significant fixed point result on soft S_b-metric spaces is established with an application. We expect that this modern thought will favour researchers in enhancing and generalizing the theory of soft metric spaces and related fields.

References

[1] A. Kharal and B. Ahmad, Mappings on soft classes, New Math. Nat. Comput., 7, 3, 471-481, 2021.
[2] C. G. Aras, S. Bayramov and V. Cafarli, A Study on Soft S-Metric Spaces, Commun. Math. & Appl., 9, 4, 713-723, 2018.
[3] C. G. Aras, S. Bayramov and V. Cafarli, Fixed Point Theorems on Soft S-Metric Spaces, Commun. Math. & Appl., 9, 4, 725-735, 2018.
[4] D. Molodtsov, Soft Set Theory-First Results, Comput. Math. Appl., 37, 19-31, 1999.
[5] D. Wardowski, On A Soft Mapping And Its Fixed Points, Fixed Point Theory Appl., 182, 1-11, 2013.
[6] D. A. Kadhim, M. K. Abd Al-Rahem and Z. H. Abood, Convergence and its Properties on Soft S_b- Metric Spaces, J. Phys. Conf. Ser, 1818, 1-9, 2021.
[7] L. A. Zadeh, Fuzzy Sets, Inform. Control, 8, 103-112, 1965.
[8] M. Mursaleen and F. Basar, Sequence Spaces: Topics in Modern Summability Theory, Series: Mathematics and Its Applications, CRC Press/Taylor & Francis Group, Boca Ratan, London, New York, 2020.
[9] N. Y. Ozgur and N. Tas, Some fixed point theorems on S-metric spaces, Mat. Vesnik, 69, 1, 39-52, 2017.
[10] P. K. Maji, R. Biswas and A. R. Roy, Soft set theory, Comput. Math. Appl., 45, 555-562, 2003.
[11] P. Majumdar and S. K. Samanta, On soft mappings, Comput. Math. Appl., 60, 2666-2672, 2010.
[12] S. Sedghi, N. Shobe and A. Aliouche, A generalization of fixed point theorem in S-metric spaces, Mat. Vesnik, 64, 258-266, 2012.
[13] S. Das and S. K. Samanta, Soft Real Set, Soft Real Number And Their Properties, J. Fuzzy Math., 20, 3, 551-576, 2012.
[14] S. Bayramov and C. Gunduz, Soft locally compact spaces and soft paracompact spaces, J. Math. System Sci., 3, 122-130, 2013.
[15] S. Das and S. K. Samanta, Soft metric, Ann. Fuzzy Math. Inform., 6, 1, 77-94, 2013.
[16] S. Sedghi and N. V. Dung, Fixed point theorem on S-metric spaces, Mat. Vesnik, 66, 1, 113-124, 2014.
[17] S. Sedghi, A. Gholidahneh, T. Došenović, J. Esfahani and S. Radenović, Common fixed point of four maps in S_b-metric spaces, J. Linear Topol. Algebra, 5, 2, 93-104, 2016.
[18] Sk. Nazmul, Some properties of soft groups and fuzzy soft groups under soft mappings, Palest. J. Math., 6, 2, 1-11, 2017.
[19] T. Mitra, Some Common Fixed Point Theorems in S-Metric Space, International Journal of Innovative Research in Science, Engineering and Technology, 4,9, 9039-9044, 2015.

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