Egypt. J. Phys. **Vol. 51,** pp. 187-195 (2023)

Egyptian Journal of Physics

https://ejphysics.journals.ekb.eg/

Fringe Visibility of Light with Fine Spectral Structure

M. M. El-Nicklawy and A. E. Elmahdy*

Department of Physics, Faculty of Science, Helwan University, 11792 Ain Helwan, Cairo, Egypt

Abstract

Analytical expression is derived for the visibility of two beams interference fringes of light with spectral fine structure. Lorentz and Gaussian spectral light are taken into consideration.

Michelson interferometer is used for studying experimentally the fringe visibility of sodium doublet spectral lines and He-Ne laser beam of multispectral axial modes as spectral fine structure. The results show that periodicity of appearing and disappearing of the interference fringes associated with decaying in their visibility along equal intervals of changes in the optical path difference between the interfering beams. This behavior of the damping characteristic enables us to know the type of the spectral line distribution whether it is Lorentz, Gauss or of Voigt profile. The experimental measurements show good agreement with the theoretical derived formula. The study gave information about the spectral spacing between the fine spectral lines forming the spectral distribution of the light beam, spectral distribution and spectral width.

Keywords: Fringe visibility, Michelson interferometer, Laser modes, Optical coherence

Introduction

Spectroscopic measurements of spectral line distribution and its spectral width are of great importance for investigating the plasma characteristics of gas discharge without its disturbance by using electric and magnetic probes. The spectroscopic technique requires a scanning spectrometer of instrumental spectral width much smaller than the spectral width of the measured spectral line. It is mostly cannot be achieved by using prism or grating spectrometer, Fabry-Perot interferometer, multiple beam interference, as an analyzer has enough small instrumental spectral width, depending on its mirror reflections [1, 2]. The disadvantage is its limited free spectral range which may be not enough to scan the required spectral range of the spectral line. Two beams interference techniques are used in the spectral domain for measuring the spectral dispersion of solids and liquids [3-7]. Michelson interferometer has high accuracy to measure the fine spectral structure of spectral line distribution and its spectral width. It is through the behavior of the fringe visibility with the optical path difference between the two interfering beams. Some spectral lines distributions having spectral fine structure represented by separated spectral peaks. The spectral distribution of the laser beam is characterized by spectral fine structure represented by its separated spectral axial modes. The sodium spectrum is characterized by fine structure represented by its doublet spectral lines. In this case the behavior of the obtained fringe visibility differs from that in case of spectral line without fine spectral structure. The interference fringes are found be periodically appear and disappear along equal intervals of change of the optical path difference and associated with damping in its visibility. Through the behavior of the damping characteristic enable to know the type of the spectral line distribution whether it is of Lorentz, Gauss or the convolution product of Lorentz and Gaussian distributions which is called the Voigt distribution. Its spectral width can also be determined through the coherence at which the interference fringes no more appear.

The aim of the present work is to give analytical formula representing the behavior of the fringe visibility with the optical path difference of two beams interference for light of spectral fine structure. Experimental verification using Michelson interferometer is carried out for sodium doublet spectral lines and laser beam of multispectral axial modes.

*Corresponding author: Adel E. Elmahdy, E-mail: adel_elmahdy@science.helwan.edu.eg.
Received: 20/07/2023; Accepted: 10/11/2023
DOI: 10.21608/EJPHYSICS.2023.222056.1089

Theoretical consideration

Laser light, for its important coherence property in many applications such as holography, information recording, image processing and optical communications, is basically considered in the present work. Sodium doublet spectral lines is presented as special case.

Laser source is an amplifying medium inside an optical resonator. The emitted radiation from the amplifying medium can be of Lorentz or Gaussian gain spectral density distribution. It depends on the physical conditions of the amplifying medium. The radiation suffers multi-reflections inside an optical resonator. It built up spectral axial modes at resonance frequencies within the gain spectral distribution. The laser beam can be of one spectral axial mode or of multi-modes occur at different resonance frequencies of equal frequency spacing. It depends on the useful spectral range of the gain spectral distribution which satisfy the laser threshold condition. The frequency spacing of the spectral axial modes and in turn its effective number depends on the resonator mirrors separation.

3. Spectral intensity of the spectral axial mode

The spectral intensity $I(\nu)$ of the spectral axial modes is given by the following formula through the conventional interference between the multi-reflected beams inside the resonator [8]

$$I(\nu) = \frac{I_o(\nu)(1-R)(1+RG)}{1-2RG\cos(4\pi L\nu/c)+R^2G^2} \tag{1}$$

$I_o(\nu)$ is the intensity inside the resonator. R is the mirrors reflection coefficient, G is the gain coefficient per one path length L of the amplifying medium.

The total laser intensity in the spectral range $c/2L$ of one laser axial mode is given by

$$P_t = \int_{\nu_r-(c/4L)}^{\nu_r-(c/4L)} \frac{I_o(1-R)(1+RG)}{1-2RG\cos(4\pi L(\nu-\nu_r)/c)+R^2G^2}\,d\nu \tag{2}$$

The radiation intensity, $I_o(\nu)$, is considered to be nearly constant (I_o) over the spectral range $c/2L$ of the axial mode. ν_r is the resonance frequency of an axial mode. $c/2L$ is the frequency spacing between two axial modes.

$$\therefore\ P_t = I_o[(1-R)/(1-RG)](c/2L) \tag{3}$$

For getting an expression for the spectral width $\delta\nu$ of a laser spectral axial mode, Eq. (1) is rewritten in the form

$$I(\nu) = \frac{I_o(1-R)(1+RG)}{(1-RG)^2+4RG\sin^2(2\pi L\nu/c)}$$

For $\nu = \nu_r + (\delta\nu/2)$, one gets

$$\delta\nu = (c/2\pi L)(1-RG)/\sqrt{RG} \tag{4}$$

The normalized spectral distribution of an axial laser mode $I_{nor}(\upsilon)$ is given through Eq. (1) and (2) by

$$I_{norm}(\upsilon) = I(\upsilon)/P_t = \frac{(1-R^2G^2)(2L/c)}{1-2RG\cos(4\pi L\upsilon/c)+R^2G^2} \tag{5}$$

Since $(4\pi L\upsilon_r/c) = 2m\pi$ where m is an integer number, Eq. (5) is rewritten in the form

$$I_{norm}(\upsilon) = \frac{(1-R^2G^2)(2L/c)}{1-2RG\cos(4\pi L(\upsilon-\upsilon_r)/c)+R^2G^2} \tag{6}$$

The effective spectral range of a laser spectral axial mode $(\upsilon-\upsilon_r)$ is very close to its resonance frequency υ_r i.e. $(\upsilon-\upsilon_r)$ is of the order $\delta\upsilon$. Thus Eq. (6) can be rewritten in the form

$$I_{norm}(\upsilon) = \frac{(1-R^2G^2)(2L/c)}{(1-RG)^2+4RG(2\pi L(\upsilon-\upsilon_r)/c)^2}$$

$$I_{norm}(\upsilon) = \left[(1+RG)/(4\sqrt{RG})\right](\delta\upsilon/\pi)/[(\delta\upsilon/2)^2+(\upsilon-\upsilon_r)^2] \tag{7}$$

Eq. (7) shows that the normalized spectral distribution of a laser axial mode is very close to Lorentz spectral distribution. Note that $RG \le 1$.

Fringe visibility of laser beam

Eq. (6) shows that the normalized intensity distribution can be of more than one spectral axial mode it depends on the used effective spectral range of the gain threshold conditions for laser oscillations.

The intensity of the interference fringes of quasi-monochromatic light, according to Winer Khinchin theorem, is the incoherent sum of the fringe intensity induced by strictly monochromatic light over the spectral range distribution of the considered quasi-monochromatic light [9].

The intensity of two beams interference fringes for strictly monochromatic light $I_{mon}(\upsilon)$ of frequency υ is given by

$$I_{mon}(\upsilon) = 2a^2\left[1 + \cos\frac{2\pi\upsilon}{c}l\right] \tag{8}$$

Where l is the optical path difference between two interfering beams of equal amplitudes a.

Note that, the absolute value of the degree of coherence $|\gamma|$ is one since the light is strictly monochromatic light. It is corresponding to the degree of fringe visibility.

The fringe intensity

The fringe intensity due to the spectral intensity distribution of laser beam consisting of N spectral axial modes according to Winer Khinchen theory is given by

$$I_{sp}(\upsilon) = \frac{1}{N}\int_{-\infty}^{\infty} I_{mon}(\upsilon)\sum_{n=0}^{N-1}\frac{\left[(1+RG)/(4\sqrt{RG})\right](\delta\upsilon/\pi)}{\left[\left((\upsilon-\upsilon_r)-n\Delta\upsilon\right)^2+(\delta\upsilon/2)^2\right]}d\upsilon \tag{9}$$

Where $\Delta\upsilon = c/(2L)$ is the frequency spacing of the laser axial modes. Assuming υ_r is the resonance frequency of axial mode of order $n = 0$. The factor $1/N$ is a normalization factor. Where N is the number of the normalized emission spectral probabilities which built up the resonance spectral laser axial modes. Eq. (8) and (9) give

$$I_{sp}(\upsilon) = \frac{2a^2}{N}\left\{N + \int_{-\infty}^{\infty}\cos\left(\frac{2\pi\upsilon}{c}l\right)\sum_{n=0}^{N-1}\frac{\left[(1+RG)/(4\sqrt{RG})\right](\delta\upsilon/\pi)}{\left[\left((\upsilon-\upsilon_r)-n\Delta\upsilon\right)^2+(\delta\upsilon/2)^2\right]}d\upsilon\right\}$$

Let $(\upsilon-\upsilon_r)-n\Delta\upsilon = x$, then

$$I_{sp}(\upsilon) = \frac{2a^2}{N}\left\{N + \int_{-\infty}^{\infty}\sum_{n=0}^{N-1}\cos\left[\left(\frac{2\pi l}{c}\right)(x+\upsilon_r+n\Delta\upsilon)\right]\frac{(\delta\upsilon/2\pi)}{[x^2+(\delta\upsilon/2)^2]}dx\right\}$$

Considering $\left[(1+RG)/(4\sqrt{RG})\right] \cong 1/2$ since $RG \cong 1$

$$I_{sp}(\upsilon) = \frac{2a^2}{N}\left\{N + \sum_{n=0}^{N-1}\cos\left[\left(\frac{2\pi l}{c}\right)(\upsilon_r+n\Delta\upsilon)\right]\exp\left(-\frac{\pi l}{c}\delta\upsilon\right)\right\} \tag{10}$$

To relate the fringe intensity to fringe visibility V, the maximum intensity $I_{sp_{max}}$ and the minimum intensity $I_{sp_{min}}$ are written in the form [10, 11]

$$I_{sp_{max}} = 2a^2[1+\gamma] \tag{11}$$

$$I_{sp_{min}} = 2a^2[1-\gamma]. \tag{12}$$

Where $\gamma = \sum_{n=0}^{N-1}\cos\left[\left(\frac{2\pi l}{c}\right)(n\Delta\upsilon)\right]\exp\left(-\frac{\pi l}{c}\delta\upsilon\right)/N$.

γ is the normalized degree of temporal coherence. The fringe visibility V is given in terms of the degree of coherence by

$$V = \frac{I_{sp_{max}} - I_{sp_{min}}}{I_{sp_{max}} + I_{sp_{min}}}. \tag{13}$$

Thus, the fringe visibility of laser light is given by

$$V = \sum_{n=0}^{N-1}\cos[(2\pi l/c)(n\Delta\upsilon)]\exp\left(-\frac{\pi l}{c}\delta\upsilon\right)/N \tag{14}$$

Eq. (14) shows that the fringe visibility oscillates periodically every change in the optical path difference $\Delta l = c/\Delta\upsilon$. It is modulated by a decaying exponential function of the optical path difference l and the spectral width $\delta\upsilon$ of the axial mode. The decaying values of the repeated maximum fringe visibility, obtained from its exponential decay with l, gives $\delta\upsilon$.

The case of $N = 2$ can be considered for the case of the sodium D doublet spectral lines. The fringe visibility changes periodically with l according to $\cos^2(\pi l\Delta\upsilon/c)$ associated with $\exp(-\pi l\delta\upsilon/c)$. The frequency spaced for the repeated oscillating visibility $c/\Delta l$ gives directly the frequency spaced $\Delta\upsilon$ of the sodium doublet spectral lines. The maximum values of the reappeared fringe visibility at position l gives, through the decaying exponential function, the spectral width $\delta\upsilon$ of the sodium doublet spectral lines.

The case of $N = 1$, one spectral axial mode, the visibility V of the interference fringes, according to Eq. (14), will be given by

$$V = \exp(-\pi l \delta\upsilon / c) \tag{15}$$

It represents the fringe visibility behavior with the optical path difference l for light of pure Lorentz line profile without fine spectral structure.

Case of Gaussian spectral light profile

Consider a Gaussian spectral light profile of spectral fine structure represented by N equal frequencies spaced $\Delta\upsilon$ Gaussian spectral lines given by

$$g(\upsilon, \upsilon_o + n\Delta\upsilon) = (2/\delta\upsilon)\sqrt{\ln(2)/\pi}\,\exp -\left[\sqrt{\ln(2)}\,(\upsilon - \upsilon_o - n\Delta\upsilon)/(\delta\upsilon/2)\right]^2 \tag{16}$$

Where $n = 0, 1, 2, \ldots$,((N-1).

Applying Winer Khinchen theorem and consider Eq. (8) of two beams interference fringes we get

$$I_{sp}(\upsilon) = \frac{2a^2}{N}\left\{N + \int_{-\infty}^{\infty}\cos\left(\frac{2\pi\upsilon}{c}l\right)\sum_{n=0}^{N-1}\frac{2}{\delta\upsilon}\exp -\left[\sqrt{\ln(2)}\,(\upsilon - \upsilon_o - n\Delta\upsilon)/(\delta\upsilon/2)\right]^2\right\} \tag{17}$$

As treated mathematically before, the incoherent sum of the monochromatic fringe intensities over the Gaussian spectrum of the light beam leads to fringe visibility behavior like that for Lorentz spectral distribution. It differs only in the decaying exponential factor. It will be $\exp[-(\pi l\delta\upsilon/c)^2/\ln(2)]$ instead of $\exp(-\pi l\delta\upsilon/c)$.

This difference in the decaying factor of the fringe visibility enable to define whether the light spectrum is of Lorentz or Gauss spectral profile. It gives straight line with slope $(\pi\delta\upsilon/c)$ or with slope $\left[\pi\delta\upsilon/\left(c\sqrt{\ln(2)}\right)\right]$ by plotting $\ln(1/V)$ against l for Lorentz profile or against l^2 for Gaussian distribution.

Experimental Results and Discussion

In order to validate the proposed theoretical model, Michelson interferometer was employed to determine the dependence of the fringe visibility on the optical path difference length l between two interfering beams. Two different light sources were used: a He-Ne laser and a low-pressure sodium lamp. Figure (1) shows a schematic diagram of the Michelson interferometer. A He-Ne laser beam (λ= 632.8 nm, 0.5 mW) is split into two beams using non-polarizing beam splitter (NPBS). The beams propagate toward a fixed mirror M1 and a movable mirror M2. The reflected beams from M1 and M2 are recombined on a detector array through an imaging lens L1 to form interference fringes. A CMOS camera C records the interference fringe patterns for each change of the optical path difference between the two interfering beams. These patterns are stored on a computer storage media and automatically processed using MATLAB code to calculate the fringe visibility. The camera is a 1280 x 1024 pixels, 8 bits, with square pixels of 5.2 µm and a maximal frame rate up to 25 Hz. To ensure that the data obtained was a true reflection of the light level, care was taken during the experiment that the camera sensor was not saturated. When the camera pixels are saturating, the linear response of the CCD will start to deviate and compromise the quantitative performance. In addition to this, the pixel value exceeds the dynamic range of the camera, in this case 255 [12]. For these reasons, a neutral density filter (NDF) was placed in the path of the original beam to reduce the high intensity of the initial beam. Also, the optical system was setup on a vibration isolation optical table to isolate the optical setup from sources of vibration. The mirrors M1 and M2 were positioned and aligned to create an optimized fringe pattern and to set l = 0.

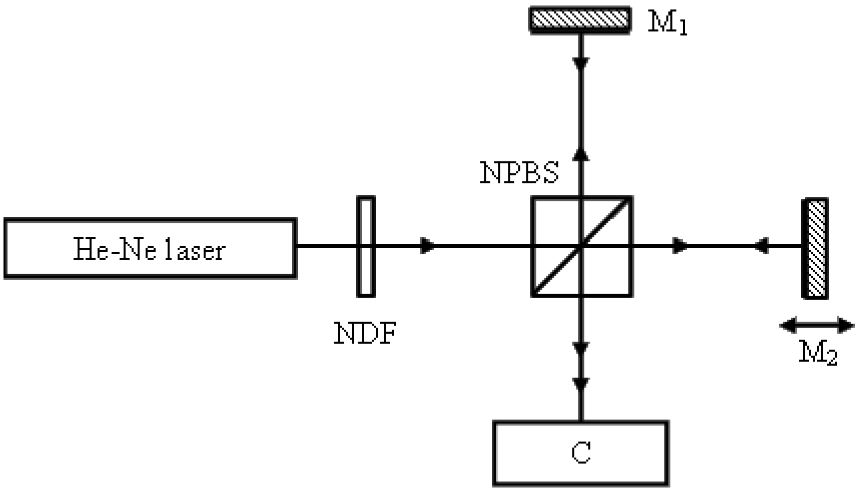
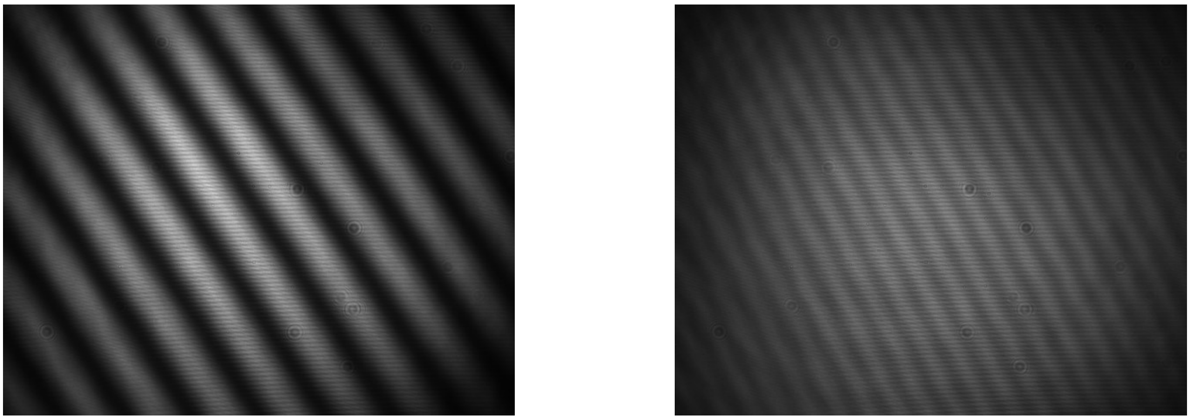


Fig. 1. Schematic of the Michelson interferometer: NDF, neutral density filter; NPBS, non-polarizing beam splitters; M1, fixed mirror; M2, movable mirror; C, CMOS camera.

Measurements using He-Ne laser

With the aid of the optical setup shown in Fig. 1, M2 was moved in step of 5 mm for a total distance of 1500 mm. At each increment, the fringe pattern was recorded and automatically processed to calculate the visibility of the interference fringe from Eq. (13). Fig. 2 shows an example of the obtained interference fringe patterns for He-Ne laser at l = 160 mm and 280 mm.



(a) (b)

Fig. 2. Interference fringe patterns obtained from Michelson interferometer using He-Ne laser with wavelength 632.8 nm; (a) l = 160 mm (b) l = 280 mm.

Fig. 3 shows the dependence of the measured visibility on the optical path difference for multimode laser line 632.8 nm (open circles). As expected, the visibility data shows a periodicity and a decrease of the visibility maxima values with increasing the optical path difference between the interfering beams, which are agree with the theory. By measuring the change in the optical path difference Δl between two visibility peaks, the value of the mode separation $\Delta\nu$ is found to be 553 MHz. The specifications of the used He-Ne laser in the current work show that it is a multimode laser source with a mode separation $\Delta\nu$ of 550 MHz.

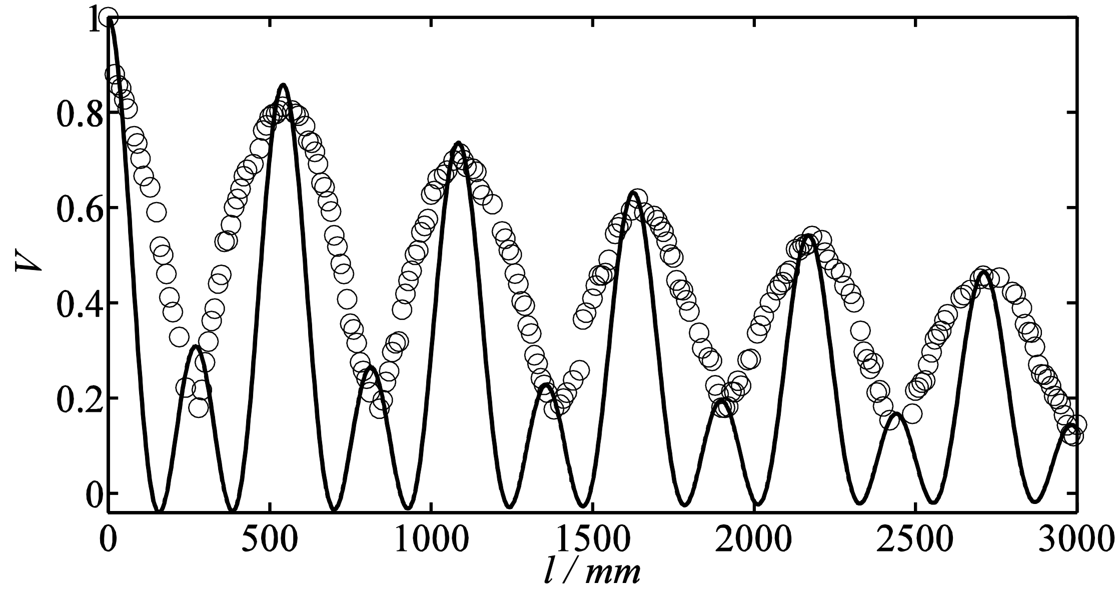


Fig. 3. Visiblity V as a function of the optical path difference l between the two interfering beams for laser light 632.8 nm using Michelson interferomter. Open circles, experimental data from which $\delta\upsilon = 27$ MHz; solid line, visiblity based on Eq. (14).

As it metioned before, the relation between $\ln(1/V_{peak})$ and the optical pathe difference l gives the information about the type of the spectral distribution of the investigated line. Fig. 4 shows that the laser light has a type of Lorentz profile as the relation is fitted as stright line. From the calculation of the slope of the stright line, the value of the spectral width of the investigated line is calculated to be $\delta\upsilon = 27$ MHz. It corresponds to a coherence length $l_c = \frac{c}{\delta\upsilon} \approx 11\ m$.

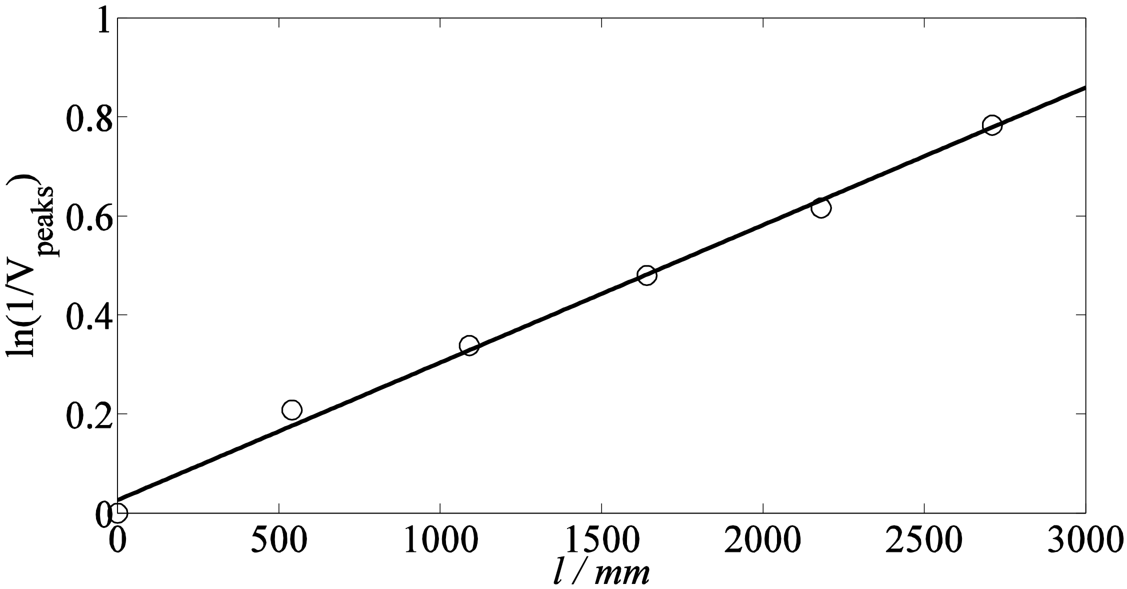


Fig. 4. $\ln(1/V_{peak})$ versus the optical path difference l measured for laser light 632.8 nm

To validate the proposed theoretical model, the experimental fringe visibility in Fig. 3 is fitted with Eq. (14) (solid line) considering the obtained experimental data as follows; the number of axial mode N=3 (shown in Fig. 5), the gain profile is of Lorentz type, the axial mode width $\delta\upsilon = 27$ MHz, and the mode spacing to be $\Delta\upsilon = 553$ MHz.

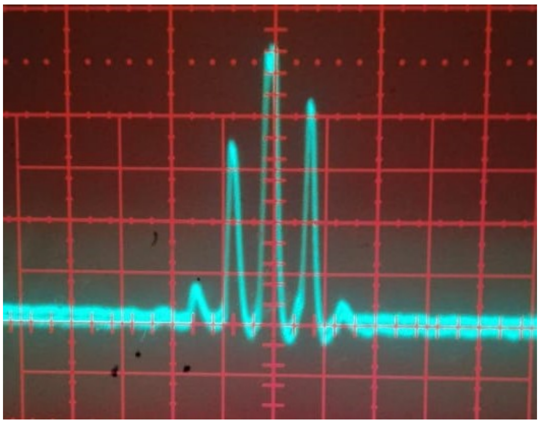
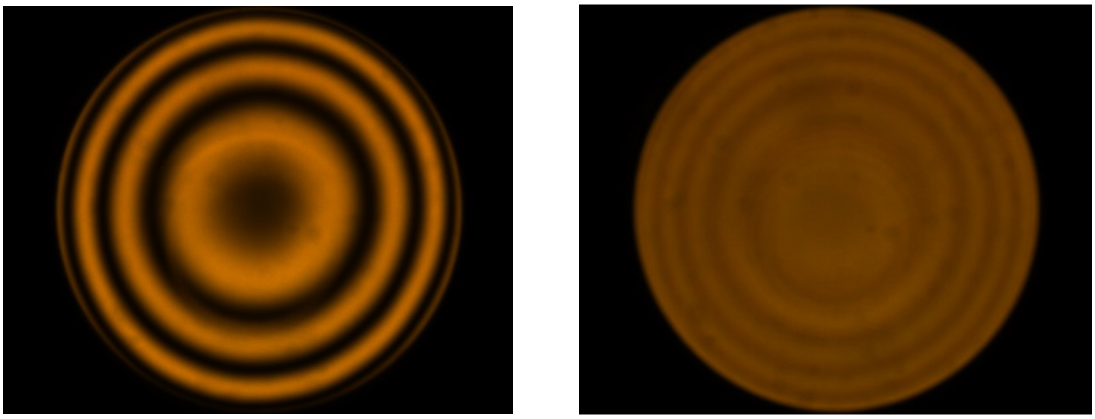


Fig. 5. Scanning Fabry-Perot interferometer display of He-Ne laser.

Measurements using Na D lines

A similar set-up shown in Fig. 1 was used except that the He-Ne laser was replaced with a low pressure sodium spectral lamp. The procedure and measurements made were the same as that for the He-Ne laser except that the mirror M2 was moved in step of 10 µm for a total distance of 4000 µm. Fig. 6 shows an example of the obtained interference fringe patterns for Na spectral lamp at optical path difference l = 480 µm and 780 µm.



(a) (b)

Fig. 6. Interference fringe pattern obtained from Michelson interferometer using Na spectral lamp; (a) $l = 480$ µm (b) $l = 780$ µm.

The visibility versus the optical path difference between the interferometer arms was plotted as shown in Fig. 7 (open circles). From the figure, the mode spacing $\Delta\upsilon$ is found to be 5.21 × 10^{11} Hz and accordingly, the doublet D-lines separation is calculated to be 6.03 A°.

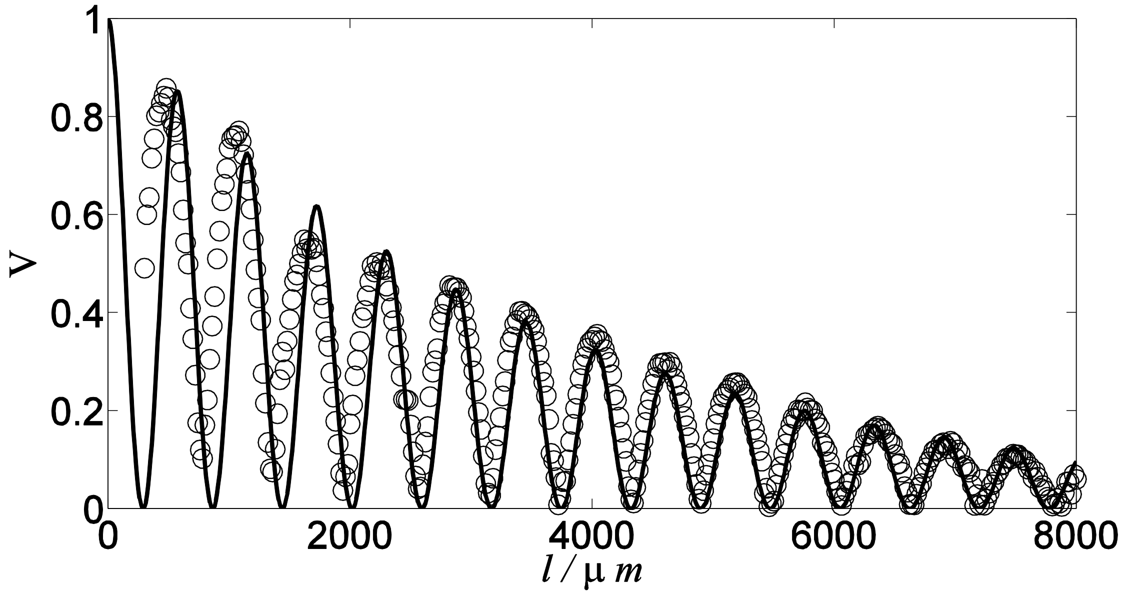


Fig.7. Visivlity as a function of the optical path difference l between the two interfering beams for Na-spectral lamp. Open circles, experimental data; solid line, visiblity based on Eq. (14) with N = 2.

Fig. 8 illustrates the relation between $\ln(1/V_{peak})$ and the optical pathe difference l for Na-spectral lamp. As the relation gives stright line, the type of the spectral line is chrachterized by Lorentz profile and its spectral width $\delta\upsilon$ is calculated from the slope of the stright line to be 2.67×10^{10} Hz . It corresponds to coherence length $l_c = \frac{c}{\delta\upsilon} \approx 1.1\ cm$.

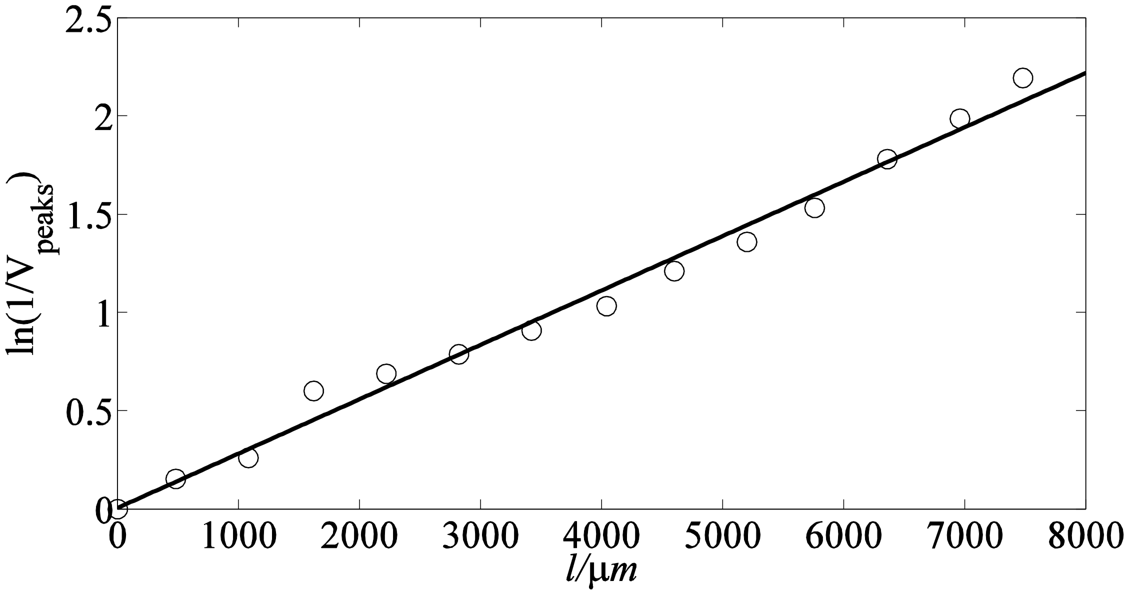


Fig. 8. $\ln(1/V_{peak})$ versus the optical path difference l measured for Na spectral lamp.

The experimental result in Fig. 7 is fitted (solid line) with the proposed theoretical model, Eq. (14), considering the data that obtained from the experimental results for the low-pressure spectral lamp as Lorentz type of the gain profile, N=2, $\delta\upsilon = 2.67 \times 10^{10}$ Hz and $\Delta\upsilon = 5.21 \times 10^{11}$ Hz

Conclusion

The presented theoretical study showed that the visibility behavior of two beams interference fringes for light beam of spectral fine structure differs from that without fine spectral structure. The fringe visibility is found to be periodically appear and disappear along equal intervals of changes in the optical path difference and associated with damping in the visibility. The behavior of the damping characteristic enables us to know the type of the spectral line distribution whether it is Lorentz or Gauss profile. Its spectral width can also be determined through the slope of the damped fringe visibility with the change of the optical path difference where the coherence length of the light beam can be calculated.

The equal intervals in the optical path difference at which the periodical fringe visibility reaches its maximum values gives the spectral spacing between the spectral fine structure. Experimental validation of the theoretical model was carried out for sodium doublet spectral lines and He-Ne laser beam of multispectral axial modes. The experimental results are in a good agreement with the theoretical calculations of the fringe visibility behavior.

References

[1] Johnson, J. R., A high resolution scanning confocal interferometer, Appl. Opt. **7** (1968) 1061-1072.

[2] Fork, R. L., Herriott, D. R. and Kogelnik, H., A scanning spherical mirror interferometer for spectral analysis of laser radiation, Appl. Opt. **3** (1964) 1471-1484.

[3] Hlubina, P., Experimental demonstration of the spectral interference between two beams of a low-coherence source at the output of a michelson interferometer, J. Mod. Opt. **44** (1997) 163-173.

[4] Hlubina, P., Fourier transform analysis of the spectral modulation obtained at the output of the uncompensated michelson interferometer Opto-Electr. Rev. **5** (1997) 117-122.

[5] Hlubina, P., Dispersive white-light spectral two-beam interference under general measurement conditions, Optik **114** (2003) 185-190.

[6] Hlubina, P., White-light spectral interferometry with the uncompensated michelson interferometer and the group refractive index dispersion in fused silica, Opt. Commun. **193** (2001) 1-7.

[7] Yamaoka, Y., Minoshima, K. and Matsumoto, H., Direct measurement of the group refractive index of air with interferometry between adjacent femtosecond pulses, Appl. Opt. **41** (2002) 4318-4324.

[8] Sinclair, D. C., Scanning spherical-mirror interferometers for the analysis of laser mode structure, Spectra-Physics laser Technical Bulletin (1968) 1-8.

[9] Pedrson, H. M., On the contrast of polychromatic speckle patterns and its dependence on surface roughness, Optica Acta: International Journal of Optics **22** (1975) 15-24.

[10] Swanson, N. L., Pham, C. N. and VanWinkle, D. H., Fringe visibility of multimode laser light scattered through turbid water, Appl. Opt. **36** (1997) 9509-9514.

[11] Illarramendi, M. A., Zubia, J., Arrue, J. and Ayesta, I., Adaption of the michelson interferometer for a better understanding of the temporal coherence in lasers, 14th Conference on Education and Training in Optics and Photonics: ETOP 2017 **1045249** (2017)

[12] Nakamura, J., Image sensors and signal processing for digital still cameras (CRC Press, Boca Raton, 2006).