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Detection and location of distribution systems faults using Wavelet packet transform based overcurrent relay

By

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Abstract

This paper presents a scheme to increase the sensitivity of overcurrent protection. The proposed scheme applies wavelet packet transform, since it can preserve both time and frequency information with high resolution. The proposed scheme demonstrates its feasible performance in detecting short circuit faults, calculating fault location with moderate accuracy and detecting nonlinear high impedance faults. In addition, the proposed scheme examines the load current continuously and has the ability of changing the relay pick up value adaptively. The examined tested cases include different internal short circuit fault conditions, different nonlinear high impedance faults, and non fault situations such as normal load changing. The obtained results indicate that the developed scheme is fast, robust, reliable and suitable for power distribution systems.

Keywords: Digital Overcurrent Relay, Fault location, High Impedance Faults, Wavelet Packet Transform.

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1. Introduction

The purpose of protective relaying systems is to isolate the faulty system as quickly as possible thus minimizing the trouble and damage caused by faults when they occur. Power lines are protected by overcurrent, distance, or pilot-relaying equipment, depending on the main requirements of reliability and selectivity. Overcurrent relays are the simplest and most widely used relays which cannot justify more expensive protection such as distance or pilot relays. However, overcurrent relaying needs readjustment or even replacement as a system changes. It is generally used as the primary protection for distribution feeders and the backup protection for transmission lines against either phase or ground faults. It is also widely used as a fault detector to enable other sensitive protective relays such as distance relays. The basic principle of overcurrent relay is that when the current flowing into the relay exceeds a predetermined amount, the relay operates with or without an intended time delay and trips the associated circuit breakers [1].
Some factors associated with overcurrent protection in distribution systems may result in incorrect discrimination between normal stable state and fault conditions such as in case of detecting nonlinear high impedance faults due to their low level of currents below the sensitivity settings of the relay, in such case, overcurrent relay will fail to trip [2].
Unfortunately, high sensitivity sometimes causes mal-trip of relay protection when there is no fault in the system due to transients, a longer delay may initiated for relay tripping; therefore as time passes and amplitude of the transient current diminishes, the mal-trip of the relay is prevented. However, this imposed delay, slows down the relay operation during faults and consequently reduces its sensitivity [3].

2. Main Features of the Proposed Overcurrent Relay:

This paper has addressed incorporating some main features for overcurrent relay resulting in more sensitivity in the protection when it is most needed. The proposed relay includes the feature of examining the load current continuously and therefore the relay pick up value is changed adaptively. Moreover, the proposed relay ensures some features to avoid some practical problems such as detecting short circuits faults with very small voltage variation, locating short circuits faults for inspection-repair purpose, and detecting non linear High-Impedance Faults (HIFs).

2.a Estimating Proper Value of RMS Fundamental:

One of the major power system problems is steady-state waveform distortion due to harmonics or subharmonics. Such distortion is produced by variable speed drives, arc furnaces, personal computers, and other non-linear devices result in severely degrading the performance of power system relaying. The proposed scheme uses Wavelet Packet Transform (WPT) to measure proper value of root mean square (RMS) fundamental and hence calculate trip time more accurately even that the signals are contaminated with other harmonics, subharmonics or interharmonics**.**

2.b Adjusting the Setting of Overcurrent Relays Adaptively:

Adjusting the setting of overcurrent relays is usually difficult where there is a wide variation in generating conditions. It cannot be adjusted based on the light load value as the relay may work under heavy load condition, in addition, it can't be adjusted based on the heavy load value or the feeder rated value as there is a great possibility that minimum faults may be below the maximum load current and hence remain undetected. Some methods have been proposed for adjusting the setting of overcurrent relay. The proposed scheme uses the method described in [4] to modify the pickup setting adaptively as a function of load current.

2.c Detecting Short Circuits Faults for Voltage Controlled Overcurrent Protection:

Discriminating between fault and full load currents when the current exceeds pickup current value and the voltage reduced to a value less than rated voltage is the main concept of the voltage dependent overcurrent protection. However in radial distribution system, some fault cases may occur very far from source remote end where the voltage maintains its normal value, in such case, conventional relay may not operate correctly and fail to trip. The proposed scheme checks the residual current effectively as a fault occurrence indicator in case of small voltage variation.

2.d Detecting Non linear High-Impedance Faults (HIFs):

In addition to bolted and linear short circuit faults, high impedance arcing faults have also a frequent occurrence on power lines. Arcing fault is a nonlinear phenomenon as the arc fault resistance is a nonlinear function of voltage and current which are difficult to detect through conventional protection such as overcurrent relays. This is principally due to relay insensitivity to the very low level fault currents which are below the sensitivity settings of the relay [5]. This type of fault usually occurs when a conductor makes a contact with a poor conductive surface. Several researchers in recent years have presented schemes aimed at detecting HIF more effectively; some of them use wavelet discrete transform for HIF detection [2]. The proposed scheme uses WPT coefficients to ensure nonlinear HIF detection and discrimination.

2.e Locating Short Circuit Faults:

Locating of short circuit faults with accepted accuracy for an inspection-repair purpose is of essential importance for reducing outage time, operating costs and customer complaints. Many techniques are proposed and implemented in both transmission and distribution systems for fault location [6-7]. Some of these techniques estimate the distance to fault using voltage and current measurements at both ends of a line. This paper uses the technique described in [8] to estimate the short circuit faults distance depends on fundamental frequency voltages and currents measured at one terminal before and during the fault.

3- Applying Wavelet Packets Transform (WPT) as a Signal Processing Tool:

Wavelet Transform (WT) is a method of analyzing transients associated with power system faults. It provides information related to the frequency decomposition of a waveform, and it is more appropriate than the familiar Fourier methods for non-periodic, wide band signals associated with electromagnetic transients [9].
However, there isn't much work on applying wavelets to RMS measurements required for overcurrent protection as the discrete wavelet transform (DWT) technique is not suitable for harmonic analysis because the resulting frequency bands do not have the same width and the results do not give easy insight into the time behaviour of the harmonics [10]. To overcome this limitation, wavelet packet transform (WPT) method is proposed that decomposes a given power system waveform into a set of harmonic bands with same frequency width, and each of these bands represents that part of the original instantaneous RMS occurring at that particular time and in that particular frequency band [11-12]. In wavelet packet analysis, the details as well as the approximations can be split as shown in Figure (1).

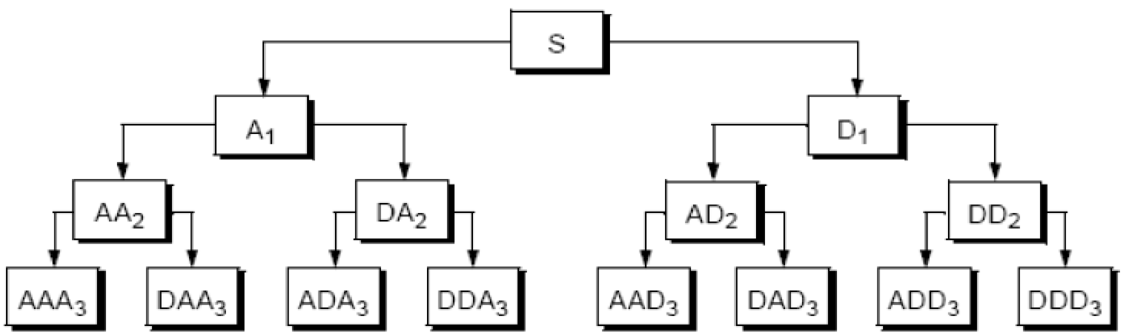


Figure (1): *The Wavelet Packet Decomposition Tree*

3.a Calculating Fundamental RMS Value Using WPT:

For the measured signal $i(t)$, its wavelet packet transform can be formulated as follows:

$$i(t) = \sum_k c_k \, \phi\,(t-k) + \sum_{j=0} \sum_k d_{j,k} \;\; \psi\,(2^j t - k) \tag{1}$$

Therefore, the total RMS value of the current signal can be calculated from the following equation:

$$i_{RMS} = \sqrt{\frac{1}{T} \; \sum_k C_0^2\,(k) \; + \; \frac{1}{T} \sum_j \sum_{k=0} d_j^2\,(k)} \tag{2}$$

As wavelet packet transform divides the signal into different frequency bands, using the wavelet coefficients of each band or component, the RMS value of each band could be calculated separately. Therefore, by selecting the appropriate sampling frequency and the wavelet analysis resolution level, each harmonic component can be allocated in an individual band and consequently, the proper RMS value can be determined. The total RMS equation can be rewritten in the following form [13]:

$$I_{RMS} = \sqrt{\frac{1}{T}\int_0^T i_t^2 \; dt} \cong \sqrt{\frac{1}{2^N}\sum_{n=0}^{2^N-1} i_n^2} = \sqrt{\frac{1}{2^N}\sum_{i=0} \sum_{k=0} \left(d_{j,k}^i\right)^2} = \sqrt{\sum_{i=0} (I_j^i)^2} \tag{3}$$

Thus, the RMS of each frequency band of node (i) and level (j) can be estimated as follows:

$$I_j^i = \sqrt{\frac{1}{2^N} \; \sum_{K=0} \; (d_j^i)^2} \tag{4}$$

Where: $2^N = 2^j * L_{dj}$

3.b Choosing Wavelet Filters:

The selection of which wavelet is used for the measurement is a formidable task. The accuracy of RMS measurements depends on the frequency response of the selected wavelet filter. In general, the frequency separation characteristic of the selected wavelet filter is dependent on the number of filter coefficients (number of vanishing moments). As the number of filter coefficients increases, more accurate measurements can be obtained [14-15]. To test the fundamental RMS measurements capability using WPT algorithm, two main cases, which are software generated waveforms, are analyzed using WAVELAB toolbox:-

- First case study: Generated waveform which contains beside the fundamental component the harmonics orders from the 2^{nd} to the 7^{th} (integer harmonics), as follows:

$X = 4.5 + \sqrt{2}\times 10\sin(2\pi\times 50\times t) + \sqrt{2}\times 9\sin(2\pi\times 50\times 2\times t) + \sqrt{2}\times 8\sin(2\pi\times 50\times 3\times t) + \sqrt{2}\times 7\sin(2\pi\times 50\times 4\times t) + \sqrt{2}\times 6\sin(2\pi\times 50\times 5\times t) + \sqrt{2}\times 5\sin(2\pi\times 50\times 6\times t) + \sqrt{2}\times 4\sin(2\pi\times 50\times 7\times t)$

The root mean square errors associated with mother wavelets of Db-4, Db-10, Db-20, Db-40 and DFT (Discrete Fourier Transform) for fundamental RMS measurements are shown in Table (1), using only one cycle of sampled signal. As obviously shown, mother wavelet (Db-20) and (Db-40) have the higher accuracy.

Table (1): *Fundamental component extraction accuracy for waveforms contaminated with harmonics*

Algorithm	Db-40	Db-20	Db-10	Db-4	DFT
Accuracy %	100	99.81	96.1	86.7	100

- Second case study: Generated waveform which contains beside the fundamental component, harmonics orders from the 2^{nd} to the 5^{th}, interharmonic components at 1.4, 2.6, 3.6 of the fundamental frequency, and subharmonic components at 0.4, 0.8 of fundamental frequency which are clearly non-integer multiple of the fundamental frequency as follows:

$X = 4.5 + \sqrt{2}\times 10\sin(2\pi\times 50\times t) + \sqrt{2}\times 9\sin(2\pi\times 50\times 2\times t) + \sqrt{2}\times 8\sin(2\pi\times 50\times 3\times t) + \sqrt{2}\times 7\sin(2\pi\times 50\times 4\times t) + \sqrt{2}\times 6\sin(2\pi\times 50\times 5\times t) + \sqrt{2}\times 4\sin(2\pi\times 50\times 0.4\times t) + \sqrt{2}\times 4\sin(2\pi\times 50\times 0.8\times t) + \sqrt{2}\times 4\sin(2\pi\times 50\times 1.4\times t) + \sqrt{2}\times 4\sin(2\pi\times 50\times 2.6\times t) + \sqrt{2}\times 4\sin(2\pi\times 50\times 3.6\times t)$

The root mean square errors associated with mother wavelets of Db-20, Db-40 and DFT for fundamental RMS measurements are shown in Table (2) for different sizes of sampling windows. As obviously shown, mother wavelet (Db-20) and (Db-40) have the higher accuracy and the measurement accuracy increases as the sampling window increase.

Table (2): *Fundamental component extraction accuracy for waveforms contaminated with harmonics, interharmonics and subharmonics*

Algorithm	Db20			Db40			DFT
	with 1 cycle	with 2 cycles	with 4 cycles	with 1 cycle	with 2 cycles	with 4 cycles	

Accuracy %	68.75	80.97	92.96	67.78	83.45	92.7	56.04

Refereeing to these above two tested cases, it can be concluded that the DFT techniques are the most susceptible to the effects of interharmonics and subharmonics, in addition, it is found that WPT technique is very adequate if sampling is carried out for several cycles of the samples, being quite resistant to the effects of interharmonics and subharmonics, but it is prone to more errors if sampling is carried out for only one cycle of the fundamental.

Db-20 will be used in the paper study as Db-40 has high numbers of filter coefficients so it needs very high hardware requirements to be implemented.

3.c Evaluating of RMS Measurements in Proposed Overcuurent Relay:

A series of tests was carried out to evaluate fundamental RMS measurements feature on trip time calculation for the proposed inverse overcurrent relay characteristic as depicted by the following expression:

$$T = TMS * \frac{C}{\left(\left(I_f/I_s\right)^{\alpha}\right)-1} \tag{5}$$

Different values of α and C for various types of inverse overcurrent characteristics defined by IEC and IEEE respectively are assigned as presented in Table (3). The results of testing different waveforms contaminated with harmonics, interharmonics and subharmonics using Db-20 are shown in Table (4). Such results clearly indicate the accepted performance.

Table (3): *Parameters for different types of inverse characteristics*

Relay characteristics type	c		α	
	IEC	IEEE	IEC	IEEE
Standard inverse(SI)	0.02	0.02	0.14	0.0515
very inverse(VI)	1	2	13.5	19.61
extremely inverse(EI)	2	2	80	28.2
long inverse(LI)	1	--	120	--

4. Fault Location Methodology:

The applied technique for fault location in this paper depends on Takagi algorithm method [8]. It requires extracting voltage and current fundamental components at a line terminal before and during the fault using the following formula:

$$D_f = \frac{Im\,(V_s\,(I''_{SF})^*)}{Im(\,Z\;I_s\,(I''_{SF})^*)} \tag{6}$$

The effect of the load flow is cancelled by using fault component current I''_{SF}, and the effect of fault resistance is reduced by eliminating fault resistance [8].

Table (4): *Evaluating of trip time calculation for proposed overcuurent relay in waveforms contaminated with harmonics, interharmonics and subharmonics*

Current (in p.u of pickup value)	**Required trip time** (in sec)	**Actual trip time** (in sec)	**Error** (in %)
1.2	4.4560	4.4380	0.403
1.4	2.0427	2.0356	0.347
1.6	1.2571	1.2533	0.302
1.8	0.8750	0.8700	0.571
2	0.6530	0.6500	0.459
2.7	0.3110	0.3100	0.321
4	0.1307	0.1303	0.306
4.9	0.0852	0.0850	0.234
5.5	0.0670	0.0669	0.149
6	0.0560	0.0559	0.178
6.7	0.0447	0.0445	0.447
7	0.0409	0.0407	0.488
7.5	0.0355	0.0354	0.281
8	0.0311	0.0310	0.321

5. Description of Proposed Scheme:

Figure (2) describes the flowchart of the proposed scheme started from WPT decomposition for current and voltages samples. The whole process is based on a moving window approach where the 1-cycle window is moved continuously by one sample. The sampling frequency is chosen to be 6400 Hz (128 samples per cycle).
Then, it calculates fundamental RMS for current and voltage as mentioned before in Section 3.a. Therefore, the relay then checks the fault conditions by:

- Comparing calculated RMS current and its instantaneous trip value, the relay trips instantaneously if the current value exceeds instantaneous trip value for specific short period.

- If the current exceeds the pickup value but less than instantaneous trip value, there is a possibility of either fault condition occurring or load increasing. In such case, the relay will utilize the voltage signals to act as a voltage controlled overcurrent relay to discriminate between fault current and full load current. If the voltage is reduced to a value less than rated voltage this means a fault case is detected. If not, the residual current "Ie" (can be calculated as the sum of the three phase currents) will be checked and used as a fault occurrence indicator in case of small voltage variation, such case may occur in radial distribution system as mentioned before in Section 2.c. If short circuit is detected, the fault location will be calculated as described in Section 4.
- In case of either not exceeding current pickup value, or exceeding pickup value but not accompanied with simultaneous occurring of either voltage decrease or residual current increase exceeding their setting values, there is a possibility of either high impedance fault condition occurring or load increasing. Therefore, the proposed index of absolute sum of WPT coefficients at node [1, 1] for current is calculated for one cycle period (*Sum_d1*). The criteria for the overcurrent relay to initiate a trip signal for HIF detection is such that the index (*Sum_d1*) must stay above the threshold level Sth continuously for three cycles (D samples). So, extensive series of studies have revealed along studied system lines and under non fault conditions, that result in the optimal settings for Sth at "73". The setting value of this threshold is dependent on the power system configuration. The criteria for the proposed relay to initiate a trip signal for HIE detection case is achieved when the counter "WI", that signifies the sample number, attains the optimal level setting that corresponding to three cycles (D samples).
- Finally, if there was not a HIF fault and still there is a considerable difference between the calculated RMS current and the stored load current value, that ensures variation in generating conditions (as mentioned before in Section 2.b). In such case, the scheme will not change its pick up value unless the new load value is settled at this condition for three consecutive cycles at power frequency.

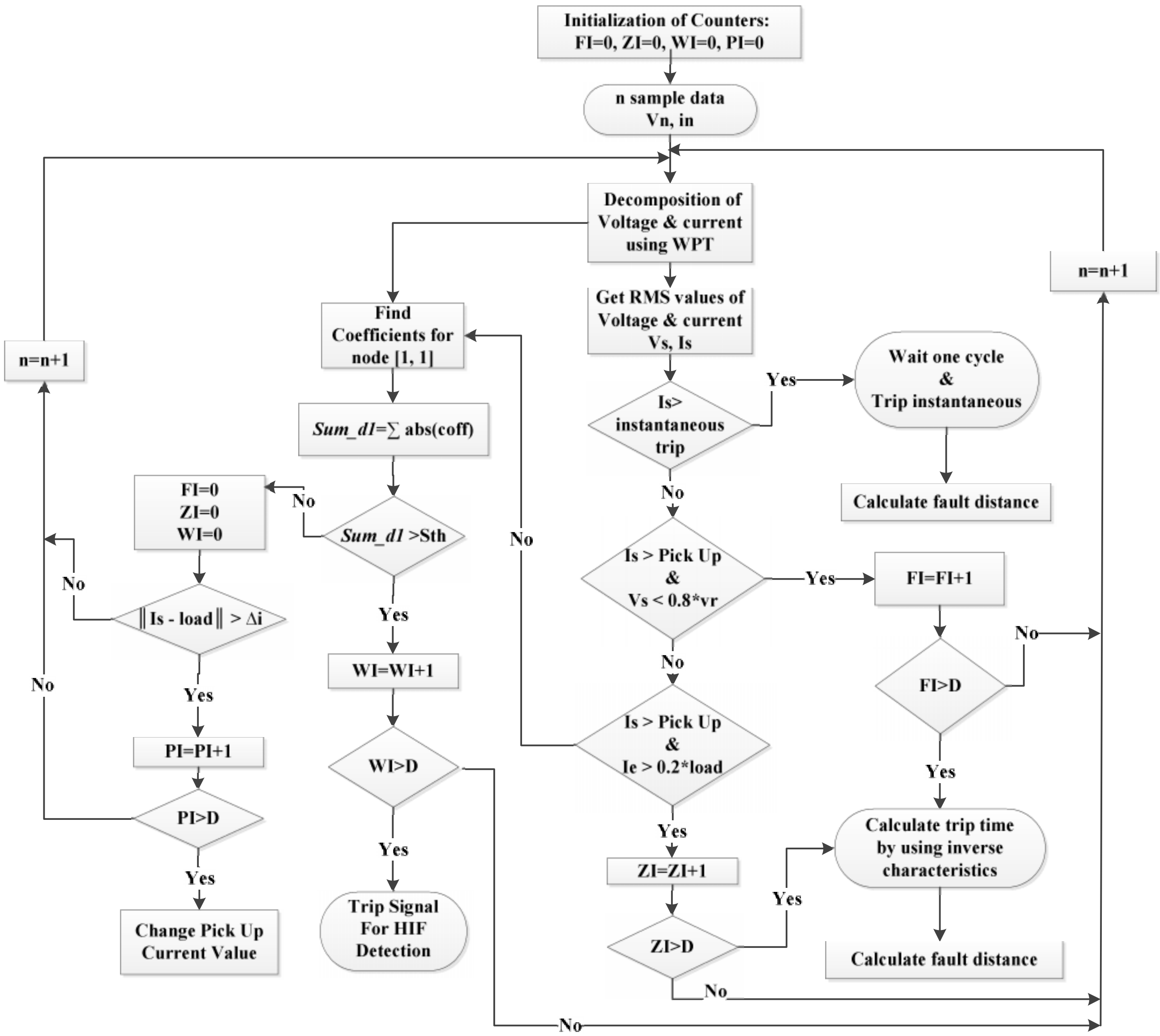


Figure (2): *Flow chart of the proposed overcurrent scheme*

6. Testing Proposed Scheme Performance:

The performance of the proposed scheme was evaluated by extensive tests carried out on radial distribution system 66/11 kV that shown in Figure (3) using ATP.

The proposed scheme at feeder F4 is considered the studied relay. Performance of the proposed relaying scheme is tested for tens times. The main tested cases investigated are: different internal short circuit fault conditions, different non linear high impedance faults (HIFs) and normal load changing. Some cases of these cases are illustrated as examples in details in the following subsequent sections.

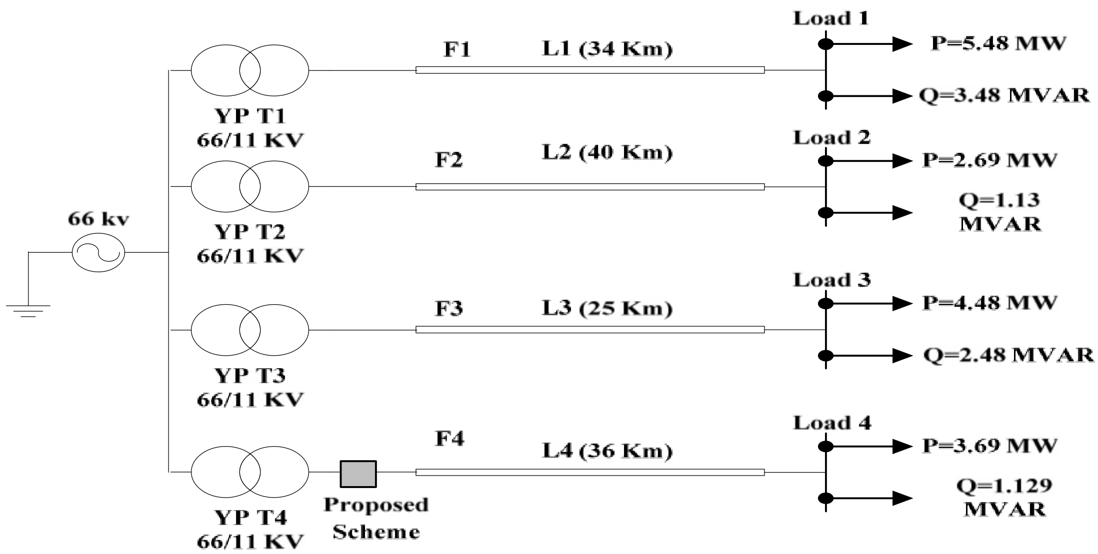


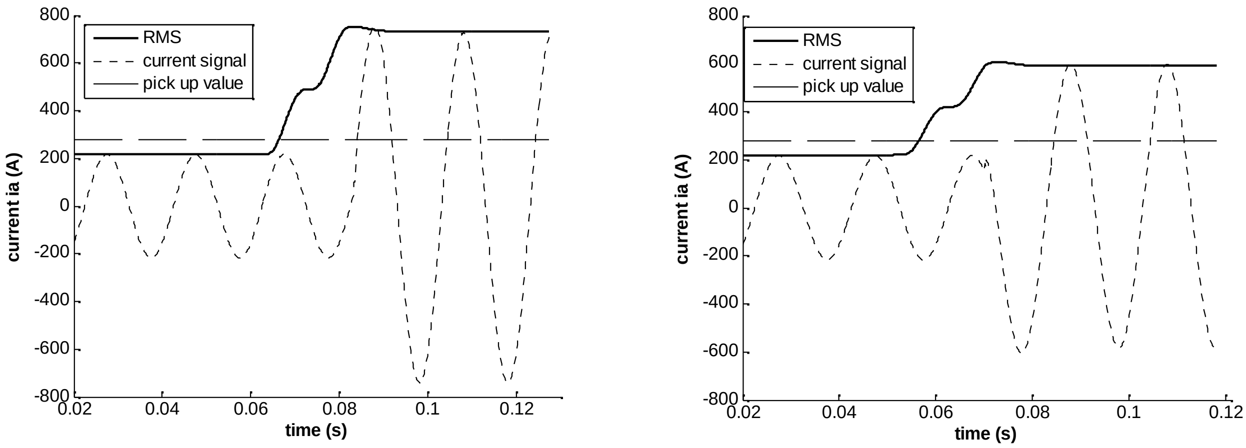
Figure (3): *Studied distribution system*

6.a Different internal short circuit fault conditions:

Several internal short circuit fault cases with different conditions were carried out ranged from small values (less than twice pick up value) to large values (8 times pick up value). As expected, the relay trips correctly at a time depending on the accurate RMS value of current measured using WPT according to tripping characteristics used. Figure (4) depicted the proposed relay detection response for examples of single line to ground faults (SLG). The proposed technique was tested for estimating the location of single-phase-to ground faults as shown in Table (5). Errors expressed as percentage of the line length indicate that the estimated distances of the faults are substantially accurate.

6.b Different nonlinear high impedance faults (HIFs) conditions:

A simplified 2-diode model of nonlinear HIF is used in the simulation as shown in Figure (5) based on arcing in sandy soil. It includes two DC sources, Vp and Vn, which represent the arcing voltage of air in soil and or between trees and the line. Two resistances, Rp and Rn, between diodes and DC voltages represent the resistance of trees and/or the earth resistance. In order to simulate asymmetric current, different values of Rp and Rn are used. When the line voltage is greater than the positive DC voltage Vp**,** the fault current starts flowing towards the ground. The fault current reverses backward from the ground when the line voltage is less than the negative DC voltage Vn. In case the line voltage is in between Vp and Vn, line voltage is counter-balanced by Vp or Vn so that no fault current flows. Figure (6) shows the responses for nonlinear HIF cases at 8 km, 16 km, 25 km and 33 km. In such cases, the criteria for the overcurrent relay to initiate a trip signal for HIF detection that the index (*Sum_d1*) must stay above the threshold level Sth continuously for three cycles.



Fault at 25.5km t=0.083sec r=3 Ω
time of trip =0.4969 sec

Fault at 30km t=0.07sec r=5 Ω
time of trip =0.887 sec

Figure (4): *SLG (AG) internal fault cases at different locations, different time (inception angles) and different fault resistance*

Table (5): *Fault location estimate for SLG fault cases with fault resistance of 1.0 ohm*

Fault distance (km)	**Calculated distance (km)**	**Error (%)**	**Fault distance (km)**	**Calculated distance (km)**	**Error (%)**
4.5	4.568	0.18	25.5	25.870	1.02
9	9.127	0.35	28	28.410	1.13
12.5	12.676	0.47	30	30.443	1.23
17	17.240	0.66	32	32.477	1.32

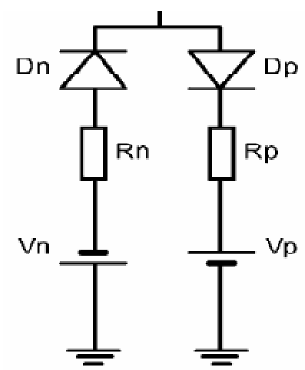
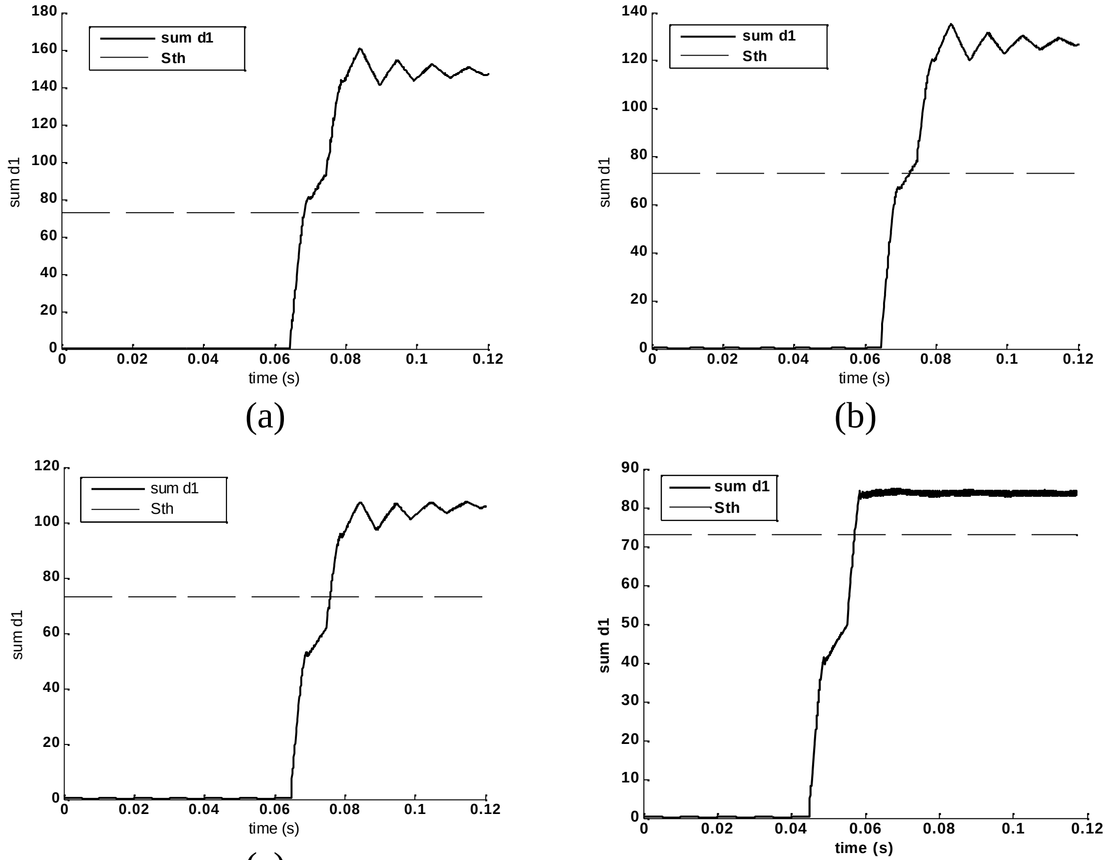


Figure (5): *Simplified 2-diode fault models of nonlinear high impedance faults (HIFs).*

6.c Normal load changing and changing the pick-up value adaptively:

Figures (7-a & 7-b) show the relay current waveform at relaying point and proposed index *Sum_d1*, respectively for a normal load changing case. In such case, there is no significant change in voltage, residual current; moreover no detection for HIF is achieved as the proposed index value does not stay above the threshold level for more than three cycles. Therefore, the significant change in RMS current value is correctly interpreted as load increasing, and therefore the pickup is changed to a new value.

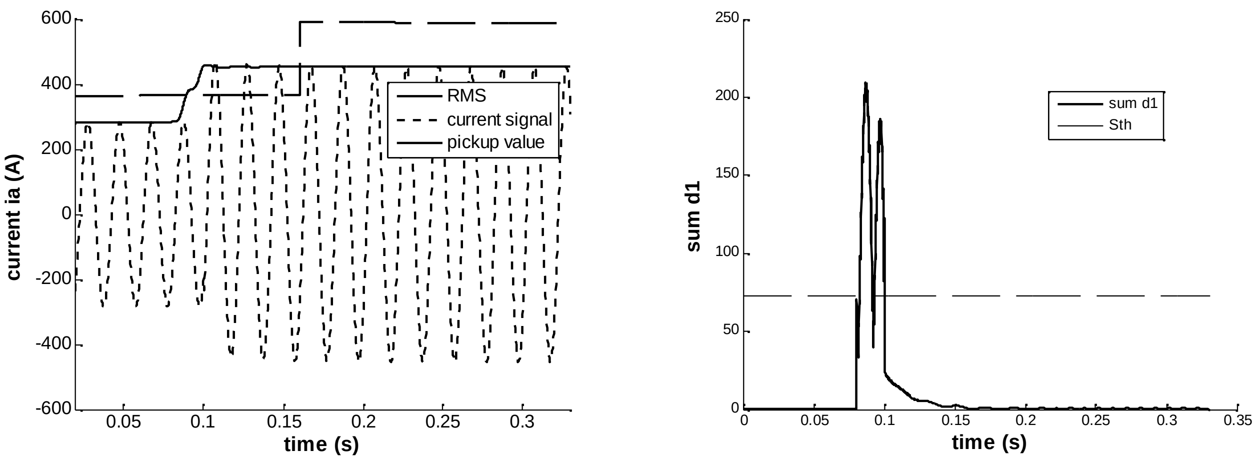


(a) (b) (c) (d)

Figure (6): *Behavior of index "Sum_d1" under HIF at different fault locations (a) 8 km, (c) 16 km, (d) 25 km, (e) 33 km from relay*

7. Conclusions:

This paper has addressed the application of a proposed overcurrent relaying scheme. The scheme uses WPT to decompose power system waveforms into a set of harmonic bands with the same frequency width, Db-20 is used since it has sharper cutoff frequency compared with others and hence the leakage energy between different resolution levels is reduced. The main achieved objectives are: estimating proper value of RMS fundamental in case of waveform distortion due to harmonics subharmonics, or interharmonics, adjusting the pickup setting adaptively, detecting and locating short circuits faults, and detecting non linear High-Impedance Faults. It is proved using extensive simulations tests that the proposed scheme is simple, accurate, and fast and can be used for updating, improving, and refurbishing of the existing overcurrent relays.



(a) (b)

Figure (7): *Load changing case (a): Current signal, (b): Sum_d1 index behavior*

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Nomenclatures:

c_k …	Approximated coefficients
$d_{j,k}$ …	Detail coefficients
$\phi(t)$ …	Scaling function
$\psi(t)$…	Wavelet function
d_j^i …	Wavelet coefficient at level (j) and node (i)
L_{dj} …	No. of points in wavelet coefficient d_j^i
D_f…	Distance to the fault point
V_s …	Voltage of source terminal
I_s …	Current of source terminal
I_{SF}'' …	Current difference between pre-fault and after-fault (fault component current)
Z …	Transmission line impedance per unit length
Im …	Imaginary component
*…	Conjugate component