Military Technical College
Kobry El-Kobbah,
Cairo, Egypt

7th International Conference
on Electrical Engineering
ICEENG 2010

Effect of XPM on WDM Optical Transmission System in Presence of First- and Second Order GVD

By

M. A. Khayer Azad * M. S. Islam *

Abstract:

Transmission in wavelength division multiplexing (WDM) optical communication system is mainly impaired and ultimately limited by group velocity dispersion (GVD) and cross-phase modulation (XPM). In this paper, we analytically determine and compare the impact of XPM in a WDM system in presence of first- and second order GVD for standard single mode fiber (SSMF) and dispersion shifted fiber (DSF). Even at high bit rate in a first order GVD compensated system, the second order GVD plays a critical role in limiting the distance of optical signal. The results show that XPM crosstalk penalty due to second order GVD is 16 dB more in DSF than that of SSMF for 0.8 nm channel spacing at 10 GHz modulation frequency. It is also found that the spectral characteristics are strongly dependent on the channel spacing and dispersion of the fiber.

Keywords:

Cross-phase modulation, Group velocity dispersion, Power transfer function, Fiber nonlinearity.

---

* Institute of Information and Communication Technology , BUET, Bangladesh
E-mail: khayer98cse@yahoo.com

1. Introduction:

Optical transmission is a preferred medium for long distance, high bandwidth communication system running at speeds in the range of gigabit per second or higher.

The process towards ever increasing speeds encounters an obstacle in the form of the group velocity dispersion (GVD) in the optical fiber that restricts bit rates [1]-[2]. On the other hand, the progress towards longer lengths of transmission has led to increasing input power and at higher power, nonlinear effect is introduced in the optical fiber that in turn also limits the transmission distance of optical signal [3]-[4]. In late 1990s wavelength division multiplexing (WDM) systems have been widely deployed as a solution for higher bit rate transmission. With the increasing demands on the capacity of WDM systems, crosstalk due to nonlinearity become important and cross-phase modulation (XPM) is one of the most significant nonlinear effects that impact the system performance[5]-[6]-[7]. GVD is a linear and XPM is a nonlinear phenomenon in optical fibers only when two or more optical channels are transmitted through the fiber simultaneously. As a result, the combined effects of GVD and XPM may cause further deterioration of transmission performance in a WDM system. However due to the presence of Group velocity dispersion of the fiber, the phase modulation can be converted into intensity fluctuations and thus can degrade the performance of intensity modulation- direct detection (IM-DD) systems.

Karfaa et.al. [8], simulate the effect of Cross-phase Modulation crosstalk in WDM Networks on Received power and number of Channels for various fiber types. *Abdul-Rashid et al.* [9], investigate the system performance limitation due to XPM and GVD in a sub-carrier multiplexing WDM passive optical network in terms of power penalty. *Rongqing, et al* [10] investigated spectral characteristics of cross-phase modulation in multi-span intensity modulation direct detection systems both theoretically and experimentally. It was shown that interference between XPM induced crosstalk components created in different amplified spans had strong impact on overall frequency response of XPM crosstalk in the system. However, the impact of second order dispersion was ignored. Since second order GVD plays a critical role in limiting the distance of optical signal at high bit rate. In this paper, we have extended results reported in *Rongqing, et al* [10] by investigating the intensity fluctuations characteristics of WDM systems including second order dispersion effects.

2. Theoretical Analysis:

Using the slowly-varying envelope approximation, the propagation of the channels along the transmission medium is governed by the nonlinear Schrödinger equation (NLS). For this analysis, we consider only the attenuation factor, the first– and second-order GVD, as well as the SPM and XPM effects in the NLS equation given for an arbitrary channel k as follows:

$$\frac{\partial A_k}{\partial z} + \frac{\alpha_k}{2} A_k + \frac{1}{v_{gk}} \cdot \frac{\partial A_k}{\partial t} + \frac{i}{2}\beta_{2k} \frac{\partial^2 A_k}{\partial t^2} - \frac{1}{6}\beta_{3k} \frac{\partial^3 A_k}{\partial t^3} = i\gamma_k \left[|A_k|^2 + 2 \sum_{j=1,\, j \neq k}^{N} |A_k|^2 \right] A_k, \qquad (1)$$

where *k* is the current channel under consideration, *N* is the total number of channels, α_k represents the attenuation parameter, β_{2k} and β_{3k} are the first -and second-order GVD parameter, respectively, γ_k is the nonlinearity coefficient and v_{jk} is the group velocity.

The first two terms on the right-hand side of Eq. (1) are the nonlinearity factors that arise due to the nonlinear refractive index of the fiber. The XPM effect is modeled in the same way as the SPM effect by introducing another term on the nonlinear part of the NLS equation. This term sums the effect of the XPM caused by all other channels. The effective nonlinear refractive index n2 of the XPM effect is twice that of the SPM effect. This leads to a factor of 2 applied to the magnitude of the sum.

In order to simplify the analysis and focus our attention on the effect of XPM induced inter channel crosstalk, we neglect the interaction between SPM and XPM and pretend that these two act independently. We will assume that the probe signal is operated in continuous wave (CW), whereas the pump signal is modulated with a sinusoidal wave at a frequency Ω. Although the effect of SPM for both the probe and the pump channels are neglected in this XPM calculation, a complete system performance evaluation must take into account the effect of SPM and other nonlinear effects separately. This approximation is valid as long as the pump signal waveform is not appreciably changed by the SPM induced distortion before its optical power is significantly reduced by the fiber attenuation. Using the substitutions $T=t-z/v_k$ and

$$A_k(t,z) = E_k(T,z)\exp(-\alpha z/2) \tag{2}$$

we have,

$$\frac{\partial E_k(T,z)}{\partial z} = -\frac{i\beta_{2k}}{2}\frac{\partial^2 E_k(T,z)}{\partial T^2} + \frac{\beta_{3k}}{6}\frac{\partial^3 E_k(T,z)}{\partial T^3} + i\gamma_k 2p_k(T-d_{jkz},0).\exp(-\alpha z)E_k(T,z) \tag{3}$$

where $d_{jk} \equiv (1/v_j)-(1/v_k)$ is the relative walk-off between the probe and the pump. Using a linear approximation, the walk-off d_{jk} can be expressed as $d_{jk} = D\Delta\lambda_{jk}$, where $D=-(2\pi c/\lambda^2)\beta_{2k}$ is the dispersion coefficient and second order GVD, $\beta_3 = \frac{\lambda^3}{2\pi^2 c^2}\left[D(\lambda)+\frac{\lambda}{2}.dispersionSlope\right]$, $\Delta\lambda_{jk}$ and λ are the wavelength spacing and the average wavelength between the probe and pump, respectively, and c is the light velocity. Here a linear approximation is used for d_{jk} for simplicity.

In general, dispersion and non-linearity act together along the fiber. However, in an infinitesimal fiber section dz, it can be assumed that dispersive and non-linear effects act independently, the same idea as used in the split-step Fourier method1. Substituting $E_k = |E_k|\exp[i\Phi_k(T,z)]$ in equation (3) gives

$$d\phi_k(T,z') = i\gamma_k 2P_j(T + d_{jk}z',0)\exp(-\alpha z')dz \qquad (4)$$

The Fourier transformation of this phase variation gives

$$d\phi_k(\Omega,z') = i\gamma_k 2P_j(\Omega,0)e^{(-\alpha+i\Omega d_{jk})z'}dz \qquad (5)$$

Neglecting the intensity fluctuations of the probe channel, this phase change corresponds to change in electrical field, $|E_k|\exp[id\phi_k(T,z')] \approx |E_k|[1 + id\Phi(T,z')]$ or in Fourier domain $|E_k|\exp[id\phi_k(\Omega,z')] \approx |E_k|[1 + id\Phi(\Omega,z')]$ Due to chromatic dispersion of the fiber, this phase variation generated at z=z' is converted into an amplitude variation at the end of the fiber z=L. Taking into account only the dispersion and source term of the phase perturbation at z=z', the Fourier transform of equation (3) becomes

$$\frac{\partial E_k(\Omega,z)}{\partial z} = -\frac{i\beta_{2k}\Omega^2}{2}E_k(\Omega,z) - \frac{\beta_{3k}\Omega^3}{6}E_k(\Omega,z) + |E_k|[1 + id\Phi(\Omega,z')]\delta(z - z') \qquad (6)$$

Where the Kronecker delta $\delta(z - z')$ is introduced to take into account the fact the source term exists only in the infinitesimal fiber section at z=z'. Therefore at z=L the probe field becomes

$$E_k(\Omega,L) = E_k + id\Phi_k(\Omega,z')E_k\exp\left[\left(\frac{i\beta_{2k}\Omega^2}{2} - \frac{\beta_{3k}\Omega^3}{6}\right)(L - z')\right] \qquad (7)$$

The optical power variation caused by non-linear phase modulation created in the short section dz at z=z' is thus

$$\Delta a_{jk}(\Omega.z',L) = |E_k(\Omega,L)|^2 - E_k^2 = -2E_k^2 d\Phi_k(\Omega,z')\sin\left[\left(\frac{\beta_{2k}\Omega^2}{2} + \frac{i\beta_{3k}\Omega^3}{6}\right)(L - z')\right] \qquad (8)$$

where linearization has been made considering that $d\phi_k$ is infinitesimal.

Using $E_k(T,z) = A_k(T + z/v_k, z)\exp(\alpha z/2)$ and equation (5) and integrating all self phase modulation and cross phase modulation contributions along the fiber, gives the total intensity fluctuations at the end of the fiber.

$$\Delta s_{jk}(\Omega,L) = -4\gamma_k E_k^2\exp(-(\alpha - i\Omega/v_k)L).\int_0^L P_j(\Omega,0).Sin\left[(\beta_{2k}\Omega^2/2 + i\beta_{3k}\Omega^3/6)(L - z')\right].\exp(-(\alpha - i\Omega d_{jk})z')dz' \qquad (9)$$

Where $P_k = |A_k|^2$ and $\Delta s_{jk}(\Omega,L) = \Delta a_{jk}(\Omega,L)e^{-\alpha L}$ represents fluctuations of A_k .Which on integration gives

$$\Delta s_{jk}(\Omega,L) = 2\gamma_k P_k(L)e^{i\Omega L/v_k}\left\{\frac{\exp(i\beta_{2k}\Omega^2 L/2 + \beta_{3k}\Omega^3 L/6) - \exp(-\alpha + i\Omega d_{jk})L}{i(\alpha - i\Omega d_{jk} - i\beta_{2k}\Omega^2/2 - \beta_{3k}\Omega^3/6}\right\} - 2\gamma_k P_k(L)e^{i\Omega L/v_k}\left\{\frac{\exp(-i\beta_{2k}\Omega^2 L/2 - \beta_{3k}\Omega^3 L/6) - \exp(-\alpha + i\Omega d_{jk})L}{i(\alpha - i\Omega d_{jk} - i\beta_{2k}\Omega^2/2 - \beta_{3k}\Omega^3/6}\right\} \qquad (10)$$

where $P_k(0)$ and $P_k(L)$ are probe optical power at input and output of the fiber.

Under the assumption that $\exp(-\alpha z) << 1$ and that modulation bandwidth is much smaller than the channel spacing, ie, $d_{jk} >> \beta_{2k}\Omega/2$ gives a simpler frequency domain description of the intensity fluctuations in the probe channel by the pump channel.

$$\Delta s_{jk}(\Omega, L) = 4\gamma_k P_k(L) P_j(\Omega, 0).\exp(i\Omega L / v_k).\frac{\sin(\beta_{2k}\Omega^2 L/2 + \beta_{3k}\Omega^3 L/6)}{\alpha - i\Omega d_{jk}} \tag{11}$$

In the time domain, the probe output optical power with XPM induced crosstalk is,

$$P_{jk}(t, L) = p_k(L) + \Delta s_{jk}(t, L) \tag{12}$$

$\Delta s_{jk}(t, L)$ is the inverse Fourier transform of $\Delta s_{jk}(\Omega, L)$ and $p_k(L)$ is the probe output without XPM. After the square law detection of photo diode, the electrical power spectral density is the Fourier transform of the correlation of the time domain optical intensity waveform. Therefore,

$$P_k(\Omega, L) = \eta^2 \left[P_k^2(L)\delta(\Omega) + \left|\Delta s_{jk}(\Omega, L)\right|^2 \right] \tag{13}$$

where η is the photodiode responsivity.

For Ω > 0, the XPM induced electrical power spectral density in the probe channel, normalized to its power level without this effect can be expressed as

$$\Delta P_{jk}(\Omega, L) = \frac{\eta^2 \left|\Delta s_{jk}(\Omega, L)\right|^2}{\eta^2 P_k^2(L)} \tag{14}$$

$\Delta P_{jk}(\Omega, L)$ is defined as the normalized XPM power transfer function.

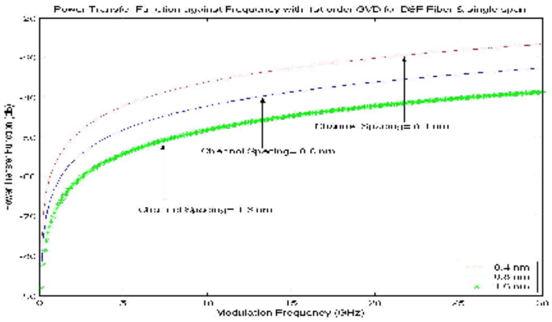
$$\Delta P_{jk}(\Omega, L) = 4\gamma_k P_j(\Omega, 0).\exp(i\Omega L d_{jk}).\frac{\sin(\beta_{2k}\Omega^2 L/2 + \beta_{3k}\Omega^3 L/6)}{\alpha - i\Omega d_{jk}} \tag{15}$$

3. Results and Discussion

To verify the theoretical expressions developed earlier, numerical solutions have been achieved by means of Matltab simulations to run the calculations and to produce the graphic relations that appear in the Fig.1 to Fig. 7. Different system parameters are shown in Table 1.

The plots for power transfer function against modulation frequency are shown in Fig. 1 and Fig. 2. Figure 1 shows the response for first- order GVD for DSF Fiber and it is found that the spectral characteristics depend on the channel spacing. Fig. 2 indicates the impact second order GVD in totally absence of first- order GVD and it is found that second order GVD plays a important roles at high bit rate. The plots for power transfer function against modulation frequency are shown in Fig. 3. Fig. 3 indicates the impact second order GVD in totally absence of first- order GVD and it is found that second order GVD plays a important roles at high bit rate.

Modulation frequency for DSF fiber is shown in Fig.6 and the plots for power transfer function against modulation frequency for various no of channel are shown in Fig. 7. The results shown in Fig. 6 and Fig. 7 are that XPM crosstalk penalty increased with increasing the no of channel as well as modulation frequency.



Figure(1): Power Transfer Function against Frequency with First order GVD for DSF Fiber.

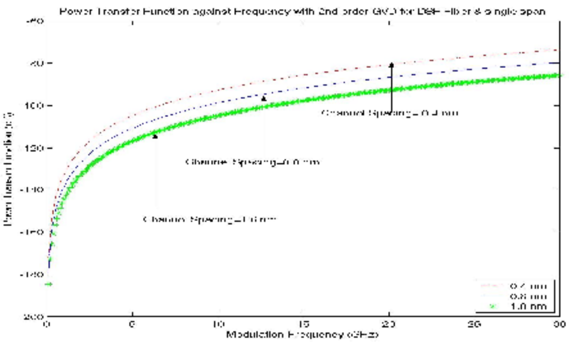


Figure (2): *Power Transfer Function against Frequency with Second order GVD for DSF Fiber*

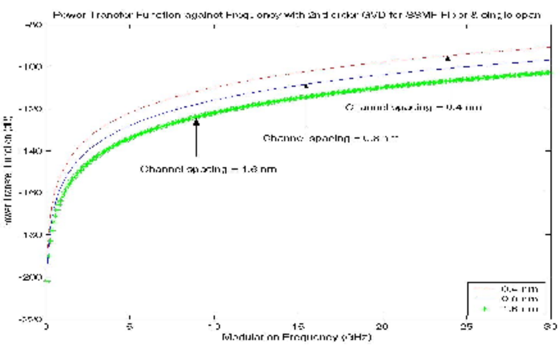


Figure (3): *Power Transfer Function against Frequency with Second order GVD for*

SSMF Fiber

The plots for power transfer function against modulation frequency with first- order GVD and second order GVD are shown in Fig. 4 and Fig. 5 respectively where DSF and SSMF XPM crosstalk penalty are compared. . The results shows that XPM crosstalk penalty due to first- order GVD is 23 dB and second order GVD is 16 dB more in DSF than that of SSMF at 10 GHz modulation frequency and 0.8nm channel spacing.

The plots for power transfer function against modulation frequency with first- order GVD and second order GVD respectively where DSF and SSMF XPM crosstalk penalty are compared. . The results shows that XPM crosstalk penalty due to first- order GVD is 23 dB and second order GVD is 16 dB more in DSF than that of SSMF at 10 GHz modulation frequency and 0.8nm channel spacing. The plots for power transfer function against no of channel for various

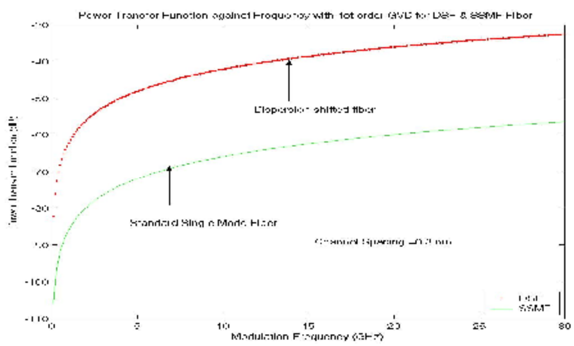


*Fig. 4**:** Power Transfer Function against Frequency with first order GVD for DSF& SSMF*

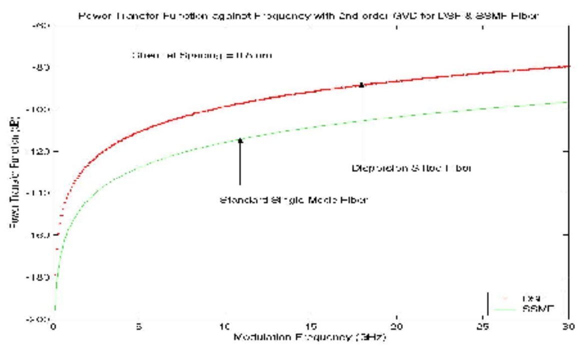


*Fig. 5**:** Power Transfer Function against Frequency with Second order GVD for DSF & SSMF*

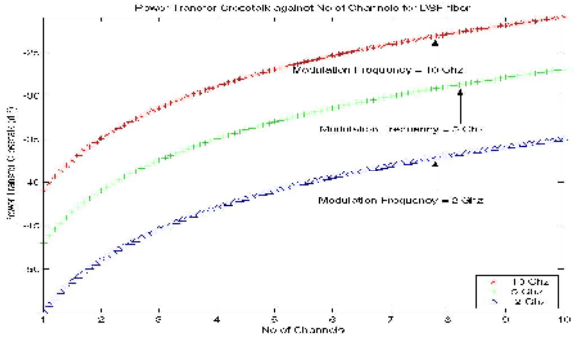


Fig. 6: Power Transfer Function against no of channel for various frequencies

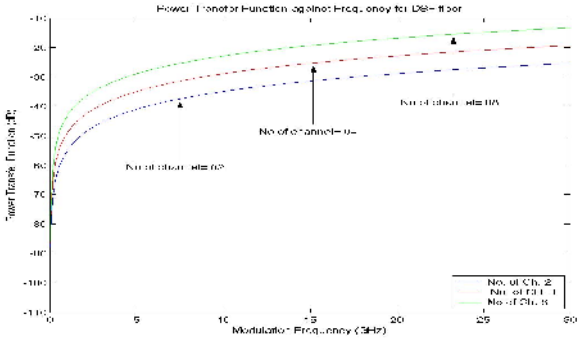


Fig. 7: Power Transfer Function against Frequency for various no of Channel

Table 1: Different system parameters

Parameter (unit)	**SSMF**	**DSF**
Probe wavelength (	1559	1559
Channel spacing	0.4, 0.8,	0.4, 0.8,
Zero dispersion	0.095	0.075
Effective Area (m^2)	8.0X10-	5.5X10-
Attenuation	0.25	0.25
Dispersion	17	3.5
First order GVD	0.206	0.4515
Second order	0.192295	0.1995
Fiber Length (km)	100	100
Input pump power	11.5	11.5
Input Probe Power	11.5	11.5

6. Conclusions:

A detailed analysis is carried out to evaluate the impact of XPM in WDM system in presence of first- and second order GVD. Results are evaluated and compared for two types of commercially available fiber SSMF and DSF respectively. We observed that XPM has more impact on DSF fiber than that of SSMF. It is also found that the second order GVD has less impact on intensity fluctuation on the probe channel on WDM system, however at high bit rate if the first order GVD is zero then there is significant impact of second order GVD on optical transmission system.

References:

[1] N. T. Dang and A.T. Pham. "*Performance degradation of spectral amplitude encoding optical code division multiple access systems due to group velocity dispersion.*" Proc. of 14th Asia-Pacific Conference on Communication 2008 (APCC 2008), vol. 1, pp. 1 – 5, 14-16 Oct. 2008.

[2] S.P. Majumder, M. R. Islam and R. Gangopadhyay. "*Effect of group velocity dispersion on a WDM optical ring network.*" Proc. of Lasers and Electro-Optics Society Annual Meeting (LEOS '98), vol. 2, pp. 236 – 237, 1-4 December 1998.

[3] G. P. Agrawal."*Nonlinear Fiber Optics*."Academic Press,2002.

[4] F. Forghieri, R. W. Tkach, and A. R. Chraplyvy. "*Fiber nonlinearities and their impact on transmission systems.*" in Optical Fiber Telecommunications IIIA, I. P. Kaminov and T. L. Koch, Eds. San Diego, CA: Academic, 1997.

[5] S. Betti and M. Giaconi. "*Effect of cross phase modulation on WDM optical systems: analysis of fiber propagation*", IEEE Photonics Technology Letters, vol. 13, pp. 305-307, April 2001.

[6] M. Nishimura. "*Optical fibers and fiber dispersion compensators for high-speed optical communication*", Journal of Optical Fiber Communication Report 2, Editor: Hans-GeorgWeber, Masataka Nakazawa, pp. 115–139, 2005.

[7] E. J. Gualda, L. C. Go´ Mez-Pavo´ And J. P. Torres, "*Compensation of third-order dispersion in a 100 Gb/s single channel system with in-line fiber Bragg gratings*", Journal of Modern Optics, vol. 52, no. 9, pp.1197–1206, 2005.

[8] Y. M. Karfaa, M. Ismail, F. M. Abbou, S. shaari, and S. P. Majumder. "*Effects of cross-phase modulation crosstalk in WDM networks on received power and number of channels for various fiber types*", Proc. of Asia Pacific Conference on Applied Electromagnetic (APCAE 2007), Malaysia, vol. 1, pp 22-27, 2007.

[9] H. A. Abdul-Rashid, F. M. Abbou, H. T. Chuah, M. B. Tayahi, M. T. Al-Qdah, and S. Lanka. "*System performance optimization in SCM-WDM passive optical networks in the presence of XPM and GVD*", IEEE Communications Letters, vol. 10, 2006.

[10] R. Hui, K. R. Demarest and C. T. Allen. "*Cross-phase modulation in multispan WDM optical fiber systems*", Journal of Lightwave Technology, vol. 17, no. 6 pp. 1018-1026, June 1999.