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Recursive Two Dimensional Adaptive Digital Filter

By
AHMED FOUAD MOHAMED ABD EL-RAZEK*

Abstract:

In this paper, a new scheme of recursive two -dimensional (2-D) adaptive digital filter is presented. It consists of a forward section, a feedback section, and decision block. Each section is realized as conventional non-recursive adaptive filter. Either 2 Ð adaptive LMS or sign function algorithm can be used for adaptation. The proposed recursive filter functions as a 2-D adaptive line enhancer (ALE) for images contaminated with noise. Its performance is compared with that of Hadhoud [1].

Keywords:

Recursive two -dimensional (R-2-D), adaptive line enhancer (ALE)

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* Electrical and Computer Engineering Dept., Higher Technological Institute, tenth of Ramadan city.

1. Introduction:

The subject of adaptive filtering has established itself as an important branch of statistical signal processing , with applications in various interesting fields particularly in image The two dimensional signal can be modeled as non-stationary 2-D random field in which the filter design is based . Thus, the resultant filter is no longer optimum. This necessitates the use of 2-D adaptive filters.

Indeed, the adaptive filtering problem is to find an algorithm for adjusting the filter parameters in a situation where complete knowledge of the relevant signal characteristics are not available, so that the performance of the adaptive filter may convergent to that of the optimum filter after a sufficiently large number of iterations of the algorithm. The pioneering work of Hadhoud Thonous [1] extended the 1-D least mean square (LMS) adaptive algorithm (Widrow's algorithm [2] to the 2-D case. In [3] Jenkins and Faust proposed a similar algorithm, joined to the McClellan transformation. Although these algorithms can successfully get the filter coefficients، they update the filter coefficients only along the horizontal direction of the 2-D planز
Consequently, these LMS algorithms are inherently 1-D adaptive algorithms which don't fully utilize the information content of the 2-D signals. Besides the performance degradation due to an isotropic processing.

Usually adaptive digital niters are classified into non-recursive or finite impulse response (FIR) filters, and recursive or infinite response (IIR) filters. Non-recursive digital filters are inherently stable, but require two times as many coefficient as recursive filters when compared to same performance index. Recursive adaptive niters may suffer from two major problems due to feed back (1) they may be unstable, and (2) the adaptation of their coefficients may drive same poles outside the unit circle. Thus recursive adaptive filters are usually susceptible to instability. Despite these difficulties, adaptive recursive filters are preferred .particularly when processing a large amount of 2-D data, because they are less complex and more fasted adaptable than non-recursive filters.

In this paper a new model of 2-D adaptive recursive digital filter is presented. It consists of two non- recursive parts: the forward section and the feedback section. The filter coefficients are adjusted in accordance with the 2-D least mean square method (2-D LSM) and the 2-D -sign - algorithm which converges to both the horizontal and the vertical directions of the 2-D plane. To evaluate the performance of the proposed scheme, the adaptive recursive filter is implemented as a 2-D adaptive line enhancer (ALE) of an image contaminated with noise. The performance the ALE presented is compared with that of the ALE presented by Hadhoud in [1]. The proposed ALE proves its superiority in noise reduction when applied to noisy images.

2. The Proposed model of the 2-D Adaptive Recursive Digital Filter:

Given the 2-D recursive filter transfer function; H [z_1, z_2] of the form

$$H[z_1, z_2] = B[z_1, z_2] / A[z_1, z_2] \tag{1}$$

and writing the denominator polynomial as follows

$$A[z_1, z_2] = k + C[z_1, z_2] \tag{2}$$

Where the polynomial C(z_1, z_2) does not contain any constant term (see [4]),then the filter transfer function will be

$$H[z_1, z_2] = B[z_1, z_2] / (k + C[z_1, z_2]) \tag{3}$$

H (z_1, z_2) can be realized by a forward section, B (z_1, z_2)/k, and a feedback section, C(z_1, z_2)/k. We exploit this property in the design of the proposed 2-D adaptive recursive filter depicted in figure 1. Both the forward and the feedback sections are non-recursive adaptive digital filters which can be realized using conventional method.
The restriction on the actual implementation of the 2-D adaptive digital filter of figure 1 is reliability. Unrealizable delay-free loops in the structure must be avoided and this can be accomplished by graph transformation of the resulting filter [5]. The purpose of the decision block shown in figure 1 is to avoid instability which might arise from adaptation of the feedback section coefficients. Stability is ensured due to the boundedness of the feedback signal.

Filter adaptation

Regardless the filter type, the filter coefficients should reach their optimum values using an adjustment method (adaptation algorithm) which involves updating each coefficient during each sequence sample. The importance of a specific algorithm is to be underlined as it will directly influence both the transient and the steady state performance of the filter as well as complexity of its implementation. In most applications, the least mean square algorithm is recommended because of the easy implementation and the favorable maladjustment versus convergence speed trade off. In our model, we use the 2-D LMS algorithm for adaptation of both forward and feedback sections.

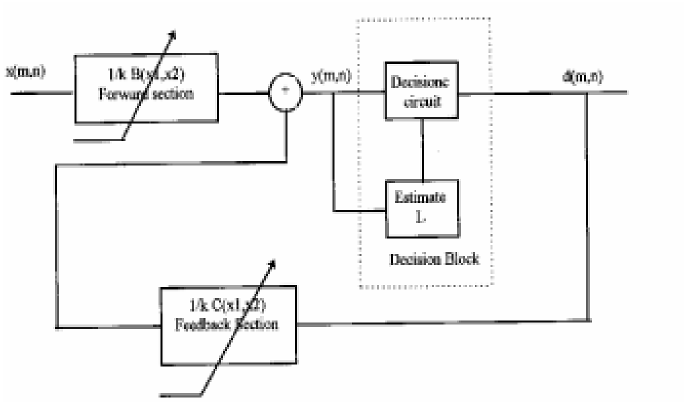

Figure (1): *The proposed 2-D adaptive recursive digital filter*

As stated in the previous section, each part of the proposed 2-D adaptive recursive digital filter is a non-recursive filter, represented by the following convolution

$$y(m,n)=\sum_{k=0}^{n-1}\sum_{l=0}^{n-1} h(k,l)x(m-k,n-l) \qquad (4)$$

Where y(m,n) and x(m,n) are the output and the reference 2-D input signals respectively. The support of the 2-D signal is assumed to be of size M x M. The filter coefficients are h(k,l) and of N x N support.

In order that the output y(m,n) tracks the desired input d(m,n), the 2-D LMS algorithm updates the filter coefficients h(k,l), via the following equation.

$$h_{m+1,n+1}(k,l) = f_h h_{m,n+1}(k,l) + f_v h_{m+1,n}(k,l) + \mu_h e(m,n+1)x(m-k,n-l+1) + \mu_v e(m+1,n)x(m-k+1,n-l), \qquad (5)$$

With the initial values h(k,1)=0, m=0,………...M and n=0,…………...M, where $h_{m+1,n+1}(k,1)$ are the filter coefficients at the number (m+1,n+1,) and f_h , f_v are the positive scaling factors satisfying the identify $f_h + f_v=1$

μ_h and μ_v are step sizes in horizontal and vertical directions, respectively and e(m,n) is the error between the filter's output and the desired signal.

e(m,n)=d(m,n)-y(m,n)

The filter used in (4) is an N x N causal filter

When the reference signal, x (m,n), and the desired signal, d(m,n), are stationary, the algorithm in equation (5) converges into mean square sense to an optimal solution as $m+n \rightarrow \infty$. The optimum coefficient vector Hopt satisfies the 2-D Weiner-Hopf equation given by

$$RH_{opt}=P,$$

Where the convariance matrix R is defined

$$R=\begin{bmatrix} R_0 & R_{-1} & \dots & R_{1-N} \\ R_1 & R_0 & \dots & R_{2-N} \\ \\ R_{N-1} & R_{N-2} & \dots & R_0 \end{bmatrix},$$

and

$$Ri=\begin{bmatrix} r_{i,0} & r_{i,-1} & \dots & r_{i,1-N} \\ r_{i,1} & ri,o & \dots & r_{i,2-N} \\ \\ r_{i,N-1} & r_{i,N-2} & \dots & r_{i,0} \end{bmatrix},$$

$$r(k,l)= E\{x(m,n)x(m-k,n-l)\},$$

E{.} is exepectation operator.

The matrix R is real, symmetric, or blocks Toeplitz and generally non negative define. It is here assumed positive define. The optimum coefficient vector H_{opt} is defined by

$$H_{opt}=\begin{bmatrix} h_{opt}(0,0) & h_{opt}(0,1) & \dots\dots & h_{opt}(0,N-1) \\ h_{opt}(1,0) & h_{opt}(1,1) & \dots\dots & h_{opt}(1,N-1) \\ \\ h_{opt}(N-1,0) & h_{opt}(N-1,1) & \dots\dots & h_{opt}(N-1,N-1) \end{bmatrix}$$

The vector P is the cross correlation vector between d(m,n) and x(m,n) given by

$P=[p(0,0),p(0,l),..,p(N-l,N-l)]$, $\quad p(k,l)=E\{d(m,n)x(m-k,n-l)\}$ (6)

The condition of the convergence of the 2-D LMS algorithm given in [6 , eq.(5)] is

$$|\ f_h - \mu_h\lambda_i\ | + |\ f_v - \mu_v\lambda_i\ | < 1 \quad , 1 \le i \le N, \tag{7}$$

Where is an Eigen value of the matrix R

Assuming f_h, = f_v =0.5 and that 4h= |-iv= Ho reduces to

$$|\ 1 - \mu\lambda_i\ | < 1, \qquad 1 \le i \le N, \tag{8}$$

which implies that X, ^0 l<i<N, i.e the matrix R must be positive definite. Furthermore the constant 4 should satisfy the set of equalities (8),

$$\text{i.e} \qquad 0<\mu<\frac{2}{\lambda_{max}} \tag{9}$$

Where implies that λ_{max} is the largest Eigen value of R. From (9) , we find that the condition of the convergence of 2-D LMS algorithm, is the same as that of 1-D LMS algorithm.

As in the 1-D case [7], the positive parameter, 4, controls the convergence of the 2-D LMS algorithm. Convergence is ensured by choosing u, according to (9). As u. increases, both the convergence speed and the mean square error increase and vice versa. Thus, the choice of p. is a compromise between the speed and the accuracy of the convergence.

One may ensure both rapid and convergence through the use of a large value of u, at the start of the adaptation and a smaller one after convergence has been approached. In practical limited precision digital implementation of adaptive filter, it is not advisable to choose a very small value of u. Indeed, if a very small value of u. is used, the coefficient correction terms in the algorithm (5) may become smaller than half of the least significant bit of the coefficient. Thus the adaptation stops and the mean square error remains the same.

As a result of using the 2-D LMS method, the filter coefficients do not exactly reach their optimum values and in the steady state the mean square error (MSE) of the output is larger than that obtained when using optimum coefficients. The MSE is calculated as

$$\text{MSE}=\frac{1}{(M-N)^2}\sum_{m=N}^{M-1}\sum_{n=N}^{M-1}e(m,n)^2 \tag{10}$$

To simplify the implementation of the 2-D LMS algorithm without seriously affecting its performance, a simple update scheme using non-linear multipliers to produce the coefficient correction terms is introduced to reduce the implementation complexity at the cost of performance. One such scheme the sign (clipped) algorithm [8] given by.

$$\begin{aligned} h_{m+1,n+1}(k,l) &= f_h h_{m,n+1}(k,l) + f_v h_{m+1,n}(k,l) \\ &+ \mu_h e(m,n+1)\operatorname{sgn}[x(m-k,n-l+1)] \\ &+ \mu_v e(m+1,n)\operatorname{sgn}[x(m-k+1,n-l)], \end{aligned} \tag{11}$$

Where sgn(x) is defined as,

$$\operatorname{sgn} = \begin{cases} +1 & \text{if } x>0\ , \\ -1 & \text{if } x\le 0 \end{cases} \tag{12}$$

In this algorithm, the signal used for updating the coefficients is corrupted by a great amount of quantization noise. Consequently, the step sizes, and, used with the sign algorithm should be smaller than those used with the 2-D LMS algorithm in order to maintain the same convergence accuracy. Hence, if the speed of convergence is not a problem, the sign algorithm may be very attractive for adaptive processing due to extreme simplicity of its implementation

Decision Block

The decision block is a conventional decision circuit with multiple threshold levels. For example, in case when the data sequence takes one of the 2M levels.

$$(\pm1,\pm3,\pm5,...,\pm(2M-1))$$

The decision circuit will have (2M-1) threshold levels, Their values will be

$$0,\pm2L,\pm4L,.,,2(M-1)L$$

In order to determine the threshold levels, the value of L must be estimated. The method we use for estimation is as follows:
In digital systems, the signal to noise ratio (SNR) is usually high and then.

$$E\{y(m,n)\}=|£|E\{d(m,n)\} \qquad (13)$$

Since E{d(m,n)} is a known quantity, given by the statistical characteristics of the input data L , can be obtained by a time averaging circuit applied to y(m,n) and whose gain is adjusted to the value 1/E {\L }. Evidently, the noise will affect the estimate of.

L For high SNR, this may be ignored. The value of L is used instead of L in assigning the values of the threshold levels. In case when L is positive, this makes no difference. However, when L is negative then using L instead of L will result in sign inversion of the whole data sequences. In most practical cases [9], sign inversion of the whole data sequences is allowed. If this is not the case, then an easy measure that depends on the protocol of data processing can recognize and correct this sign inversion.

3. 2-D Adaptive Line Enhancer

Here, we consider the 2-D adaptive Ine enhancer (ALE) as an application to the proposed 2-D adaptive recursive digital filter described in the previous section. The 2-D ALE which depicted in figure 2, is an adaptive filter for noise reduction.

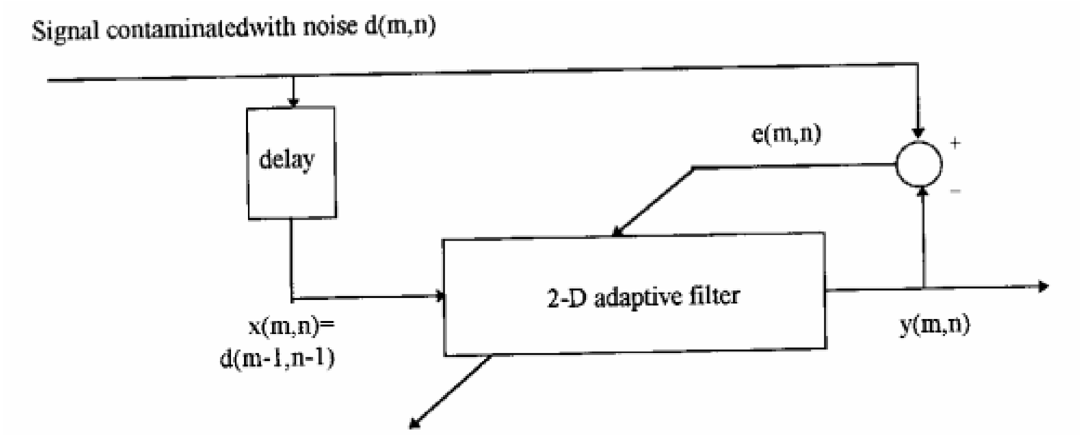

Figure (2): *The 2-D adaptive line enhancer*

As shown in figure 2, the desired input d(m,n) is the input signal and the reference signal x(m,n) is the delayed version of d(m,n).The signal y(m,n) represents the output of the ALE, The ALE is implemented and its performance is studied. The used input signal is an image of size 128 x 128 and the filter mask size is 3 x 3 i.e. M=128 and N=3. The images of a girl (128 x 128) produced by the applied Mathematic and Computing Group laboratory of Cranfield institute of Technology used in our experiments. The ALE based on the proposed 2-D adaptive digital filter is designed using 2-D LMS algorithm with $f_h = f_v = 0.5$ and $\mu_h = \mu_v = \mu$ and we call it ALE one. For the proposed of comparison the ALE based on Handhold's 2-D LMS algorithm [1] is also implemented and called ALE2. In both ALE'S the value of the parameter μ is chosen to be the optimum value that makes the mean square error (MSE) between the desired input and the output minimum and it is decided by trail and error.

To evaluate and compare the performance of the ALE based on the proposed adaptive filter, first the ideal image of figure 3 is applied to ALE1 and ALE2. The values taken by μ and MSE are 2.1×10^{-6} and 225 respectively, while for the ALE2 they take the values $\mu = 1.6 \times 10^{-6}$ and MSE =169. The results are shown in figure 4 and 5. The image processed by the ALE'S is blurred to some extend. The image processed by the ALE1 is sharper than that of the ALE2. One can find that the MSE of the ALE2 is smaller than that of the ALE1. That is because the traceability of ALE1 to local statistic of images whose statistics vary gradually is superior to that of the ALE2. The blur caused by ALE1 is greater than that of ALE2.

In the second experiment, the performance of the ALE'S is evaluated and compared in the presence of noise. The ALE1 with $\mu = 1.9 \times 10^{-6}$ and MSE-341, and the ALE2 with $\mu = 6.5 \times 10^{-7}$ and MSE =325 are designed. The noisy image of figure 6 is applied to the two ALE'S. The noise is additive Gaussian white noise with zero mean and its variance is adjusted to make the signal to noise ratio (SNR) 6dB. The results are shown in figures 7 and 8. The ALE1 produces better results of noise reduction in flat areas than that of ALE2, and the effect of the ALE2 for noise reduction in edge areas is almost equal to ALE1. The ALE1 does not impose any degradation as in the case of ALE2.

Finally, the ALE is implemented using the 2-D sign algorithm with $f_h = f_v = 0.5$, $\mu_h=\mu_v=\mu$ and compared with ALE1. Again, the parameter u. is chosen by trail and error to obtain the minimum MSE. μ takes the value 1.5 x 10^{-7} and MSE = 192. It is noticed that the sign algorithm converges more slowly than the 2-D LMS algorithm.
However, when 4 are quite small both algorithms ensure almost the same MSE. It should be reminded that the main advantage of the sign algorithm is its simple implementation. Consequently, the sign algorithm is recommended for the cases, in which the interest is the technical simplicity rather than the convergence speed.….

4. Conclusions:

The 2-D adaptive filter is usually used when the statistical characteristics of the data processed do not mach prior information on which the design of 2-D adaptive recursive digital filters. The new model consists of two sections: the forward section and the backward section. Each section is a conventional non-recursive adaptive filter. The stability of the filter is ensured due to the fact that the use of the decision block bounds the feedback signal. The filter coefficients are adjusted in accordance with the 2-D least mean square method (2-D LMS) which converges to both the horizontal and the vertical directions of the 2-D plane. Hence, the proposed filter does not impose degradation on the data being processed. To evaluate the performance of the proposed adaptive filter, it is implemented in the form of adaptive line enhancer (ALE). The ALE is capable of blurring images without imposing degradation. It also shows a good performance when it is used for noise reduction. When comparing the ALE based on the proposed adaptive filter with that of Hadhoud. It proves to be superior. The ALE is implemented using the 2-D sign algorithm, and compared with ALE adapted with the 2-D LMS algorithm. When the control parameter 4 is quite small both algorithm ensure the same MSE. The sign algorithm has the advantage of simple implementation. In a conclusion, the sign algorithm is recommended for the cases in which the main interest is the technical simplicity rather than convergence speed.

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Figure 3: The original image of a girl

Figure 4: The image of the girl processed by ALE1

Figure 5: The image of the girl processed by ALE2

Figure 6: The noisy image (SNR = 6 dB)

Figure 7: The effect of noise reduction with ALE1

Figure 8: The effect of noise reduction with ALE2