

Assessment of some global geopotential models over Egypt and the Kingdom of Saudi Arabia

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Abstract: Orthometric heights related to the geoid, are important in many engineering projects. The use of Global Positioning System (GPS) does not give the heights of the points on the surface of the geoid but gives them on the surface of the reference ellipsoid. Geoid undulations should be offered to obtain the needed orthometric heights from their corresponding available ellipsoidal heights. The aim of this research is to obtain the best among the Global Geopotential Models (GGMs) that give the results closest to the observed geoid heights over two study areas. Firstly, in Egypt and secondly in the Kingdom of Saudi Arabia (KSA). In the first study area, RMS of the differences between the observed undulations (GPS/levelling) and those from GOCO05c, XGM2016, and GECO are the lowest values among the ten tested GGMs. In the second study area, RMS of the differences between the undulations from KSA-GEOID21 and those from XGM2019e_2159, XGM2016, and XGM2019 are the lowest among the ten tested GGMs.

Keywords: GGMs, GNSS, ICGEM, HARN, KSA-GEOID21.

1. Introduction

Precise determination of the orthometric height (H) is required in many fields like construction, geodesy, and geophysics. Orthometric heights can be obtained using the GNSS positioning and a global geopotential model. It replaces conventional levelling techniques because it is faster and there is no restriction in distances. For these reasons, the behavior of these models, in terms of accuracy is always a concern [4].

The GGMs use estimated spherical harmonic coefficients to depict the Earth's gravitational field at various wavelengths. The GGMs created by scientists were published by the International Centre for Gravity Earth Models (ICGEM). Accuracy and resolution are influenced by the model's degree and the data used to create the GGM. The GGMs are often divided into three categories: tailored, combination, and satellite-only models. The satellite only GGMs coefficients came from studies of satellite orbital deviations. In comparison to other models, the degree and accuracy of these models are typically lower. Different data categories, including satellite and terrestrial data, are integrated to create the combined model. These models are more accurate and have a higher degree than the satellite-only models. The tailored models, however, are based on the recalculation and optimization of previously acquired GGMs within the framework of some mathematical concepts [5].

The majority of applications that need a datum to calculate topographic heights or ocean depths use the geoid surface as a reference. Our understanding of the Earth's gravity field must be greatly improved to determine orbits and height systems in science and engineering, both in terms of precision and spatial resolution. The GGM is a set of fully normalized, spherical harmonic coefficients that are

derived from geopotential solutions and is used to determine the long wavelength component of the earth's gravity field. These coefficients are calculated by merging the satellite observations, gravity data from land and ship tracks, marine gravity anomalies obtained from satellite radar altimetry, and aerial gravity data [14].

[12] gave an assessment of the five global geopotential models (EIGEN-6C4, EIGEN-6C2, EGM2008, EGM96 and GECO) with GPS/levelling points. The results for assessment of these GGMs show that the GECO GGM gives the best results compared with GPS/Levelling data, where the standard deviation of the undulation differences from 17 HARN stations (GPS/Levelling data) and GECO GGM is ± 0.42 m.

[2] Presented a study to assess the accuracy of five GGMs along coastal zones of Egypt. The RMS of geoid undulation differences, at 145 points, between GNSS/levelling undulations and their corresponding values from the five GGMs (XGM2016, XGM2019e, EIGEN-6C4, GO_CONS_GCF_2_TIM_R6e, and EGM2008) are estimated. The results indicated that EGM2008 gives the highest RMS of all models; the lowest values were 0.670 m from GO_CONS_GCF_2_TIM_R6e and 0.303 m from XGM2019e for regions A and B, respectively. Area B is situated in Egypt's eastern zone along both sides of the Red Sea Suez Gulf and stretches to the Gulf of Aqaba, whereas area A is situated in Egypt's northern zone along the Mediterranean Sea.

[1] Evaluated two Global Geopotential Models (GGMs), EGM96 and EGM08 with GPS/Levelling at 17 points of High Accuracy Reference Network (HARN) over Egypt. The results showed that EGM08 is better where the standard deviation of the geoid undulation differences

between GPS/Levelling and EGM96 is ± 1.212 m and that of EGM08 is ± 0.543 m.

[3] Compared KSA-GEOID17 and the geoid heights computed from GNSS/levelling at 3465 points in KSA. The result of the comparison showed that the standard deviation of the geoid height differences is 0.074 m.

In this research, two study areas are used to assess ten GGMs. The first study area is in Egypt, where the geoid heights of the 17 points of the 30 points of the HARN are obtained from their ellipsoidal and orthometric heights.

Those geoid heights will be compared with their corresponding values from the ten tested GGMs. The second study area is in the Kingdom of Saudi Arabia, where the geoid heights obtained from KSA-GEOID21 will be compared with their corresponding values from the ten GGMs.

These ten models were chosen based on the best accuracy of the results obtained from these models in many countries such as Mexico, USA, Canada, Brazil, Australia, Europe, and Japan, see Table (1).

Table (1): The Root Mean Square Error (RMSE) of the Ten Models about Mean of GPS/Levelling Minus Gravity Field Model Derived Geoid Heights [10].

No.	Model	Australia (7224 points)	Brazil (1154 points)	Canada (2706 points)	Europe (1047 points)	Japan (816 points)	Mexico (4898 points)	USA (6169 points)
1	SGG-UGM-1	0.092 m	0.241 m	0.141 m	0.121 m	0.076 m	0.189 m	0.245 m
2	SGG-UGM-2	0.091 m	0.234 m	0.139 m	0.121 m	0.074 m	0.19 m	0.249 m
3	XGM2016	0.104 m	0.213 m	0.16 m	0.14 m	0.125 m	0.178 m	0.263 m
4	XGM2019	0.103 m	0.213 m	0.159 m	0.14 m	0.125 m	0.178 m	0.264 m
5	XGM2019e_2159	0.097 m	0.208 m	0.139 m	0.127 m	0.09 m	0.173 m	0.248 m
6	GECO	0.095 m	0.233 m	0.142 m	0.123 m	0.08 m	0.186 m	0.246 m
7	EIGEN-6C	0.106 m	0.242 m	0.146 m	0.128 m	0.082 m	0.195 m	0.247 m
8	EIGEN-6C3stat	0.095 m	0.237 m	0.14 m	0.121 m	0.078 m	0.197 m	0.247 m
9	EIGEN-6C4	0.091 m	0.234 m	0.137 m	0.121 m	0.079 m	0.197 m	0.247 m
10	GOCO05C	0.105 m	0.219 m	0.163 m	0.138 m	0.217 m	0.221 m	0.262 m

2. Background

2.1 Geoid, Orthometric, and Ellipsoidal Heights

The points with GPS ellipsoidal heights are related to a reference ellipsoid (WGS84), and those of orthometric heights are related to an equipotential reference (geoid). They can be incorporated to compute the geoid height by geometrical approach. The GPS/Levelling geoid heights are computed by the following equation.

$$N = h - H \quad (1)$$

Where N is the geoid height, h is the ellipsoidal height determined from GPS, and H is the orthometric height determined from the levelling process, see Figure (1).

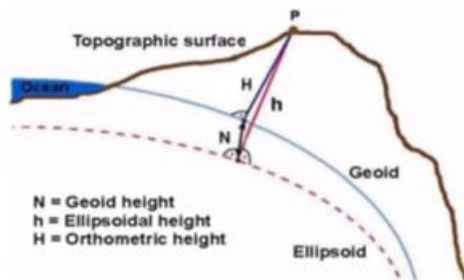


Fig (1): Relation between Ellipsoidal, Orthometric, and Geoid Heights [13].

2.2 Global Geopotential Models

The geoid height (N) can be represented by a set of spherical harmonic coefficients in spherical approximation with the following equation (2).

$$N(\theta, \lambda) \approx R \sum_{l=2}^{l_{\max}} \sum_{m=0}^l \bar{P}_{lm}(\sin \theta) [\bar{C}_{lm} \cos m\lambda + \bar{S}_{lm} \sin m\lambda] \quad (2)$$

where (θ, λ) co-latitude and longitude of the computation point, R is the Earth mean radius, \bar{P}_{lm} s the associated Legendre polynomials, \bar{C}_{lm} and \bar{S}_{lm} are the spherical harmonic coefficients for degree l and order m , respectively [14].

3. Study Areas

The first study area is in Egypt, and 17 GPS/Levelling HARN points are used. The ellipsoidal heights of these points are known on the WGS84, and the orthometric heights of them are also known as first-order levelling related to the mean sea level, see Figure (2). The second study area comprises 86 points distributed over KSA by a $1^\circ \times 1^\circ$ grid. These points have their geoid heights from KSA-GEOID21 see Figure (3).



Fig (2): The Locations of 17 HARN Points Over Egypt.

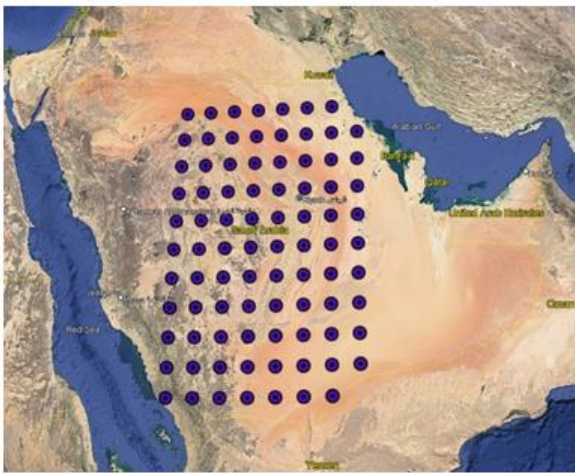


Fig (3): Distribution of Geoid Heights Points Over the Kingdom of Saudi Arabia.

4. Data used

4.1 GPS/Levelling data

The ten GGMs were validated using 17 GPS/levelling stations as a part of a High Accuracy Reference Network (HARN). The HARN consists of 30 stations approximately 200 km apart, covering all of Egypt with a standard accuracy of 1:10,000,000 [11]. Figure (2) shows the positions of these points.

4.2 KSA-GEOID21

KSA-GEOID21 is the most recent Geoid Model in the Kingdom of Saudi Arabia after Previous geoid models. KSA-GEOID21 is a hybrid geoid model for the Kingdom of Saudi Arabia determined using land and ship-borne pointwise data, GASGI GPS levelling data, airborne gravity, and satellite altimetry gridded data. The KSA-GEOID21 is a main component of KSA-Vertical Reference Frame (KSA-VRF), which allows the computation of orthometric heights for the points over KSA different from KSA- National Vertical Network (KSA-NVN) benchmarks. The accuracy of KSA-GEOID21 is more homogeneous through the Kingdom of Saudi Arabia compared to KSA-GEOID17 due to the better distribution of input data. The

average accuracy of derived geoid height from KSA-GEOID21 is better than 2 cm for the entire Kingdom of Saudi Arabia [8].

4.3 The Global Geopotential Models Used for Assessment

In this study, ten global geopotential models were chosen for comparison based on the best accuracy of their results obtained in many countries such as Mexico, USA, Canada, Brazil, Australia, Europe, and Japan. Some of the data about these models are illustrated in Table (2).

5. Methodology

- 1- The ICGEM web page is used to compute the geoid heights of the stations from the Global Geopotential Models (GGMs) using the user-defined points in the study area [6].
- 2- Computing the geoid heights at the 17 HARN points in the first study area (Egypt) using equation (1).
- 3- Estimating the geoid heights of the 86 points of the second study area (KSA) from KSA-GEOID21 [9].
- 4- Computing the differences between the geoid heights determined from the ten GGMs and GPS/Levelling using equation (3).

$$\Delta N = N_{\text{GPS/Levelling}} - N_{\text{GGM}} \quad (3)$$

Where,

- $N_{\text{GPS/Levelling}}$: The geoid height, the difference between the ellipsoidal height and the orthometric height at the station.
 - N_{GGM} : The geoid height of the station obtained from the Global Geopotential Model.
- 5- Determining the differences between the geoid heights from KSA-GEOID21 and those obtained from ten GGMs at the 86 points of KSA using equation (4).

$$\Delta N = N_{\text{KSA-GEOID21}} - N_{\text{GGM}} \quad (4)$$

Where,

- $N_{\text{KSA-GEOID21}}$: The geoid height of the KSA-GEOID21 at the station.
- 6- Validating the obtained differences of equations (3) and (4) using the statistical values, minimum, maximum, mean, standard deviation and root mean square error.
 - 7- The Root Mean Square Error (RMSE) is determined for each GGM by equation (5). This equation represented the closeness between the actual values of geoid heights and the estimated values of geoid heights from GGM.

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^n \Delta N_i^2}{n}} \quad (5)$$

Where,

- ΔN_i : The differences in geoid heights between the actual values determined from (GPS/Levelling or KSA-GEOID21) and estimated values obtained from GGM at each point in the study area.
- n : The number of points used in assessment in the study area.
- 8- The standard deviation is determined using equation (6). The equation concerns the differences between the actual and estimated values against their mean value.

$$SD = \sqrt{\frac{\sum_{i=1}^n (\Delta N_i - \Delta N_{\text{mean}})^2}{n-1}} \quad (6)$$

Where,

- ΔN_{mean} : The mean of the geoidal height differences, ΔN_{mean} is calculated from the following equation:

$$\Delta N_{\text{mean}} = \frac{1}{n} \sum_{i=1}^n \Delta N_i \quad (7)$$

[1].

6. Results

The aim of this research is to evaluate the different Global Geopotential Models that gave the best results in different study areas around the world in two study areas. The first study area is in Egypt and the second study area is in the Kingdom of Saudi Arabia.

The statistical analysis for each GGM has been performed at the two study areas in the form of the minimum, maximum, mean, standard deviation, and root mean squared errors see Tables (3) and (4).

With respect to the first study area, results in table (3), the mean values range from -0.663 to -0.604 m except the mean of SGG-UGM 1 which is -0.744 m. the standard deviation values range from 0.396 to 0.470 m and those of RMSE range from 0.726 to 0.774 m except of SGG-UGM 1 which is 0.868 m. so nine GGMs are close to each other's except SGG-UGM 1.

The best GGM is GOCO05C produced an RMSE equals to 0.726 m, and the arrangement of the GGMs which gave the smallest to the highest values of RMSE are XGM2016, GECO, EIGEN-6C4, EIGEN-6C3stat, XGM2019, XGM2019e_2159, SGG-UGM-2, EIGEN-6C, and SGG-UGM-1.

The results in table (1) showed that the biggest RMS values of the ten tested GGMs was in USA, and they range from 0.246 to 0.264 m. Recalling that [1] tested EGM2008 at the 17 HARN stations and the standard deviation of the differences was 0.543 m, and [12] tested GECO at the 17 HARN stations and the standard deviation of the differences was 0.42 m which matches the results here.

The results of the statistical analysis of the ten GGMs over the second study area, Table (4), showed that the mean values range from -0.051 to -0.030 m and SD values range from 0.135 to 0.200 m and RMS values range from 0.142 to 0.208 m. the tested GGMs are close to each other's except GOCO05c.

Anyhow, the best of GGMs in the study area are XGM2019e_2159, XGM2016, and XGM2019, respectively, where RMS in XGM2019e_2159 was 0.142 m, in XGM2016 it was 0.147 m, and in XGM2019 it was 0.147 m.

Table (2) The Specifications of the ten tested Global Geopotential Models [7].

No.	Model	Degree	Year	Data
1	SGG-UGM-1	2159	2018	EGM2008, S(Goce)
2	SGG-UGM-2	2190	2020	A, EGM2008, S(Goce), S(Grace)
3	XGM2016	719	2017	A, G, S(GOCO05s)
4	XGM2019	760	2019	A, G, T, S(GOCO06s)
5	XGM2019e_2159	2190	2019	A, G, S(GOCO06s), T
6	GECO	2190	2015	EGM2008, S(Goce)
7	EIGEN-6C	1420	2011	A, G, S(Goce), S(Grace), S(Lageos)
8	EIGEN-6C3stat	1949	2014	A, G, S(Goce), S(Grace), S(Lageos)
9	EIGEN-6C4	2190	2014	A, G, S(Goce), S(Grace), S(Lageos)
10	GOCO05C	720	2016	A, G, S

Where A is altimetry data, S is satellite data (e.g., GRACE, GOCE, LAGEOS), G is ground data (e.g., terrestrial, shipborne, and airborne measurements), and T is topography data.

Table (3): Statistical Analysis for the Ten GGMs at First Study Area in Egypt.

GGM Model	Min (m)	Max (m)	Mean (m)	SD (m)	RMSE (m)
EIGEN-6C	-1.400	0.308	-0.625	0.470	0.774
EIGEN-6C3stat	-1.488	0.178	-0.625	0.444	0.759
EIGEN-6C4	-1.477	0.192	-0.620	0.441	0.754
GECO	-1.467	-0.053	-0.635	0.418	0.753
GOCO05C	-1.257	0.249	-0.604	0.415	0.726
XGM2016	-1.469	0.176	-0.646	0.397	0.752
SGG-UGM-1	-1.502	0.251	-0.744	0.461	0.868
XGM2019	-1.469	0.091	-0.660	0.396	0.764
XGM2019e_2159	-1.514	0.124	-0.663	0.403	0.769
SGG-UGM-2	-1.409	0.226	-0.639	0.444	0.770

Table (4): Statistical Analysis for the Ten GGMs at the Second Area in the Kingdom of Saudi Arabia.

GGM Model	Min (m)	Max (m)	Mean (m)	SD (m)	RMS (m)
EIGEN-6C	-0.362	0.499	-0.032	0.172	0.174
EIGEN-6C3stat	-0.366	0.439	-0.034	0.166	0.168
EIGEN-6C4	-0.353	0.44	-0.030	0.162	0.164
GECO	-0.322	0.44	-0.035	0.158	0.161
GOCO05c	-0.564	0.566	-0.059	0.200	0.208
XGM2016	-0.385	0.511	-0.051	0.138	0.147
SGG-UGM-1	-0.303	0.47	-0.030	0.165	0.167
XGM2019	-0.388	0.49	-0.050	0.139	0.147
XGM2019e_2159	-0.335	0.466	-0.048	0.135	0.142
SGG-UGM-2	-0.334	0.441	-0.046	0.162	0.167

7. Conclusion

The evaluation of the ten GGMs by comparing them with GPS/Levelling data over Egypt showed that the GOCO05C model gave the best results, as the RMSE for this model was 0.726 m. In the second study area over the Kingdom of Saudi Arabia, the results of comparing the ten models with KSA-GEOID21 showed that the XGM2019e_2159 model is the best model with RMSE of 0.142 m.

The best GGMs that gave close results to the real results in the first and second study areas were the GGMs that contained multi-source data. Where in the first study area, the best GGM was GOCO05C. As this model contains A, G, and S data. In the second study area, the model XGM2019e_2159, which contains A, G, S, and T data, was the best model that gave results close to the terrestrial data.

The degree of the GGM does not affect the accuracy of the results obtained from this model, but the accuracy of the GGM is affected by the sources of the data constituting this model.

The RMS resulting from the comparison of the ten GGMs with the KSA-GEOID21 geoid in the second study area is better than those from the same GGMs with GPS/Levelling in the first study area because KSA-GEOID21 was created through huge data from different compatible sources that were merged to get an accurately created geoid. In the first study area over Egypt, the RMS values were higher might be because most of the used 17 HARN stations are existing in desert areas distant from the Egyptian Benchmarks network which affected the accuracy of the obtained orthometric heights of those 17 points.

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