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Performance evaluation of detection of Direct Sequence Spread Spectrum signals over a flat fading channel

By

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Abstract:

This paper characterizes the performance of direct sequence spread spectrum (DS-SS) signals from the detection probability point of view. The detection probability of DS-SS signals is estimated using wideband radiometer receiver over flat fading channel. Simulations are performed to evaluate detection probability of DS-SS signals over flat fading channel for various time bandwidth product values. The results are compared with the detection probability of DS-SS signals over AWGN channel. The results show that the fading parameter degrades the detection probability of DS-SS signals. The performance of DS will be discussed later in the presence of imperfect channel estimation errors.

Keywords:

Detection, spread spectrum, radiometer.

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1. Introduction:

The requirement for low probability of intercept (LPI) and jam resistant communications has led to the development of modulation techniques known as spread spectrum (SS). The two primary spread spectrum techniques are direct sequence (DS) and frequency hopping (FH). In DS-SS, the carrier is modulated by a pseudo random binary code signal operating at a much higher rate than the information signal.[5],[6]
When spread spectrum signals are properly designed they can prevent an interceptor to detect the presence of radio frequency transmission by dispersing the transmitted energy below the noise level. The transmitted signal is indistinguishable from the environment noise level, the optimum approach to detect the signal is by using signal energy detecting device (radiometer).
A radiometer is a simple energy measuring device designed to detect the presence of radiated radio frequency energy within a specified bandwidth. There are many types of radiometers configured to detect any particular type of SS transmission. The wideband radiometer is the best to be used to detect unknown DSSS transmissions. The wideband radiometer is shown in Figure (1). [1],[2]
In Figure (1) (a) the modulated received signal is the input to a filter whose bandwidth is 'W' matched to the spectrum over which signal energy is being searched. The filter is followed by an envelope detector (a square-law device) and an integrator. The integrator output is sent to a comparator. If the integrator output is higher than the threshold, a signal is declared present. In Figure (1) (b) and (c) the received signal is multiplied by the carrier to be down converted to base-band signal, the demodulated signal is the input to a low pass filter whose bandwidth is 'W' matched to the spectrum over which signal is being searched. In (c) output signal from the filter is sampled at rate $1/W$ to obtain digital signal. The filter is followed by a square-law device and an integrator for (b) or a summer for (c). The output from the summer is sent to a comparator.

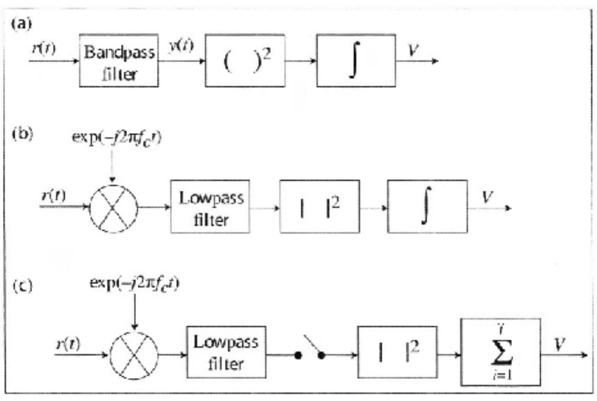

Figure (1): Radiometers: (a) passband, (b) baseband with integration, and (c) baseband with sampling at rate $1/W$ and summation

Assuming that the noise at the input to the radiometer is zero-mean, Gaussian random process with a flat power spectral density $\frac{N_o}{2}$, the total signal energy measured in a filter of bandwidth W in time T is ε and it is independent of the signal waveform.
The performance of radiometer (in terms of ε / N_o) from the probability of detection (P_d) point of view is[1]

$$P_d \approx Q\left[\frac{V_t - N_o TW - \varepsilon}{(N_o^2 TW + 2N_o \varepsilon)^{1/2}}\right] \qquad (1)$$

where V_t is the threshold level $\approx \hat{N}_o \sqrt{TW}\, Q^{-1}(P_f) + \hat{N}_o TW$, $Q^{-1}(\ )$ is the inverse Q function defined as [4],[6] $Q(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{t^2/2} dt$, where P_f is the probability of false alarm (if the integrator (summer) output is higher than the threshold and no signal is present), and $\hat{N}_o$ is the estimated noise power spectral density. The expression is valid when detecting low level signals having large time bandwidth product (TW>>1) over additive white Gaussian noise AWGN channel.
The relationship between P_d and ε / N_o for DS over AWGN channel using wideband radiometer for various values of time-bandwidth product TW , probability of false alarm 10^{-3} and $\hat{N}_o = N_o$ (analytical and simulated) is shown in Figure (2).
Comparing the intercept ability of DS for different TW, appears that as TW increases the energy-to-noise (ε / N_0) increases, consequently the probability of detection is improved.
The paper is organized as follows, in section 2 ideal detection of DS-SS signals is discussed and the performance evaluation of DS-SS signals using wideband radiometer as a detector over flat fading channel is estimated. In section 3 simulations for the performance evaluation of DS-SS signals over flat fading channel are shown and compared with performance over AWGN channel. In section 4 the conclusions are introduced.

2. Detection of DS-SS signal

Detection theory leads to various detection receivers depending on precisely what is assumed to be known about the signal to be detected.

2.1 Ideal detection

For ideal detection it is assumed that the chip timing of the spreading wave form is known and whenever the signal is present, it is present during the entire observation

interval. The spreading sequence is modeled as a random binary sequence.
Consider the detection of a DS signal with PSK modulation [1],[4], the DS-SS signal is given by:

$$s(t) = \sqrt{2S}\, p(t)\cos(2\pi f_c t + \phi) \qquad (2)$$

where S is the average signal power, f_c is the known carrier frequency, and ϕ is the carrier phase assumed to be constant over the observation interval $0 \le t \le T$. And p(t) is the spreading wave form given by: $p(t) = \sum_{i=0}^{N-1} p_i \psi(t - iT_c)$, where $p_i = \pm 1$, is the code sequence and $\psi(t)$ is a rectangular pulse of duration T_c.
To determine the signal s(t) is present based on the observation of the received signal, classical detection theory is applied, choosing between two hypotheses H_o (noise only) and H_1 (signal + noise).

$$r(t) = \begin{cases} s(t) + n(t) & H_1 \\ n(t) & H_o \end{cases} \qquad (3)$$

where n(t) is zero-mean, white Gaussian noise with two-sided power spectral density No/2.
Let θ denote the vector of random values that characterize the signal to be detected. The average likelihood ratio, which is compared with a threshold for a detection decision, is [1],[8]

$$\Lambda(r) = \frac{E_\theta[f(r|H_1,\theta)}{f(r|H_o)} \qquad (4)$$

where $f(r|H_1,\theta)$ is the conditional density function of r given the hypotheses H_1 and the value of θ, $f(r|H_o)$ is the conditional density function of r given hypotheses H_o, and E_θ is the expectation over the random vector θ. In case of AWGN the considered probability density functions are defined as:[8]

$$f(r|H_1,\theta) = \prod_{l=1}^{\upsilon} \frac{1}{\sqrt{\pi N_o}} \exp\left[-\frac{(r_l - s_l)^2}{N_o}\right] \qquad (5)$$

$$f(r|H_o) = \prod_{l=1}^{\upsilon} \frac{1}{\sqrt{\pi N_o}} \exp\left[-\frac{(r_l)^2}{N_o}\right] \qquad (6)$$

where{ s_l } are the coefficients of the signal, along the l^{th} orthonormal basis. Where it is assumed that the signal is expanded over a set of υ orthonormal basis thus the received signal is represented as a vector r=[$r_1 \quad r_2 \ldots\ldots\ldots\ldots\ldots r_\upsilon$]. If $\upsilon \to \infty$, the average likelihood ratio may be expressed in terms of the signal waveforms as [1],[8]

$$\Lambda[r(t)] = E_\theta\left\{\exp[\frac{2}{N_o}\int_0^T r(t)s(t)dt - \frac{\varepsilon}{N_o}\right\} \qquad (7)$$

If N is the number of chips, each of duration T_c received in the observation interval, then there are 2^N equally likely patterns of spreading sequence. For coherent detection, it is assumed that $\phi = 0$ in (2), substitute into (7) and evaluate the expectation to obtain [1],[8]

$$\Lambda(r(t)) = \exp\left(\frac{\varepsilon}{N_o}\right)\sum_{j=1}^{2^N}\exp[\frac{2\sqrt{2S}}{N_o}\sum_{i=0}^{N-1}p_i^{(j)}r_i^{'} \qquad (8)$$

where $p_i^{(j)}$ is chip i of pattern j and

$$r_i^{'} = \int_{iT_c}^{(i+1)T_c} r(t)\psi(t-iT_c)\cos(2\pi f_c t)dt \qquad (9)$$

One can see from (9) that ideal detection of DS-SS requires the knowledge of the spreading code p_i and perfect synchronization to maximize the ENR and consequently the probability of detection. However such knowledge is not available for an interceptor. For that reason the wideband radiometer is introduced in next section to tolerate these requirements.

2.1Wide-band radiometer detection

The wideband radiometer requires no detailed information about the signals to be detected other than their rough spectral location, even the modulation type is not essential parameter for radiometric detection. Suppose that the signal to be detected is approximated by a zero-mean, white Gaussian process, the channel is slow varying flat fading. Now consider two hypotheses that both assume the presence of a zero-mean, band limited white Gaussian process over an observation interval $0 \le t \le T$. Under H_o noise only is present, and the one-sided power spectral density over the signal band is N_o. While under H_1, both signal and noise are present and the one sided power spectral density is N_1 over the band. Using υ orthonormal basis functions and ignoring the effects of the band limiting, we find that the conditional densities are approximated by [1],[8]

$$f(r|H_i) = \prod_{l=1}^{\upsilon}\frac{1}{\sqrt{\pi N_i}}\exp\left[-\frac{(r_l)^2}{N_i}\right] \quad , i = 0,1 \qquad (10)$$

Then calculating the likelihood ratio and merging constants with the threshold, leads to the decision rule to compare the summer output

$$V = \sum_{l=1}^{\upsilon} r_l^2 \qquad (11)$$

to a threshold V_t, as $\upsilon \to \infty$ and use the properties of orthonormal basis functions, it is found that

$$V = \int_0^T r^2(t)dt \qquad (12)$$

The filter of the radiometer is assumed to be an ideal filter that passes the desired signal while limiting the noise. It produces the output

$$y(t) = \alpha s(t) + n(t) \qquad (13)$$

where $\alpha = a.e^{j\theta}$ is the fading parameter, a is the amplitude of fading parameter, it is Rayleigh distributed and θ is the phase shift of fading parameter which is uniformly distributed over $[0,2\pi]$. Then α follows Gaussian distribution with zero mean and variance σ^2.

The band limited signal can be represented as [1],[3]

$$s(t) = s_c(t)\cos 2\pi f_c t - s_s(t)\sin 2\pi f_c t \qquad (14)$$

where s_c(t) and s_s(t) are the in-phase and quadrature components of s(t) respectively. These components are low pass signals confined for $|f| \le W/2$. Also the noise passing through the filter can be represented as

$$n(t) = n_c(t)\cos 2\pi f_c t - n_s(t)\sin 2\pi f_c t \qquad (15)$$

squaring and integrating y(t), and assuming that $f_c >> W$ and $f_c >> 1/T$, we get

$$V = \frac{1}{2}\int_0^T [\alpha s_c(t) + n_c(t)]^2 dt + \frac{1}{2}\int_0^T [\alpha s_s(t) + n_s(t)]^2 dt \qquad (16)$$

The sampling theorem for deterministic and stochastic processes provide expansions of $s_c(t)$, $s_s(t)$, $n_c(t)$ and $n_s(t)$ that facilitate the analysis. For example [1],[5]

$$s_c(t) = \sum_{i=-\infty}^{\infty} s_c(\frac{i}{W})\sin c(Wt - i) \qquad (17)$$

Let $\gamma = \lfloor TW \rfloor$, where $\lfloor x \rfloor$ denotes the integer part of x. Substituting expansions similar to (17) in (16) and use approximations and it is always assumed TW≥ 1.

$$V = \frac{1}{2W}\sum_{i=1}^{\gamma}\left[\alpha s_c\left(\frac{i}{W}\right) + n_c\left(\frac{i}{W}\right)\right]^2 + \frac{1}{2W}\sum_{i=1}^{\gamma}\left[\alpha s_s\left(\frac{i}{W}\right) + n_s\left(\frac{i}{W}\right)\right]^2 \qquad (18)$$

The power spectral density of $n_c(t)$, $n_s(t)$ are

$$N_c(f) = N_s(f) = \begin{cases} N_o & |f| \le W/2 \\ 0 & |f >| W/2 \end{cases} \qquad (19)$$

and the associated autocorrelation functions are

$$R_c(\tau) = R_s(\tau) = N_o W \sin c(W\tau) \qquad (20)$$

which indicates that $n_c\left(\frac{i}{W}\right)$ and $n_c\left(\frac{j}{W}\right)$ are statistically independent, $i \ne j$ similarly $n_s\left(\frac{i}{W}\right)$ and $n_s\left(\frac{j}{W}\right)$. Therefore (16) becomes

$$V = \frac{N_o}{2}\left\{\sum_{i=1}^{\gamma} A_i^2 + \sum_{i=1}^{\gamma} B_i^2\right\} \qquad (21)$$

where

$$A_i^2 = [\frac{\alpha}{N_o W} s_c\left(\frac{i}{W}\right) + \frac{1}{N_o W} n_c\left(\frac{i}{W}\right)]^2 \qquad (22)$$

$$B_i^2 = [\frac{\alpha}{N_o W} s_s\left(\frac{i}{W}\right) + \frac{1}{N_o W} n_s\left(\frac{i}{W}\right)]^2 \qquad (23)$$

The signal is assumed to be a Gaussian process with unit variance and mean $\frac{1}{\sqrt{N_o W}}\left(s_c\left(\frac{i}{W}\right) + s_s\left(\frac{i}{W}\right)\right)$ thus the signal energy has a noncentral chi-squared distribution with 2γ degrees of freedom and non central parameter $\lambda = \frac{2\varepsilon}{N_o}$, ε is the signal energy. Then the signal energy probability density function is[1]

$$f_X(x) = \frac{1}{2}\left(\frac{x}{\lambda}\right)^{(\gamma-1)/2} \exp\left(-\frac{x+\lambda}{2}\right) I_{\gamma-1}\left(\sqrt{x\lambda}\right) u(x) \qquad (24)$$

where $I_n(\ )$ is the modified Bessel function of the first kind and order n defined by

$I_n(x) = \sum_{i=0}^{\infty} \frac{\left(\frac{x}{2}\right)^{n+2i}}{i!\Gamma(n+i+1)}$, Γ is the gamma function.

Also the fading parameter is assumed Gaussian with zero mean and variance σ^2, thus the square of fading parameter is central chi-squared distribution with zero mean. Its probability density function is[4],[5]

$$f_Y(y) = \frac{1}{\sqrt{2\pi y}\sigma} e^{\frac{-y}{2\sigma^2}} \qquad (25)$$

To obtain the probability density function of $Z = 2V / N_o$, assume $z = x.y$ and $p = y$, then the joint probability density function is

$$f_{Z,P}(z,p) = \frac{1}{\sqrt{8\pi p^3}\sigma}\left(\frac{z}{\lambda p}\right)^{\frac{\gamma-1}{2}} e^{-\frac{1}{2}\left(\frac{z}{p}+\lambda+\frac{p}{\sigma^2}\right)} I_{\gamma-1}\left(\sqrt{\frac{z\lambda}{p}}\right) u(p) \qquad (26)$$

and the probability function of Z is given by

$$f_Z(z) = \int_0^{\infty} f_{Z,P}(z,p)dp \qquad (27)$$

Since the detection probability P_d is declared if $V > V_t$ when the signal is present indicates that

$$P_d = \int_{\frac{2V_t}{N_o}}^{\infty} f_Z(z)dz \qquad (28)$$

The above integration is hard to evaluate, for TW>100, if V is approximated by a Gaussian random variable, then the direct application of the statistics of Gaussian variables to (21) yields

$$E[V] = \sigma^2 \varepsilon + N_o TW \quad (29)$$

$$\mathrm{var}[V] = 2N_o \sigma^2 \varepsilon + N_o^2 TW \quad (30)$$

This implies that

$$P_d \approx \frac{1}{\left(2\pi\left(2N_o\sigma^2\varepsilon + N_o^2 TW\right)\right)^{\frac{1}{2}}} \int_{V_t}^{\infty} \exp\left[-\frac{\left(z - N_o TW - \sigma^2\varepsilon\right)^2}{2\left(2N_o\sigma^2\varepsilon + N_o^2 TW\right)}\right] \quad (31)$$

then

$$P_d \approx Q\left[\frac{V_t - N_o TW - \sigma^2\varepsilon}{\left(N_o^2 TW + 2N_o\sigma^2\varepsilon\right)^{\frac{1}{2}}}\right], \quad TW >> 1 \quad (32)$$

When V_t is specified the value of energy-to-noise ratio necessary to achieve a specified value of P_d may be obtained by inverting (34) and substitute by V_t in the inverted function yielding to

$$\frac{\sigma^2\varepsilon}{N_o} \approx h\sqrt{TW}\beta + (h-1)TW + \xi^2 - \sqrt{TW}\xi\left[2h - 1 + \frac{2\beta h}{\sqrt{TW}} + \frac{\xi^2}{TW}\right]^{\frac{1}{2}}, \quad TW >> 1 \quad (33)$$

where $\beta = Q^{-1}(P_f)$, $\xi = Q^{-1}(P_d)$, $h = \frac{N'_o}{N_o}$. It is clear that as TW increases, The required energy to noise ratio increases.

3. Computer Simulations

Simulation program is performed using matlab to show the performance of DS from the probability of detection point of view, the probability of detection is plotted versus the required energy-to-noise ratio for various time-bandwidth product values. Assuming that the noise power spectral density estimated to calculate the threshold level matches the noise power spectral density at the receiver input used to calculate the probability of detection. In Figure (2) comparison between analytical and simulation results for the performance of DS over AWGN channel is declared using wideband radiometer, ε / N_o is set to be [0:30 dB], the probability of false alarm is set to be 10^{-3}, it is seen that as TW has a small value the analytic and simulated curves are closed to each other, and when TW increases there is a small difference between the analytic and simulated curves, it is also seen that increasing TW requires an increasing in the energy-to-noise ratio to maintain the same probability of detection. In Figure (3) comparison between analytical and simulation results for the performance of DS over flat fading channel is declared, ε / N_o is set to be [0:30 dB], the probability of false alarm is set to be 10^{-3}, and the fading parameter is set to be 0.457, it is seen that as TW has a small value the analytic and simulated curves are closed to each other, and when TW increases there is a small difference between the analytic and simulated curves, it is also seen that increasing TW requires an increasing in the energy-to-noise ratio to maintain the same probability of detection. In Figure (4) comparison between the performance of DS Simulation program is performed using matlab to show the performance of DS from the probability of detection point of view, the probability of detection is plotted versus the required energy-to-noise ratio for various time-bandwidth product values. Assuming that the noise power spectral density estimated to calculate the threshold level matches the noise power spectral density at the receiver input used to calculate the probability of detection. In Figure (2) comparison between analytical and simulation results for the performance of DS over AWGN channel is declared using wideband radiometer, ε / N_o is set to be [0:30 dB], the probability of false alarm is set to be 10^{-3}, it is seen that as TW has a small value the analytic and simulated curves are closed to each other, and when TW increases there is a small difference between the analytic and simulated curves, it is also seen that increasing TW requires an increasing in the energy-to-noise ratio to maintain the same probability of detection. In Figure (3) comparison between analytical and simulation results for the performance of DS over flat fading channel is declared, ε / N_o is set to be [0:30 dB], the probability of false alarm is set to be 10^{-3}, and the fading parameter is set to be 0.457, it is seen that as TW has a small value the analytic and simulated curves are closed to each other, and when TW increases there is a small difference between the analytic and simulated curves, it is also seen that increasing TW requires an increasing in the energy-to-noise ratio to maintain the same probability of detection. In Figure (4) comparison between the performance of DS detection over flat fading channel and AWGN channel is presented ε / N_o is set to be [0:30 dB], the probability of false alarm is set to be 10^{-3}, and the fading parameter is set to be 0.457, it is shown that the fading parameter degrades the probability of detection. In Figure (5) Comparison between the required energy to noise ratio versus different values of TW to specify the desired probability of detection and defined threshold level is declared, the probability of detection is set to be 0.99 and various values of probability of false alarm, also it is shown the sensitivity of the radiometer to errors in noise power spectral density estimation as TW increase due to the bias term $N_o TW$.

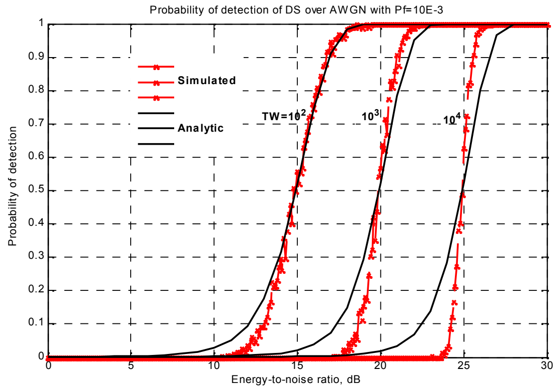

Figure (2): Probability of detection versus ε / N_o for DS over AWGN channel for wideband radiometer with $P_f = 10^{-3}$ and various values of TW, $\hat{N}_o = N_o$

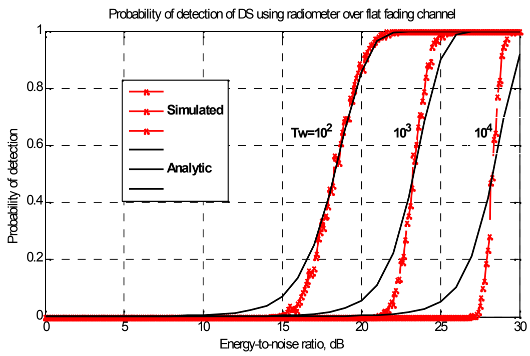

Figure (3): Probability of detection versus ε / N_o for DS over flat fading channel for wideband radiometer with $P_f = 10^{-3}$ and various values of TW, $|\alpha|^2 = 0.457$, $\hat{N}_o = N_o$

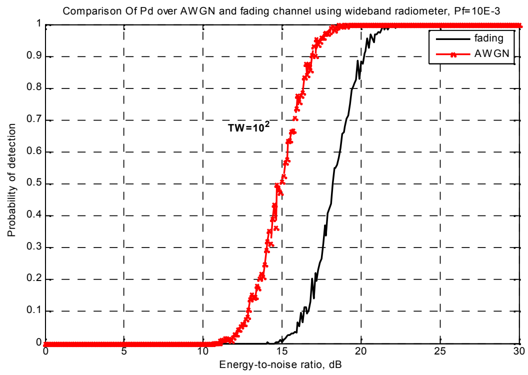

Figure (4): Probability of detection versus ε / N_o for DS, over flat fading channel $\hat{N}_o = N_o, |\alpha|^2 = 0.457$ for wideband radiometer with $P_f = 10^{-3}$ and compared with AWGN channel.

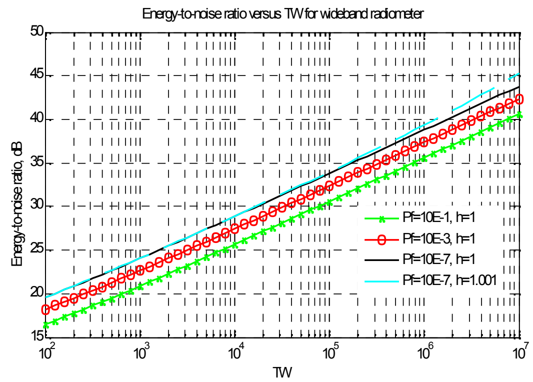

Figure (5): Energy-to-noise ratio versus TW for wideband radiometer with $P_d = 0.99$ and various values of P_f and h.

4. Conclusions:

This paper discusses the performance evaluation using wide-band radiometer for detection of DS-SS signal over flat fading channel. Ideal detection requires previous knowledge about the received signal. The wideband radiometer requires no detailed information about the signal to be detected even the modulation. Also, it is seen that as the time bandwidth product (TW) increases the energy-to-noise ratio required to maintain the same probability of detection increases. It is clear that the fading effect degrades P_d compared with the AWGN. Single radiometer is incapable of determining whether one or more than one signal has been detected. It is shown that the radiometer is sensitive to errors in noise estimations as TW increases due to the bias term $N_o TW$.

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