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GPS correlated jammer mitigation utilizing MUSIC algorithm

By

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Abstract:

This paper study the behavior of multiple signal classification (MUSIC) algorithm for suppressing correlated and uncorrelated jammers to GPS receiver. Simulation indicates that MUSIC can cancel dramatically high power jammers for both types of signals and perfectly detects their direction of arrivals. It also can suppress both jammers of decreased powers up to -120 dBW. At that power level the suppression behavior regarding to uncorrelated jammer is better than that for correlated one but the algorithm still detects the correlated and uncorrelated jammers direction of arrivals.

Keywords:

Adaptive antenna, MUSIC, eigen decomposition, GPS anti-jamming.

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1. Introduction:

Global Positioning system (GPS) receiver is the best device in the field of navigation to give a very accurate user position. So it is used in many civilian and military applications. Interference from radar systems and other devices affect the civilian use, otherwise intentionally used jammers affect the military use, so increasing the protection against intentional and unintentional interferences is required. The received GPS signal is in the range of -160dBW i.e., it is below the receiver thermal noise power by about 20-30 dB. High power jammers like FM, impulsive noise and CW, degrade the behavior of the GPS receiver and cause their code tracking loop and carrier tracking loop to be out of look. Adaptive antenna is suitable to cancel these types of jammers. It utilizes some techniques of cancellation based on determining the jammer directions like MUSIC algorithm [1-5] or power inversion [6]. The main purpose of adaptive antenna is to reduce the jammer signals up to a level where the spread spectrum mechanism can extract the signal. Section 2 is assigned for MUSIC (Multiple Signal Classification). Section 3 is assigned for the simulation results. The conclusion is summarized in section 4.

2. Multiple signal classification (MUSIC)

One of the high resolution methods for estimating the direction of arrival (DOA) of a narrow band signal in presence of noise is multiple signal classification (MUSIC) algorithm. It is used to describe experimental and theoretical techniques involved in determining the parameters of multiple wavefronts arriving at an antenna array by measuring the signal received at the antenna elements. MUSIC algorithm provides asymptotically unbiased estimate of the number of signals and there directions of arrival [1], [2], and [3]. This algorithm is very useful when dealing with GPS anti-jamming. Consider the received signal at *M*-elements uniformly spaced linear array as shown in figure (1), is linear combination of all the far field incident signals and noise. Thus,

$$\mathbf{X} = \mathbf{V}\,\mathbf{u} + \mathbf{N}$$

or

$$\begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_M(t) \end{pmatrix} = \begin{pmatrix} \boldsymbol{\alpha}(\theta_1) & \boldsymbol{\alpha}(\theta_2) & \cdots & \boldsymbol{\alpha}(\theta_L) \end{pmatrix} \begin{pmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_L(t) \end{pmatrix} + \begin{pmatrix} n_1(t) \\ n_2(t) \\ \vdots \\ n_M(t) \end{pmatrix} \quad (1)$$

$$y = \mathbf{w}^H\,\mathbf{X} \quad (2)$$

$$\mathbf{w} = \begin{pmatrix} w_1 & w_2 & \Lambda & w_M \end{pmatrix}^T$$

Where $\mathbf{X}$ is $M \times 1$ vector represents the antenna array received signal, **V** is a matrix contains the steering vectors associated to the incident signals, **u** is a vector represents the incident signals amplitudes, **w** is the $M \times 1$ complex vector represents the array weight vector, **y** is the output of the array antenna given by the weighted sum of the array antenna received signal, **N** is $M \times 1$ vector consists of an independent Gaussian noise of variance σ^2 includes channel noise, receiver noise and antenna elements noise, L is the number of incident signals and

$$\boldsymbol{\alpha}(\theta_i) = \left[1 \quad \exp\left[-j(\frac{2\pi\, l \sin\theta_i}{\lambda}) \right] \quad \Lambda \quad \exp\left[-j(\frac{2\pi(M-1)\, l \sin\theta_i}{\lambda}) \right] \right]^T$$

Where θ_i is the incident signals directions, i=1,2, … L and l is the distance between each two antenna elements.

It is assumed that the incident signals are independent each other and independent on the thermal noise. The covariance matrix of the array received signal **R** is given as,

$$\begin{aligned} \mathbf{R} &= E[\mathbf{X}\,\mathbf{X}^{\mathrm{H}}] \\ &= \mathbf{V}\,\mathbf{P}\,\mathbf{V}^{\mathrm{H}} + \sigma_n^2\,\mathbf{I} \end{aligned} \tag{3}$$

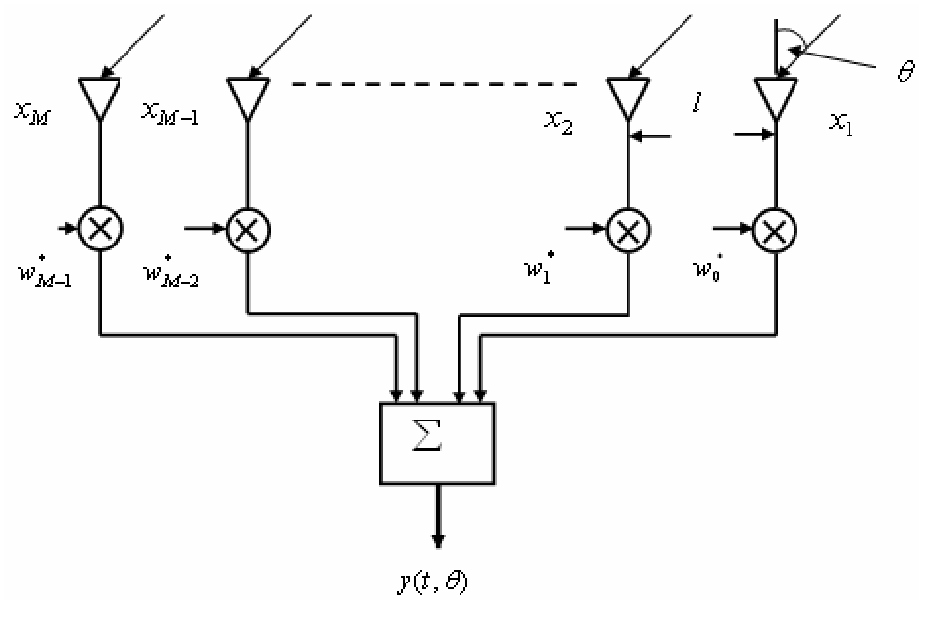

Figure (1): *Adaptive array*

$$\mathbf{P} = diag\begin{pmatrix} p_1 & p_2 & \mathrm{K} & p_L \end{pmatrix}$$

$p_i = E\left(u_i(t)\ u_i^*(t)\right)$ is the power of i^{th} incident signal.

The autocorrelation matrix can be written in terms of its eigenvalues and eigenvectors as follows,

$$\mathbf{R} = \mathbf{Q}\,\mathbf{\Lambda}\,\mathbf{Q}^{\mathrm{H}} \tag{4}$$

Where $\mathbf{Q} = \left(\mathbf{q}_1 \quad \mathbf{q}_2 \quad \cdots \quad \mathbf{q}_M\right)$, λ_i, represents the i^{th} eigenvalue of **R,** i=1, 2, … M and $\mathbf{q}_i$ represents the i^{th} eigenvector associated to the i^{th} eigenvalue of **R,** i=1, 2, …M. The eigenvalues of **R** are in order of decreasing size $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_L > \lambda_{L+1} = \lambda_{L+2} = \ldots = \sigma^2$.

The signal subspace is represented by the eigenvectors correspond to the L largest eigenvalues. The remaining [$L+1 : M$] eigenvectors span the noise subspace. So,

$$\mathbf{Q} = \left(\mathbf{Q}_{Signal} \quad \mathbf{Q}_{Noise}\right) \tag{5}$$

$\mathbf{Q}_{Signal} = \left(\mathbf{q}_1 \quad \mathbf{q}_2 \quad \ldots \quad \mathbf{q}_L\right)$ and $\mathbf{Q}_{Noise} = \left(\mathbf{q}_{L+1} \quad \mathbf{q}_{L+2} \quad \ldots \quad \mathbf{q}_M\right)$

.

It is proved in [1] that the columns of $\mathbf{Q}_{Noise}$ are orthogonal to the space spanned by the columns of **V**. i.e.,

$$\mathbf{q}_l \perp span(\mathbf{V}) \tag{6}$$

$l = L+1,\ L+2, \ldots\ldots\ldots M$

The direction of the incident signals can be obtained by getting the minimum of the following formula.

$$\xi(\theta) = \left(\boldsymbol{\alpha}^{\mathrm{H}}(\theta)\,\mathbf{Q}_{Noise}\mathbf{Q}_{Noise}^{\mathrm{H}}\,\boldsymbol{\alpha}(\theta)\right) \tag{7}$$

Since the GPS signal power is lower than the thermal noise power, so the received signal can be detected by the incident jammers and the noise [3]. The received signal vector can be given by modifying (1) as follows,

$$\begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_M(t) \end{pmatrix} = \begin{pmatrix} \boldsymbol{\alpha}(\theta_{J1}) & \boldsymbol{\alpha}(\theta_{J2}) & \cdots & \boldsymbol{\alpha}(\theta_{JL}) \end{pmatrix} \begin{pmatrix} J_1(t) \\ J_2(t) \\ \vdots \\ J_L(t) \end{pmatrix} + \begin{pmatrix} n_1(t) \\ n_2(t) \\ \vdots \\ n_M(t) \end{pmatrix}$$

Where θ_{Ji} is the i^{th} jammer direction, i=1,2, … L and

$\boldsymbol{\alpha}(\theta_{Ji}) = \left[1 \quad \exp\left[-j(\frac{2\pi \; l \sin\theta_{Ji}}{\lambda}) \right] \quad \Lambda \quad \exp\left[-j(\frac{2\pi(M-1) \; l \sin\theta_{Ji}}{\lambda}) \right] \right]^T$ is the steering vector associated with the i^{th} jammer. It is clear form the previous discussion that, the columns of **V** are represented by the steering vectors associated with the jammers.

In case of the adaptive array, it is required that the optimum weight vector $\mathbf{w}_o$ be orthogonal to all the jammers directions to achieve *L*-nulls in the jammers directions. From (6) it is obvious that any eigenvector in the noise subspace can be considered as an optimum weight. So,

$$\mathbf{w}_o \perp span(\mathbf{V}) \tag{8}$$

In general the optimum weight vector can be given as a linear combination of the eigenvectors of the noise subspace [3] as follows,

$$\mathbf{w}_o = \sum_{i=L+1}^{M} \beta_i \, \mathbf{q}_i \tag{9}$$

Where β_i is an arbitrary complex constant belongs to the *M-L* dimensional complex space.

3. Simulations

Computer simulations were performed using 5 elements uniform linear array arranged in the y-axis with elements spaced half wave length apart. There are five useful GPS signals each with power -160 dBW come from directions (-60° -36° 0° 30° 45°). Two jamming signals come from fixed directions (-18° 50°). The jamming signal coming form 50° is chosen to be the same as the first GPS signal (correlated jammer) coming form direction -60° but with higher power. A sinusoidal signal of frequency 1575.42 MHz with higher power is chosen to be as the second uncorrelated jamming signal. Simulation is performed using 200 snap shot. Four different power levels for jamming signals at the GPS receiver were simulated.

3.1 -100dBW power level for both jammers

From figure.(2), it is obvious that MUSIC algorithm introduces deep nulls to the two jammers. It assigns -125.2 dB null for the uncorrelated jammer from direction -18° and assigns -86.77 dB null for the correlated jammer from direction 50^o. It assigns two false nulls to directions 78° and -26° of depths -52.23 dB and -67.08 dB respectively. It is obviously that MUSIC assigns higher attenuation for the true nulls than the false. In this case MUSIC not only is used as jammer suppression but also as jammer direction finder. For GPS signals form directions -60°, -36°, 45° it introduces attenuations -13.4 dB, -13.04 and -14.13 respectively. For GPS signal from direction 30° is introduces a gain of 2.29 dB. Power pattern levels for the desired and the jammer signals of figure (2) are summarized in table (1).

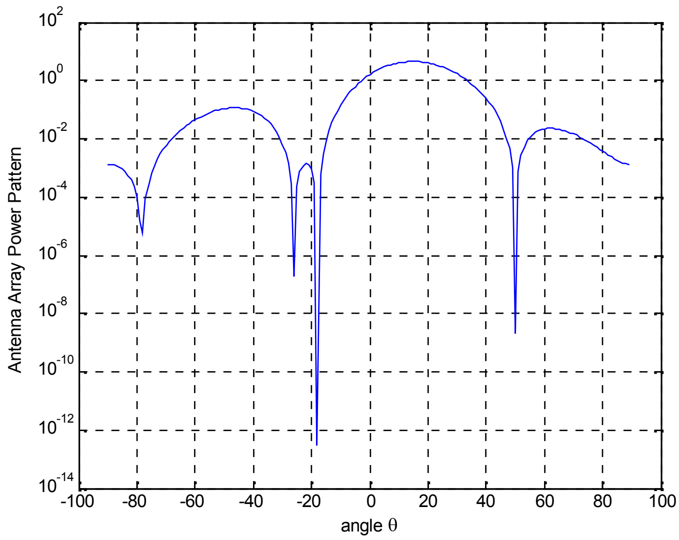

***Figure (2)**: Antenna array power pattern in case of using MUSIC for -100 dBW correlated and uncorrelated fixed jammers.*

***Table (1):** Summarizing the Power Patten Level of Figure (2)*

Angle in degrees	**Power Pattern Level in dB for MUSIC**
-60°	-13.4
-36°	-13.04
0°	2.14
30°	2.29
45°	-14.13
-18°	-125.2
50°	-86.77

3.2 -120dBW power level for both jammers

In this case as seen from the power pattern figure 3, MUSIC introduces suppression -85.2 dB for the uncorrelated jammer and -46.78 dB for the correlated one. Two false nulls of depths (-62 dB,-52.65 dB) in the directions -26° and -78° respectively are introduced. MUSIC can be used as a jammer suppression algorithm for both jammers. Power pattern levels for the desired and the jammer signals of figure (3) are summarized in table (2).

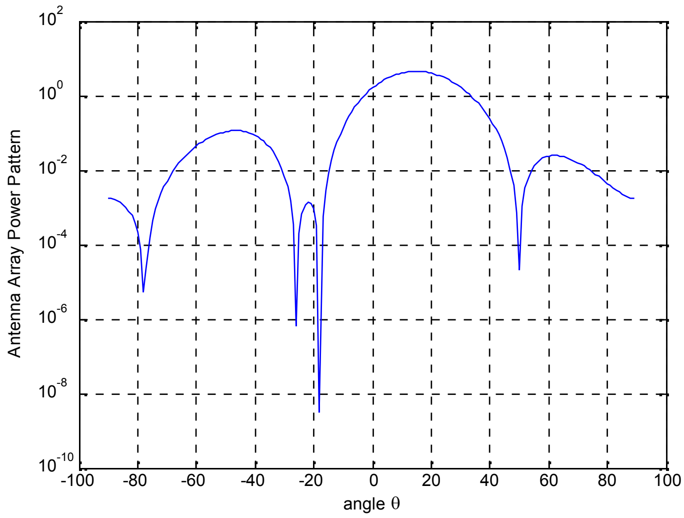

***Figure (3)**: Antenna array power pattern in case of using MUSIC for -120 dBW correlated and non correlated fixed jammers.*

***Table (2):** Summarizing the power pattern levels of Figure (3)*

Angle in degrees	Power Pattern Level in dB for MUSIC
-60°	-13.52
-36°	-12.98
0°	2.13
30°	2.29
45°	-14.29
-18°	-85.2
50°	-46.78

3.3 -125dBW power level for correlated jammer and -130dBW for uncorrelated jammer

MUSIC introduces -65 dB suppression to the uncorrelated jammer while introduces -17.2 dB attenuation to the correlated one as shown in figure (4). It is clear that the null of -36 dB depth which must be directed to the correlated jammer is shifted left by 3° to the direction 47°. MUSIC introduces attenuation varies between -1.83 dB and -21 dB to the useful GPS signal as shown in table (3). It is clear from figure (4) and table (3) that MUSIC fails to detect DOA of the correlated jammer while it detects the DOA of the uncorrelated lower power jammer. Power pattern levels for desires and the jammers of figure (4) are summarized in table (3).

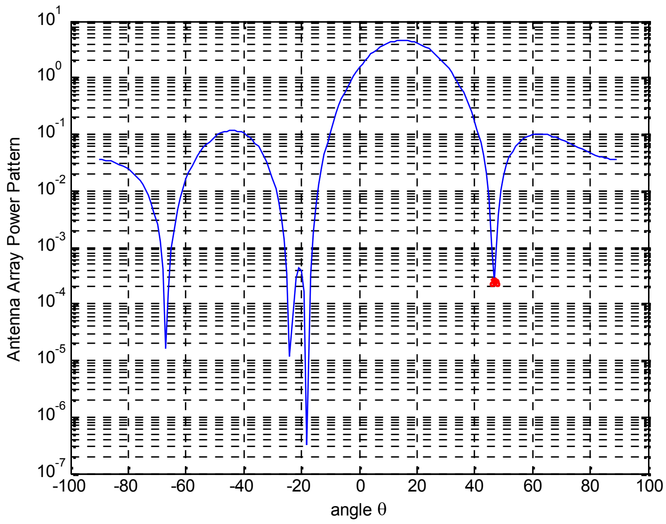

Figure (4): *Antenna array power pattern in case of using MUSIC for -125 dBW correlated and -130 dBW non correlated fixed jammers.*

Table (3): *Summarizing The Power Pattern Levels Of Figure (4).*

Angle in degrees	Power Pattern Level in dB for MUSIC
-60°	-18
-36°	11.54
0°	1.83
30°	2.27
45°	-21
-18°	-65
50°	-17.2

3.4 -130dBW power level for correlated jammer and -130dBW for uncorrelated jammer.

In this situation the algorithm can suppress the uncorrelated jammer by 67.3 dB while it fails to introduce any suppression for the correlated jammer, on the contrary it introduces gain about 3.8 dB. GPS signals from (0° 30° 45°) directions have gains (0.67 dB, 5.67 dB, 5.52 dB) while that from directions (-60° -36°) complain about -1.76 dB and -11.58 dB attenuation as shown in figure (5) and table (4).

Power pattern levels for the desired and the jammer signals of Figure (5) are summarized in table (4).

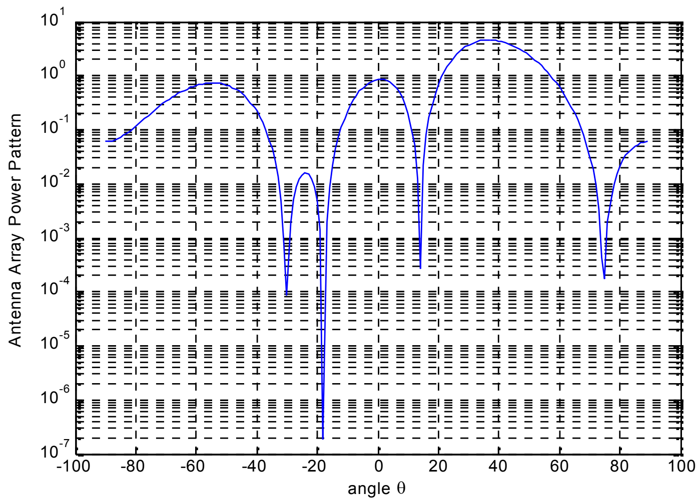

Figure (5): *Antenna array power pattern in case of using MUSIC for -130 dBW correlated and -130 dBW uncorrelated fixed jammers.*

Table (4): *Summarizing the Power Pattern Level of Figure (5).*

Angle in degrees	Power Pattern Level in dB for MUSIC
-60°	-1.76
-36°	-11.58
0°	0.67
30°	5.67
45°	5.52
-18°	-67.3
50°	3.8

4. Conclusions:

MUSIC algorithm is one of the best methods used for canceling uncorrelated jammer. Through this paper the behavior of MUSIC with correlated and uncorrelated jammers was studied. For high power jammers MUSIC suppresses dramatically both jammers and detects their directions. For lower power jammers up to -120 dBW, it suppresses both jammers and detects their directions. If MUSIC is used as jammer DOA estimation at that power level it will be confused with that deeper false nulls introduced by the algorithm. At power level -125 dBW for correlated jammer the algorithm shifts the null which must be assigned to the jammer and fails to suppress the jammer, contrary it can suppress the uncorrelated jammer and detects its direction. For lower power level than -125 dBW MUSIC introduces gain for the correlated jammer but work perfectly to suppress the uncorrelated jammer.

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