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New Conclusions of Strong Decay Widths for $\eta_b(5S)$ State via Non-Relativistic Quark Model Frame

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ABSTRACT
A detailed analysis is conducted on the sources of Beauty mesons (B mesons) through the strong decay of $\eta_b(5S)$, which represents one of the singlet-spin states S-wave bottomonium mesons. That is considered a significant step in understanding the physics behind the strong decay of bottomonium mesons, which is demonstrated to furnish us with a plentiful source of Beauty mesons. Their rare decay is regarded as a gateway to a new realm of physics, commonly called New Physics or Beyond Standard Model. The expected masses for higher bottomonium mesons are determined within the framework of the nonrelativistic quark model, and the recently obtained experimental data exhibit substantial concordance with our findings. Additionally, other theoretical forecasts align with our outcomes. The Quark Pair Creation model is employed to compute the strong decay of the $\eta_b(5S)$ meson. Moreover, the strong decay branching ratio is determined. Novel findings regarding the partial strong decay widths of the $\eta_b(5S)$ are obtained, while the total decay width aligns with previous studies. The strong decay width of the B*B* mesonic pairs channel is expected to be the greatest width by the ratio (~ 56.37 %) concerning the higher scalar $\eta b(5S)$ state. Additionally, the Bs*Bs* channel is predicted to have the next-to-dominant width relative to $\eta_b(5S)$ state.

INTRODUCTION

Nowadays, the newest challenge that faces particle physics scientists is to investigate New Physics (NP) or the so-called Beyond Standard Model (BSM) [1-5] that discusses revolutionary issues in Physics beyond modern physics. Beyond Standard Model theories include perfectly novel interpretations like String Theory, M-Theory, and extra dimensions, and it involves loop quantum gravity, Supersymmetry, and the various standard model extensions of Supersymmetry, such as the Minimal Supersymmetric Standard Model (MSSM) and the Next-to-Minimal Supersymmetric Standard Model (NMSSM). Most of such new physics theories (BSM theories) foresee the being of novel heavy particles [6] that until now have been considered hidden particles like massive versions of the W and Z bosons [7] and the fourth generation of quarks [8], a SUSY companion for every SM particle, and a diverse array of five Higgs bosons which include a doublet of charged scalar Higgs bosons [9], etc. BSM theories endeavor to establish a comprehensive theory that is so-called Theory of Everything (ToE), in which this awaited theory fully elucidates physical phenomena by amalgamating these natural phenomena. Furthermore, in principle, the Theory of Everything anticipates the outcomes of experimental endeavors.

Without a doubt, the field of B-meson physics plays a crucial and significant role in delving deep into the realm of the New Physics, which is commonly referred to as the Beyond Standard Model. This exploration is made possible through the investigation of rare B meson decays, encompassing both inclusive rare decays, such as $B \rightarrow X_s\ \mu^+\ \mu^-$ and exclusive rare decays, like $B \rightarrow K^*\ \mu^+\ \mu^-$ [10], as well as exceedingly rare decays, for instance, $B_s \rightarrow \mu^+\ \mu^-$, with a ratio of ($\mathfrak{B}\,(B_s \rightarrow \mu^+ + \mu^-) \cong 3 \times 10^{-9}$) [10, 11]. Simultaneously, the field of B-meson physics exerts a profound impact on unraveling certain enigmas within the framework of the Standard Model, like CP violation, as well as determining specific CKM matrix elements. Furthermore, it plays a significant role in advancing our understanding of the properties of bottomonium states and *B* mesons. Consequently, the search for sources of $B\bar{B}$ mesons represent a paramount objective for particle colliders in both the present and upcoming years. Undoubtedly, this objective can be achieved through a study of the strong decays for higher $b\bar{b}$ bottomonium mesons, which decay to *BB* mesons, thus furthering our knowledge in this field.

Amid the various states in which the bottomonium mesons exist, the ϒ(4S) meson stands out with its intriguing characteristics, particularly due to it being the lighter mass concerning the bound state for bottomonium mesons. Which possesses a mass is enough to enable the decay into pairs of *B* mesons, a phenomenon that has been extensively studied [12]. In the realm of literature, the detectors known as CLEO at Cornell and ARGUS at DESY (Deutsches-Elektronen-Synchrotron) in Hamburg emerged as the pioneering instruments [13-20] that were employed to investigate B-physics through the utilization of the $\Upsilon(4S)$ resonance. The significant observation of the B°-$\overline{B}^{\circ}$ mixing was initially documented [17] during the operational period of the ARGUS detector, spanning from 1982 to 1992 [17-20]. The primary purpose of this decay $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\overline{B}$ was to acquire pure B-pairs, and the exploration of B_s physics in relation to this phenomenon was undertaken by Bell at the $\Upsilon(5S)$ state.

In reality, there is a scarcity of research conducted on the strong decays for bottomonium mesons sectors functioning as sources for beauty mesons, thus necessitating the undertaking of additional studies in these particular sectors, particularly in light of the presence of more advanced and forthcoming generations from the particle colliders [21-23]. In this study, we examine the strong decay of $\eta_b(5S)$ bottomonium mesons as probable sources for *B* mesons, representing an advancement in our understanding of the strong decay physics of bottomonium mesons. Where their strong widths (B mesons) are regarded as our means of accessing a novel realm of physics through their rare decay. This investigation serves as a stride toward this captivating and demanding objective. In the subsequent section of this research, the theoretical framework of a non-relativistic quark model and a $^3P_0$ (QPC) model are offered. The evaluation of our findings regarding the strong decay phenomenon is explicated in Section 3. Ultimately, Section 4 is dedicated to the discussion of our results and conclusion.

2. The utilized theoretical frame of calculations

The quark model (QM) is highly efficacy in conducting calculations that facilitate the study of hadron properties and the exploration of its characteristics [24-30]. Various types of decay, such as hadronic transitions, annihilation decay, leptonic and semileptonic decay, radiation transitions, and strong decay, contribute significantly to our comprehension of hadrons. In this context, our focus lies on the strong decay of $\eta_b(5S)$ bottomonium mesons, where $\eta_b(5S)$ mesons are considered the initial state. These mesons eventually separate into two B mesons in the final states through the production of quark pairs in the $^3P_0$ state.

There exist numerous strong decay models, such as the Cornell model [31, 32], the fluxtube model [33], the $^3S_1$ model [34, 35], the microscopic models [36, 37], and the $^3P_0$ model [38]. However, the latter is the most widely and uncomplicated model, as it effectively describes the phenomenon of strong decay. In this study, we employ the non-relativistic quark model to compute the mass spectrum and employ the $^3P_0$ model to calculate the strong decay widths of the $\eta_b(5S)$ bottomonium mesons.

2.1 The Predicted Mass spectrum

In this segment, we undertake the task of presenting the theoretical framework for calculating the anticipated masses within the non-relativistic quark model for some higher bottomonium mesons including $\eta_b(5S)$, as delineated in Table 2. To achieve this, we employ a conventional potential, which combines the Coulomb potential with a linear potential [39]. Furthermore, we incorporate spin-dependent corrections that arise from vector gluon exchange as well as an effective scalar confinement interaction [40-42]. The utilized potential can be expressed mathematically in the subsequent formula:

$$\boldsymbol{V}(r) = \left(br - \frac{4\alpha_s}{3r}\right) + \frac{1}{m_b^2}\left[\frac{32\pi\alpha_s\delta_\sigma(r)S_bS_{\bar{b}}}{9} + \left(\frac{2\alpha_s}{r^3} - \frac{b}{2r}\right)\boldsymbol{L}.\boldsymbol{S} + \frac{4\alpha_s}{r^3}\boldsymbol{T}\right] \tag{2.1}$$

The meson reduced mass μ is calculated by the next equation due to the meson is compound of two tiny particles (quark-antiquark):

$$\mu = \frac{m_b m_{\bar{b}}}{m_b + m_{\bar{b}}} \tag{2.2}$$

δ_σ is obtained it from formula:

$$\delta_\sigma(r) = \left(\frac{\sigma}{\sqrt{\pi}}\right)^3 e^{-\sigma^2 r^2} \tag{2.3}$$

$\boldsymbol{S}_b\boldsymbol{S}_{\bar{b}}$ is gotten from:

$$\langle SM_S|\boldsymbol{S}_b\boldsymbol{S}_{\bar{b}}|SM_S\rangle = \frac{S(S-1)}{2} - \frac{3}{4} \tag{2.4}$$

S represents the total spin [43, 44] for the meson.

$\boldsymbol{T}$ expresses the tensor operator [42] as follows:

$$\boldsymbol{T} = \boldsymbol{S}_b.\hat{r}\,\boldsymbol{S}_{\bar{b}}.\hat{r} - \frac{1}{3}\boldsymbol{S}_b.\boldsymbol{S}_{\bar{b}} \tag{2.5}$$

$\boldsymbol{L.S}$ gives the orbit-spin operator:

$$\langle \boldsymbol{L.S}\rangle = [\frac{J(J+1)}{2} + \left(-\frac{S(S+1)}{2} - \frac{L(L+1)}{2}\right)] \quad (2.6)$$

2.2 The QPC model

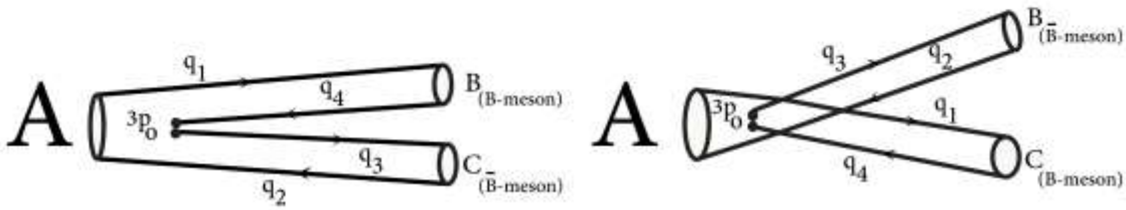

Figure.1: The two possible figures take part to A → B + C strong decay process as the QPC Model ($^3P_0$ model) simulates it (Author)

The use of the Quark Pair Creation model (QPC), commonly referred to as the $^3P_0$ model, was made possible thanks to Micu [38]. The Orsay group has credit for their contributions to advancing the $^3P_0$ model as well [45-49]. The $^3P_0$ model is widely applied as the most straightforward and used method for describing and computing the strong decay of various hadrons spectra into two hadronic particles [50-55] under the OZI-allowed rule. The basic mathematical formula of the $^3P_0$ model is introduced in this section to accurately determine strong decay widths. This model requires only one normalization parameter for the quark pair production process, making it superior to other models. Additionally, it has a good phenomenological approach and provides insight into the production of quark pair ($q\bar{q}$) from a vacuum.

In general, we can express any hadron strong decay A → B + C process using the helicity amplitude as follows:

$$\langle BC|T_r|A\rangle = \delta^3(\vec{k}_i - \vec{k}_f)\mathcal{A}_m^{M_{J_A}M_{J_B}M_{J_C}}(\vec{K}) \quad (2.7)$$

T_r is the transition operator: this operator presents a significant imaginary for the pair production process from the vacuum, the quark pair is in the $^3P_0$ state, and it has the quantum number 0^{++}.

$\vec{k}_i$, $\vec{k}_f$ denote the mesons three-momenta in the initial and final states, correspondingly as follows:

$$\vec{k}_i = \vec{k}_A \ \& \ \vec{k}_f = \vec{k}_B + \vec{k}_C \tag{2.8}$$

$M_{J_A}, M_{J_B}, M_{J_C}$ exemplify the orbital magnetic momenta.

The transition matrix contains the most significant operator concerning this model, and it is the transition operator T_r that defines from:

$$T_r = -3\sum_m \langle 1m; 1-m|00\rangle \, \gamma \int b_3^\dagger(k_3) d_4^\dagger(k_4) \delta^3 \left(\vec{k}_3 + \vec{k}_4\right) \times \mathcal{Y}_{1m}\left(\frac{\vec{k}_3-\vec{k}_4}{2}\right) \chi_{1-m}^{34} \phi_0^{34} \, \omega_0^{34} \, d^3k_3 d^3k_4 \tag{2.9}$$

γ: is an undetermined dimensionless parameter of this model.

$\mathcal{Y}_{1m}$, χ_{1-m}^{34}, ω_0^{34} and ϕ_0^{34} exemplify the wave functions for the quark pair creation from the vacuum concerning solid harmonic oscillator, spin, flavor, and color, respectively. 3 and 4 represent the quark pair ($q\bar{q}$), respectively.

The helicity amplitude $\mathcal{A}_m^{M_{J_A} M_{J_B} M_{J_C}}(\vec{K})$ is a significant part for this model, using Eq.2.7, Eq.2.9 and with the normalization [56], we can write it as follows:

$$\begin{aligned}\mathcal{A}_m^{M_{J_A} M_{J_B} M_{J_C}}(\vec{K}) = \gamma \sum_{\substack{M_{L_A}, M_{S_A} \\ M_{L_B}, M_{S_B} \\ M_{L_C}, M_{L_C}, m}} & \langle L_B M_{L_B}; S_B M_{S_B} | J_B M_{J_B} \rangle \langle L_C M_{L_C}; S_C M_{S_C} | J_C M_{J_C} \rangle \\ & \times \langle L_A M_{L_A}; S_A M_{S_A} | J_A M_{J_A} \rangle \langle 1m; 1-m|00\rangle \\ & \times \left\langle \chi_{S_B M_{S_B}}^{q_1 q_4} \chi_{S_C M_{S_C}}^{q_3 q_2} \middle| \chi_{S_A M_{S_A}}^{q_1 q_2} \chi_{1-m}^{q_3 q_4} \right\rangle \\ & \times [\langle \phi_B^{q_1 q_4} \phi_C^{q_3 q_2} | \phi_A^{q_1 q_2} \phi_0^{q_3 q_4} \rangle I(\vec{K}, m_1, m_2, m_3) \\ + (-1)^{1+S_A+S_B+S_C} & \langle \phi_B^{q_3 q_2} \phi_C^{q_1 q_4} \phi | \phi_A^{q_1 q_2} \phi_0^{q_3 q_4} \rangle I(-\vec{K}, m_2, m_1, m_3)]\end{aligned} \tag{2.10}$$

Just for clarification, (L_i, S_i & J_i) ; $i = A, B, or\ C$ are the orbital angular momentum, the total spin, and the total angular momentum for $A,\ B,\ and\ C$ hadron, correspondingly. In Figure 1; the right part, we can easily substitute B ⟷ C, $m_1 \leftrightarrow m_2$ and $\vec{K} \longrightarrow -\vec{K}$. In the meson A center-of-mass framework, we consider $\vec{K} \equiv K_B = -K_C$.

$\langle L_A M_{L_A} S_A M_{S_A} | J_A M_{J_A} \rangle$, $\langle 1m; 1-m|00\rangle$, $\langle L_B M_{L_B};\ S_B M_{S_B} | J_B M_{J_B} \rangle$, and $\langle L_C M_{L_C};\ S_C M_{S_C} | J_C M_{J_C} \rangle$ present the Clebsch-Gordan coefficients of the orbital angular momentum, the total spin and the total angular momentum for the initial meson and quark pair creation, and the final mesons (B & C), correspondingly, during the strong decay processes of mesons.

We will present the momentum-space integral of the left part of Figure 1 as follows:

$$I(\vec{K}, m_1, m_2, m_3) = \int d^3k\ \mathcal{Y}_{1m}(\vec{k})\psi_{n_A, L_A, M_{L_A}}(\vec{k}+\vec{K})\psi_{n_B, L_B, M_{L_B}}\left(\vec{k}+\frac{m_1+m_3}{m_3}\vec{K}\right) \times\ \psi^*_{n_C, L_C, M_{L_C}}\left(\vec{k}+\frac{m_1+m_3}{m_3}\vec{K}\right) \quad (2.11)$$

For clarity,

m_1 and m_2 present the quark masses of the initial meson, and $m_3 = m_4$ are the quark pair creation masses from vacuum, the same quarks supply us the constituent quarks mass of B and C meson.

We use the wavefunction $\psi^{SHO}_{nLM_L}(\vec{k})$ of the simple harmonic oscillator (SHO) in momentum space:

$$\psi^{SHO}_{nLM_L}(\vec{k}) = R^{SHO}_{nL}(k) Y_{LM_L}(\theta_k, \phi_k) \quad (2.12)$$

where;

$$R^{SHO}_{nL}(k) = ((-1)^n(-i)^L \beta^{-\frac{3}{2}}) \sqrt{\frac{2n!}{\Gamma\left(n+L+\frac{3}{2}\right)}} \times \left(\frac{k}{\beta}\right)^L L_n^{L+\frac{1}{2}}\left(\frac{k^2}{\beta^2}\right) e^{-k^2/(2\beta^2)} \quad (2.13)$$

The formulation of the partial decay amplitude is by using the Jacob-Wick formula within the helicity amplitudes $\mathcal{A}_m^{M_{J_A} M_{J_B} M_{J_C}}(\vec{K})$ [57, 58] as next:

$$\mathcal{A}_m^{LS}(K) = \frac{\sqrt{4\pi(2L+1)}}{2J_A+1} \sum_{M_{J_B}, M_{J_C}} \langle L0SM_{J_A} | J_A M_{J_A} \rangle \langle J_B M_{J_B} J_C M_{J_C} | SM_{J_A} \rangle \mathcal{A}_m^{M_{J_A} M_{J_B} M_{J_C}}(K_{\hat{z}}) \quad (2.14)$$

Taking into account:

$$M_{J_A} = M_{J_B} + M_{J_C} \quad (2.15)$$

$$\vec{L} = \vec{J}_A - \vec{S} \text{ and } \vec{S} = \vec{J}_B + \vec{J}_C \quad (2.16)$$

So, we obtain:

$$|J_A - S| \le L \le J_A + S \quad (2.17)$$

$$|J_B - J_C| \le S \le J_B + J_C \quad (2.18)$$

$\vec{K} \equiv K_{\hat{z}}$ is the outgoing momentum of the final producing mesons (B or C), that is in the $\hat{z}$-axis through the A meson Centre-of-mass framework.

Now, the partial width can be formulated in the framework of relativistic phase space [37, 59] as follows:

$$\Gamma_{A\rightarrow BC}^{LS} \ (K) = 2\pi \ K \frac{E_B(K)E_C(K)}{M_A} \ |\mathcal{A}_m^{LS}(K)|^2 \tag{2.19}$$

The A is an initial meson, B and C are final mesons, these mesons have masses M_A , M_B, and M_C, correspondingly. The momentum K is written by:

$$K = \frac{\sqrt{[M_A{}^2-(M_B+M_C)^2][M_A{}^2-(M_B-M_C)^2]}}{2M_A} \tag{2.20}$$

For B and C mesons, we can obtain the E_B and E_C that are total energy for these mesons by the next equations:

$$E_B = \sqrt{M_B^2 + K^2}, \quad E_C = \sqrt{M_C^2 + K^2} \tag{2.21}$$

In the final, the equation for the total decay width of the A meson is as following:

$$\Gamma^{total} = \sum_{L,S} \Gamma_{A\rightarrow BC}^{LS} \ (K) \tag{2.22}$$

3. RESULTS and discussionn

We solve numerically the Schrödinger equation utilizing the matrix method. The parameters obtained from Quantum Chromodynamics theory, in addition to the nonrelativistic quark model, as they are listed in Table 1, can be used to determine the higher bottomonium mesons masses. These parameters are extracted from Ref. [60]. In accordance with this, the determined masses are compared with current empirical data [61]. The computed masses for the bottomonium mesons are listed in Table 2, indicated by the state notation (n $^{2S+1}L_J$). The other groups' studies for the bottomonium mesons spectra utilized many potential models, which involved the variational approaches with the single Gaussian wavefunction in the Cornell potential relativistic quark model by (Virendrasinh Kher et al.) [62]. Also, the Screened potential in the frame of the non-relativistic quark model was conducted by (W.J. Deng et al.) [63].

Table 1: The parameters values for the predicated masses of the bottomonia mesons [60]

The parameter	The value
α_s	0.4040
m_b	4.8087 GeV
b	0.1620 GeV^2
σ	2.2927 GeV

Table 2: The expected mass of bottomonia of the higher nS states by our Non-relativistic (QM)Quark model. The measured data are given from PDG [61]. The other theoretical results are from Refs. [62, 63]

Bottomonia mesons	Notation of states	Our expected masses (MeV)	Experimental masses (MeV)	Results from Ref.	Results from Ref.
ϒ(5S)	$5\,^3S_1$	10.8290	10.8852	10.912	10.8110
η_b(5S)	$5\,^1S_0$	10.8210	–	10.901	10.8000
ϒ(6S)	$6\,^3S_1$	11.0360	11.000	11.151	10.9970
η_b(6S)	$6\,^1S_0$	11.0290	–	11.140	10.9880
ϒ(7S)	$7\,^3S_1$	11.2290	–	–	–
η_b(7S)	$7\,^1S_0$	11.2220	–	–	–

The masses of B mesons are taken from measured data [61] as in Table 3, and the mass of η_b(5S) S-wave higher bottomonia meson state have been extracted from the calculated values as seen in Table 2 where its empirical data not available. It is clear from Table 2 that our Non-Relativistic Quark model (NRQM) provides us with the predicted masses for higher bottomonium mesons states that include η_b(5S) state, we note that the findings agree with the available recent measured data by PDG [61] and with the other results for other groups in Refs.[62, 63].

Table 3: The mass of B mesons that obtained from the recent measured data PDG [61]

Beauty	Notation of	Our expected masses (MeV)	Experimental masses
B	1^1S_0	5273	5279.65 ± 0.12
B^*	1^3S_1	5322	5324.70 ± 0.21
B_s	1^1S_0	5367	5366.88 ± 0.14
B_s^*	1^3S_1	5413	$5415.4^{+1.8}_{-1.5}$

The S-wave bottomonia has two sectors; the first one is Υ(nS), and the second sector is η_b(nS). The second sector has two singlet-spin states above the threshold $B\bar{B}$ mesons. In fact, these states in the second sector have J^{PC} quantum numbers are 0^{-+}, and these states are the η_b(5S) state and η_b(6S) state. The bottomonium states in two sectors decay to $B\bar{B}$ mesons by strong decay mode. In this paper, we concentrate on the strong decay of η_b(5S) meson, we will perform analyzing and detailed studies of its strong decay to give a clear insight into the sources of $B\bar{B}$ mesons pairs from the strong decay for η_b(5S).

Here, we utilize the method introduced by Barnes and Swanson to calculate the decay amplitude for the $^3P_0$ model [37, 40], which have been constructed as in the prior section to furnish us with the decay widths of the η_b(5S) mesons. We use the previous techniques, that have been constructed as in the last section to give us the strong decay widths of the η_b(5S) mesons. In this section, we are entering the parameters, that make this model applicable to calculate the strong decay widths of the η_b(5S) meson. Analytical evaluation of the strong decay amplitude was conducted in this study based on the Barnes and Swanson technique, and numerical values were subsequently determined based on specific parameters, including β = 0.5 GeV, γ = 0.89, and constituent quark masses of $m_{u,d}$ = 0.33 GeV, m_s = 0.55 GeV, and m_b = 4.8089 GeV [64]; the flavor symmetry of meson wave functions allows for the use of a fixed width of the simple harmonic oscillator parameter β, which has been validated in previous literature [40, 65] and aligns with experimental results.

We adopted using the SHO wavefunctions as Barnes and Swanson technique [37, 40] with γ equal to 0.89, also (S. Godfrey and K. Moats) [56] applied the SHO wavefunctions, but with γ = 0.6. And (J.Z. Wang et al.) [66] applied realistic wavefunctions that are with the γ strength pair creation value equal to 7.09. We utilized the $^3P_0$ model, which enabled us to expect a total decay width for η_b(5S) to be 26.417 MeV.

Table 4: The expected values concerning strong decay widths of η_b(5S) bottomonium state by our QPC model and from Ref.[56], and Ref.[66]

The decaying meson	Notation of states	Partial strong decay	Our expected results by QPC model by (MeV)	expected results from Ref. [56] by (MeV)	expected results from Ref. [66] by (MeV)
η_b(5S)	5 $^1S_0$	BB	—	—	—
		$B\,B^*$	2.684	13.10	20.30
		$B^*\,B^*$	14.892	0.914	0.771
		B_sB_s	—	—	—
		$B_sB_s^*$	1.496	0.559	1.110
		$B_s^*B_s^*$	7.345	5.490	2.910
		Total width	26.417	20.063	25.091

3.1 η_b(5S) state

The state is denoted by η_b(5S), characterized by notation (n $^{2S+1}L_J$= 5 $^1S_0$). The quantum numbers of this state, $J^{PC} = 0^{-+}$, make it a scalar state. Furthermore, this state is considered above the threshold of the $B\bar{B}$ mesons pair. The strong decays of the η_b(5S) are:

η_b(5S) $\rightarrow BB^*$ & η_b(5S) $\rightarrow B^*B^*$

η_b(5S) $\rightarrow B_sB_s^*$ & η_b(5S) $\rightarrow B_s^*B_s^*$

In reality, η_b(5S) is distinguished own more varied partial decay widths, which is very important in the search for sources of B mesons, we will establish detailed studies for each alone partial strong decay width by applying our QPC model in calculating the theoretic partial width value, and then finding the branching ratio $\mathcal{BR}$ to every partial strong decay. Thereafter, we compare our theoretical values with others in Refs. [56], and [66]. The strong decay widths for η_b(5S) bottomonium state are listed in Table 4. Unfortunately, there are no experimental total width and experimental partial widths of the strong decay [61] of the η_b(5S) state, so our study is very important to fill this lack in the measured data.

Our theoretic total strong decay width value $\Gamma_{\eta_b(5S)}^{\text{total value.}} = 26.417$ MeV, whereas the other theoretical values are approximately equal to 20.063 MeV, and 25.091 MeV in Refs. [56], and [66], respectively. From that, we infer that our theoretical value concurs significantly with other theoretical total decay width values. The present study offers new conclusions about the partial strong decay widths, which differ from the predictions made by Refs. [56] and [66] as well as differ from each other. These conclusions can be tested by upcoming particle colliders in the future. We will offer new conclusion as the following:

i. $\boldsymbol{\eta_b(5S) \to BB^*}$

The computed value for this partial decay width amounts to $\Gamma_{\eta_b(5S)\ \to BB^*}^{our\ ^3P_0\ \text{model}} \cong 2.684$ MeV, accompanied by a branching decay width ratio $\mathfrak{BR}_{\eta_b(5S)\ \to BB^*}^{^3P_0\ \text{model}} = \frac{\Gamma_{\eta_b(5S)\ \to BB^*}}{\Gamma_{\text{total}}} \cong$ 10.16 %. Thus, based on our calculations, this strong decay does not exhibit the highest branching decay width within this state. Instead, it represents the third partial decay width and is not the dominant one in comparison to the other partial strong decays of this state. It is noteworthy that this partial decay of this state provides a reasonable source for BB^*mesons pairs.

ii. $\boldsymbol{\eta_b(5S) \to B^*B^*}$

The calculated value of the partial strong decay width is $\Gamma_{\eta_b(5S)\ \to B^*B^*}^{^3P_0\ \text{model}} \cong 14.892$, accompanied by a branching decay width ratio of $\mathfrak{BR}_{\eta_b(5S)\ \to B^*B^*}^{^3P_0\ \text{model}} = \frac{\Gamma_{\eta_b(5S)\ \to B^*B^*}}{\Gamma_{\text{total}}} \cong$ 56.37 %. As a result, this specific partial strong decay exhibits the highest branching decay among all others, as for our estimations. As such, it holds dominance over the other partial decays associated with this state. It is evident from our calculations that this particular partial strong decay serves as a significant source for generating the B^*B^* mesons pairs.

iii. $\boldsymbol{\eta_b}$**(5S)** $\rightarrow \boldsymbol{B_s B_s^*}$

The value estimated for the width of this particular decay is $\Gamma^{^3P_0\ \mathrm{model}}_{\eta_b(5S)\ \rightarrow B_s B_s^*} \cong 1.496$ MeV, and the ratio of the branching decay width is $\mathfrak{BR}^{^3P_0\ \mathrm{model}}_{\eta_b(5S)\ \rightarrow B_s B_s^*} = \frac{\Gamma_{\eta_b(5S)\ \rightarrow B_s B_s^*}}{\Gamma_{\mathrm{total}}} \cong$ 5.66 %. As a result, this specific decay has the lowest branching decay among the decays of this state, according to our calculations. Therefore, it holds the fourth position in terms of the width of other partial decays of this state. Nonetheless, it serves as a valid source for the $B_s B_s^*$ pairs of mesons.

$\boldsymbol{\eta_b}$**(5S)** $\rightarrow \boldsymbol{B_s^* B_s^*}$

The calculated value for this particular decay width is $\Gamma^{^3P_0\ \mathrm{model}}_{\eta_b(5S)\ \rightarrow B_s^* B_s^*} \cong 7.345$ MeV, along with a branching decay width ratio of $\mathfrak{BR}^{^3P_0\ \mathrm{model}}_{\eta_b(5S)\ \rightarrow B_s^* B_s^*} = \frac{\Gamma_{\eta_b(5S)\ \rightarrow B_s^* B_s^*}}{\Gamma_{\mathrm{total}}} \cong 27.8$ %. Consequently, it is evident that this specific decay does not possess the highest branching decay among all the decays associated with this state, as for our calculated results. Hence, this decay cannot be considered as the dominant one relative to the other partial decays of this state. However, it is noteworthy that its value is deemed as the subsequent-to-dominant partial decay of this state. Furthermore, this strong decay holds good potential as a source for generating $B_s^* B_s^*$ mesons pairs.

In light of our previous analysis of the results in relation to the $\eta_b(5S)$ state, we obtained the following conclusions:

- $\eta_b(5S)$ is a good source of non-strange B mesons type: BB^*, and B^*B^* in addition to the strange B mesons type: $B_s B_s^*$, and $B_s^* B_s^*$.
- $\eta_b(5S)\ \rightarrow\ B^*B^*$ owns the dominant strong decay partial width in our theoretic calculations of $\eta_b(5S)$ state strong decays.
- $\eta_b(5S)\ \rightarrow\ B_s^* B_s^*$ provides us a sizable value relative, and it is considered the next-to-dominant regarding our calculations of private strong decay partial widths of $\eta_b(5S)$ state.

- The strong decay widths for $\eta_b(5S)$ can be arranged according to their width value to B^*B^*, $B_s^*B_s^*$, BB^*, and $B_sB_s^*$ respectively.
- The discrepancies in the theoretical calculations of partial strong decay widths values, (concerning us with the other works, and among themselves), can be attributed to several factors. Firstly, the practical measurements conducted may be not sufficient in accurately determining the decay values. Additionally, there exists the possibility of latent errors within the experimental setup. Secondly, the computed partial width of the strong decay is directly influenced by the mass of the particle under consideration. Therefore, a more precise determination of the mass value could potentially reduce the potential deviation in the decay model. Finally, it is plausible that there are other physical effects and mechanisms that are currently unknown to us at play in the strong decays of the $\eta_b(5S)$ and other hadrons.

4. CONCLUSION

In this work, we offered analysis and detailed study around the strong decays of $\eta_b(5S)$ as the source of B mesons. This study is very important to the recent developments of the Standard Model (SM) in addition to its necessity to New Physics (NP) or Beyond Standard Model (BSM) physics. Where our inlet to a new world of physics is the rare decays of B mesons. So, we searched for the sources of the B mesons through the strong decays of $\eta_b(5S)$. The non-relativistic quark model was used to obtain higher bottomonium states, including the mass of $\eta_b(5S)$. Additionally, the ${}^3P_0$ model was used to acquire the strong decay partial widths of $\eta_b(5S)$. The present study shows that our result for the total decay width is consistent with the findings of other research groups. Additionally, we present new conclusions regarding the partial decay widths. Our study is particularly important because of there are currently no experimental data available for either the total width or the partial width.

Based on our study, it has been determined that the $\eta_b(5S)$ state serves as a source for BB^*, B^*B^*, $B_sB_s^*$ and $B_s^*B_s^*$ pair mesons. However, it cannot supply us the BB pair mesons and B_sB_s pair mesons, respectively. The $\eta_b(5S)$ supplies us with a significant source of the B^*B^*, and it furnishes with a sizeable source of the $B_s^*B_s^*$ mesons pairs that the first is considered the dominant source, and the second is next-to-dominant, respectively, it also provides a reasonable source of BB^* pair mesons and $B_sB_s^*$ pair mesons, correspondingly.

We focus our conclusions about the source of Beauty mesons from the $\eta_b(5S)$ between the experimental high energy physics scientists' hands and theoretical high energy physics scientists' hands to help them to know the nature of the *B* mesons and the $\eta_b(5S)$ meson, examine several issues of SM, and open new horizons in a BSM using currently and upcoming hadrons colliders. We recommend utilizing the QPC model to calculate the strong decay widths of $\eta_b(5S)$ and other hadrons. In addition, we strength exhort using the non-relativistic quark model to get the mass of $\eta_b(5S)$, the rest of the bottomonium mesons, and any other heavy hadron.

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