Assiut University Journal of Multidisciplinary Scientific Research (AUNJMSR)
Faculty of Science, Assiut University, Assiut, Egypt.
Printed ISSN 2812-5029
Online ISSN 2812-5037
Vol. 53(1): 190- 198 (2024)
https://aunj.journals.ekb.eg

Simulating the Nonlinear Propagation Effects in Biological Materials Using Westervelt Equation

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ARTICLE INFO

Article History:
Received: 2023-10-30
Accepted: 2023-12-06
Online: 2023-12-28

Keywords:
Acoustic
nonlinear effects
Westervelt equation
tissue interaction
FDTD method.

ABSTRACT

Numerical simulations of the nonlinear acoustic waves for medical applications are performed by using Westervelt equation. In this paper, FDTD has been introduced to solve Westervelt equation. Nonlinear propagation scheme is applied on different biological media where a 1 kpa transducer pressure wave of 1 MHz and interactions with tissues are analyzed. Our results show the importance of considering nonlinear interactions in understanding the behavior of sound waves in biological tissues.

INTRODUCTION

Nonlinear acoustic wave equations pose complex mathematical problems in various scientific and engineering fields, such as physics, acoustical engineering, sound science, and acoustics [1]. They are of paramount importance because linear wave equations often fall short in accurately describing high-intensity acoustic waves [2]. One of the most widely used and important nonlinear acoustic wave equations is Westervelt equation, introduced by P.J. Westervelt in 1963 [3], which describes the propagation of finite-amplitude acoustic waves in fluids [4], accounting for the nonlinear effects of the acoustic pressure and particle velocity on the wave propagation [5]. Solving these equations remains an ongoing research challenge, as they can exhibit intricate phenomena due to nonlinear interactions between acoustic waves and the surrounding medium.

Effective and accurate solution methods for nonlinear acoustic wave equations are of utmost importance in understanding acoustic phenomena and their practical applications.

Over the years, Westervelt equation has been extensively studied and widely used in various applications due to its ability to capture the nonlinear effects of sound propagation accurately [6]. Researchers have explored its behavior, properties, and numerical solution methods to gain a deeper understanding of the nonlinear phenomena exhibited by acoustic waves. Simulating - nonlinear - acoustic wave propagation in biological materials is a critical area of research with applications in medical ultrasound imaging and therapeutic ultrasound [7- 9]. To address the complexities of Westervelt equation, three principal methods have been employed [10]: finite difference schemes, a direct and pragmatic approach [11, 12]; Greens functions, designed for exact solutions with linear operators and well-defined forcing functions; and finite element method (FEM) which offers a unique perspective. Although significant progress has been made in understanding Westervelt equation and its nonlinear behavior, some gaps in the existing literature still remain. For instance, further investigations are needed to explore the impact of nonlinear interactions in more realistic and complex scenarios, such as the presence of multiple scattering sources and heterogeneous media. Moreover, while the Finite Difference Method has proven effective for numerical simulations in terms of The Finite - Difference Time - Domain scheme (FDTD).The development of more advanced numerical techniques, such as the Finite Element Method and the Spectral Method, merits further exploration to enhance the accuracy and efficiency of solving Westervelt equation.

MATERIALS AND METHODS

A nonlinear acoustic field can be simulated by Equation 1. This equation is known as Westervelt equation and can be used to simulate wave propagation in tissue,

$$\nabla^2 p - \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} + \frac{\delta}{c_0^4}\frac{\partial^3 p}{\partial t^3} = -\frac{\beta}{\rho_0 c_0^4}\frac{\partial^2 p^2}{\partial t^2} \qquad (1)$$

Where p is the pressure of wave; c_0 is the propagation speed; $\beta=1+B/2A$ is the nonlinearity coefficient where B/A is the nonlinearity parameter of the medium [13]; and δ is the sound diffusivity, which depends on viscosity and thermal conductivity of the medium. The first and second terms of Equation (1) represent linear propagation of waves in a medium without any loss. The third term is related to the losses due to the thermal conductivity and viscosity of the fluid. The last term of the equation is related to the nonlinear factors influencing the simulation of wave propagation which may cause thermal and mechanical changes within the tissue.

The system of equation 1 is solved using (FDTD) algorithm. The FDTD is a set of discrete points in time that will sample our functions. This is done by fixing a grid spacing of Δt in time and Δx in space. The points that lie on the mesh are then defined as,

$$t_j = j\Delta t \quad j = 1,2, \ldots \ldots, N_t \qquad (2)$$

$$x_i = i\Delta x \quad i = 1,2, \ldots \ldots, N_x \qquad (3)$$

Where Δt and Δx is the constant length of the time and space steps. The mesh function will be computed from algebraic equations derived from the equations (1) using the central-difference method as follows [14, 15],

$$\frac{\partial p}{\partial t} = \frac{1}{\Delta t}\left(p_{i,j} - p_{i,j-1}\right) \tag{4}$$

$$\frac{\partial^2 p}{\partial t^2} = \frac{1}{\Delta t^2}\left(p_{i,j+1} - 2\,p_{i,j} + p_{i,j-1}\right) \tag{5}$$

$$\frac{\partial^3 p}{\partial t^3} = \frac{1}{2\Delta t^3}\left(3\,p_{i,j+1} - 10\,p_{i,j} + 12\,p_{i,j-1} - 6\,p_{i,j-2} + p_{i,j-3}\right) \tag{6}$$

$$\frac{\partial^2 p}{\partial x^2} = \frac{1}{12\Delta x^2}\left(-p_{i+2,j} + 16\,p_{i+1,j} - 30\,p_{i,j} + 16\,p_{i-1,j} - p_{i-2,j}\right) \tag{7}$$

The term $\frac{\partial^2 p^2}{\partial t^2}$ is calculated by making use of the chain rule and product rule:

$$\frac{\partial^2 p^2}{\partial t^2} = 2\left(\left(\frac{\partial p}{\partial t}\right)^2 + p\,\frac{\partial^2 p}{\partial t^2}\right) \tag{8}$$

Substitution in eq.(1) we get

$$P_{i,j+1} = \Big[\frac{1}{12\Delta x^2}(-p_{i+2,j} + 16\,p_{i+1,j} - 30\,p_{i,j} + 16\,p_{i-1,j} - p_{i-2,j}) + p_{i,j}\left(\frac{2}{c_0^2\,\Delta t^2} - \frac{5\,\delta}{c_0^4\,\Delta t^3}\right) + p_{i,j-1}\left(\frac{6\,\delta}{c_0^4\,\Delta t^3} - \frac{1}{c_0^2\Delta t^2}\right) - p_{i,j-2}\left(\frac{3\,\delta}{c_0^4\,\Delta t^3}\right) + p_{i,j-3}\left(\frac{1}{2}\,\frac{\delta}{c_0^4\,\Delta t^3}\right) + \frac{2\beta}{\rho_0 c_0^4}\,\frac{1}{\Delta t^2}\left(p_{i,j} - p_{i,j-1}\right)^2 + \frac{2\beta}{\rho_0 c_0^4}\,\frac{p_{i,j}}{\Delta t^2}\left(p_{i,j+1} - 2\,p_{i,j} + p_{i,j-1}\right)\Big] \Big/ \left(\frac{1}{c_0^2\Delta t^2} - \frac{3}{2}\,\frac{\delta}{c_0^4\Delta t^3}\right) \tag{9}$$

Where in one dimensions $\nabla^2 p = \frac{\partial^2 p}{\partial x^2}$. Note that c is actually dependent on (i) as it varies depending on the properties of the medium at each position. For simplicity we assume reflecting boundary conditions (i.e. $p_{i,j} = 0$ for all (i, j) which lie outside the grid) [16]. The Boundary condition can be described as

$$F(t) = p_0\, e^{-\left(\frac{t-td}{0.5\,t\omega}\right)^2}\ \sin\,\omega(t - td) \tag{10}$$

$$P(0, t) = f(t), \qquad P(L, t) = 0$$

$$P(x, 0) = 0, \qquad \left.\frac{\partial p}{\partial t}\right|_{t=0} = 0, \qquad \frac{c_0\,\Delta t}{\Delta x} < 1$$

f (t) is the source function at x = 0. P has essential boundary conditions at x = 0 and x= L [17].

Table (1). Complication of acoustic tissue properties [18].

Tissue Name (units)	C m/s	∝ dB/m/MHz	ρ Kg/m^3	Z MRayls	B/A
Blood	**1584**	**14**	**1060**	**1.68**	**6**
Fat	**1430**	**60**	**928**	**1.33**	**10.3**
Liver	**1578**	**45**	**1050**	**1.66**	**6.75**
Muscle	**1580**	**57**	**1041**	**1.64**	**7.43**

RESULTS AND DISCUSSION

The proposed method was implemented in FORTRAN and used $\Delta x = 3 \times 10^{-4}$ cm, $\Delta t = 5 \times 10^{-8}$ s and the step of grid, which λ/5, where λ is the wavelength of acoustic for the given frequency and medium. The source function is shown in fig. (1). the different biological materials used in this research are shown in Table (1). In the beginning, the computational start with the linear equation, i.e. β=0 and δ=0 in eq. (1). Fig. (2) shows that pressure amplitude at distance 1.2 cm in blood medium. Secondly, we study the effect of the nonlinear coefficient while neglecting the coefficient of absorption. Fig. 3 shows the effect of the β value, In addition to the initial pressure value, on the shape of the spectrum. It is clear from this figure that as the β value and the pressure value increase, the value and number of harmonic frequencies increases. Fig. (4) shows the change in pressure with frequency for the biological materials mentioned in Table (1). Due to the closeness of the β value between the materials, we find that the change is slight in harmonics frequencies and the fundamental frequency is constant for all biological materials. Thirdly, we study the effect of coefficient of absorption on the shape of the wave while neglecting beta. Fig. (5) shows the waveform of the fat substance at different depths, as well as Figure No. 6 shows the change in the shape of the spectrum for biological materials with a variable value of the absorption coefficient at depth 1.2 cm. The observed decrease in wave amplitude at specific distances suggests the influence of factors such as absorption. This understanding aids in optimizing ultrasound imaging and therapeutic procedures at varying depths within tissues. Fig. (6) shows the change in the shape of the spectrum for biological materials with a variable value of the absorption coefficient and nonlinear coefficient at depth 0.9 cm. By manipulating the nonlinearity coefficient (β) and adjusting the source pressure, distinct frequency-domain patterns were observed. These variations emphasized the sensitivity of the acoustic signals to changes in nonlinear parameters, which is crucial in applications like medical ultrasound where accurate signal interpretation is essential.

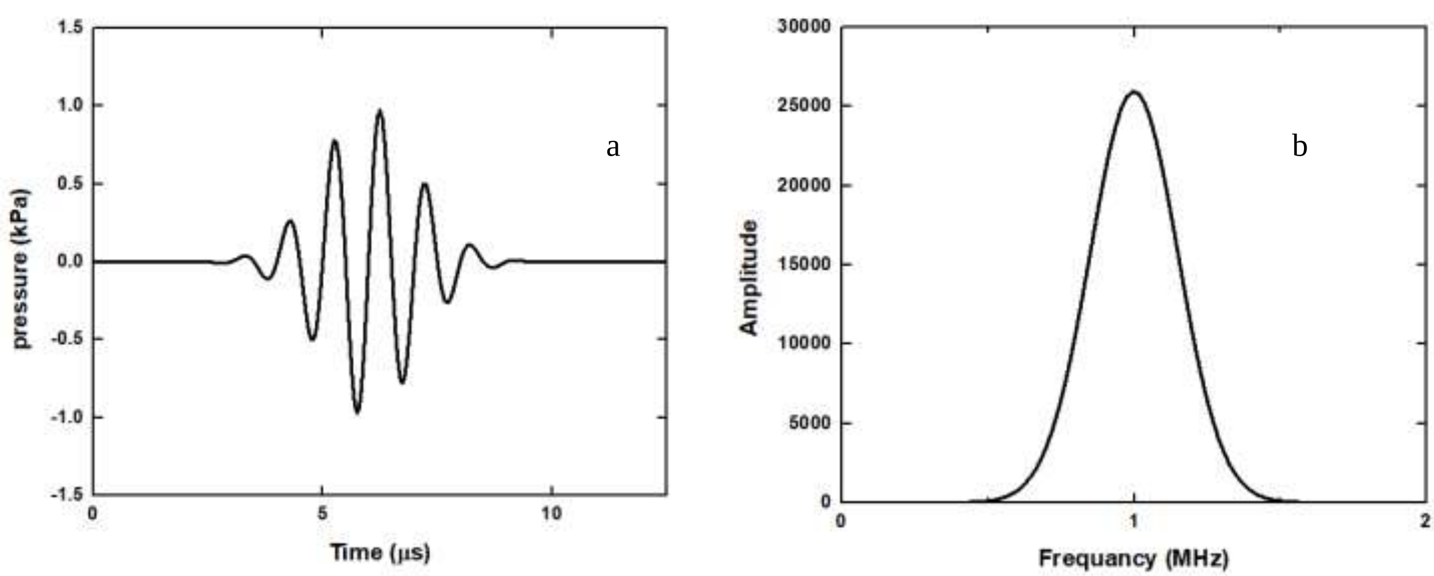

Figure 1 (a) pressure field as function of the time from the source and (b) Frequency domain.

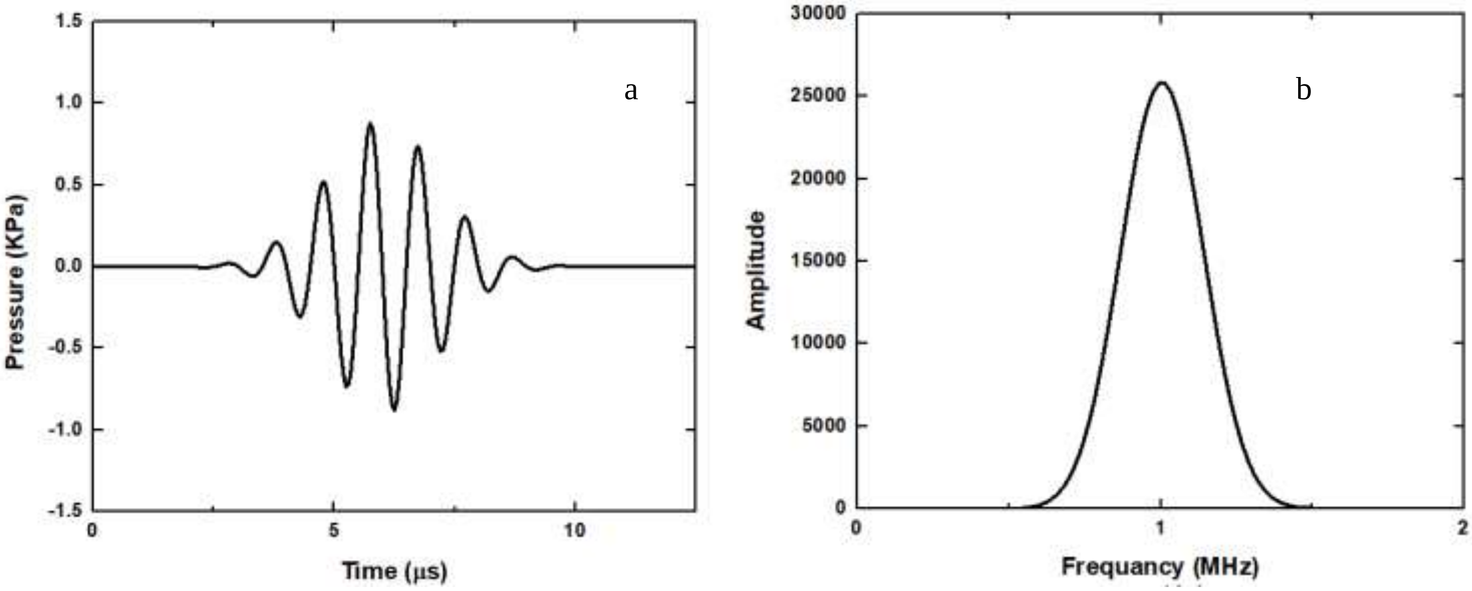

Figure 2 (a) pressure field as function of the time from the source at distance x = 1.2 cm and (b) Frequency domain.

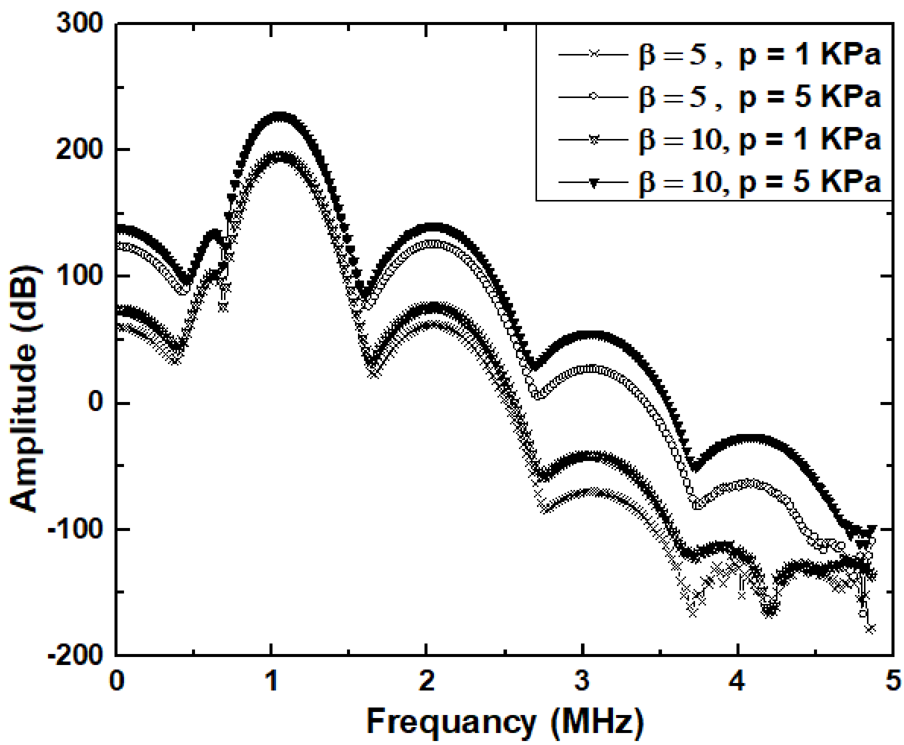

Figure 3 Frequency-domain results at different coefficient of nonlinearity and source pressure.

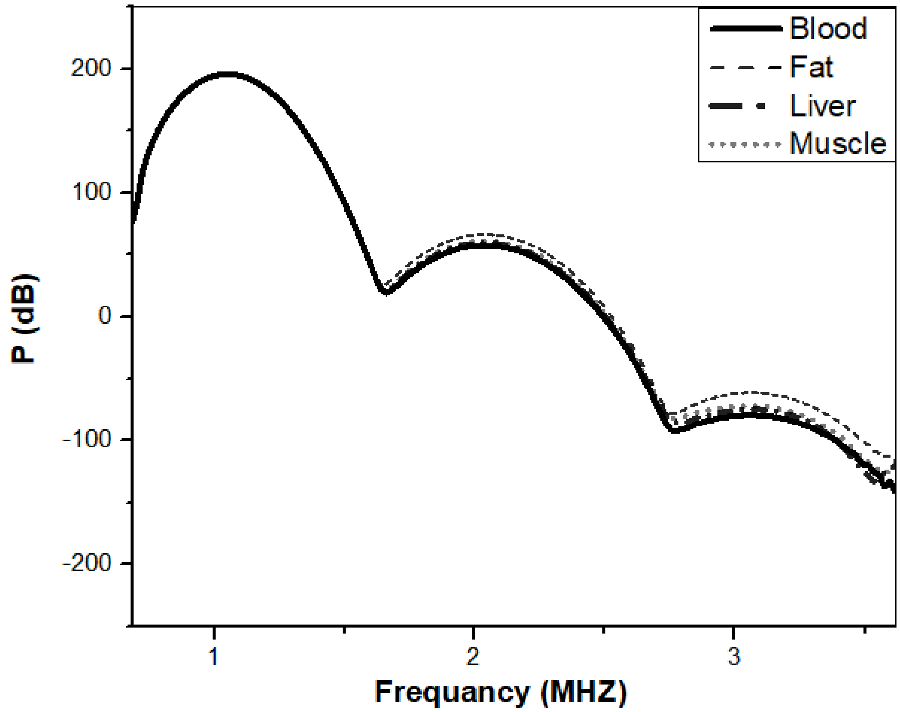

Figure 4 Frequency-domain results at different tissues at $\delta = 0$ with the effect of nonlinearity.

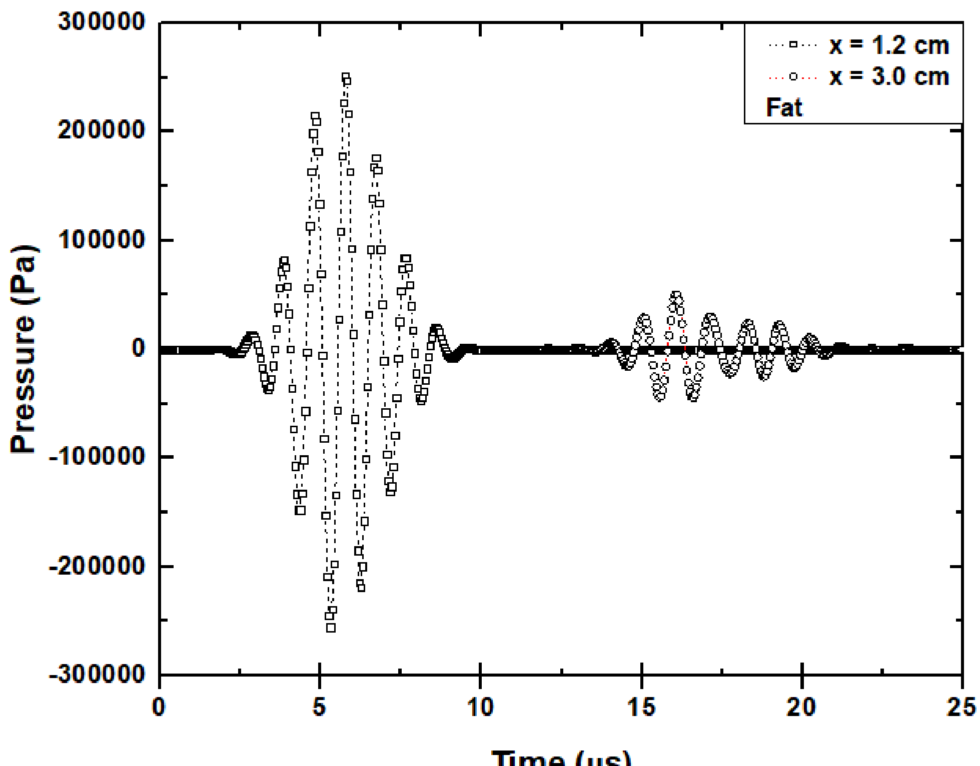

Figure 5 time domain signal for fat at different distance with the effect of absorption and $\beta = 0$.

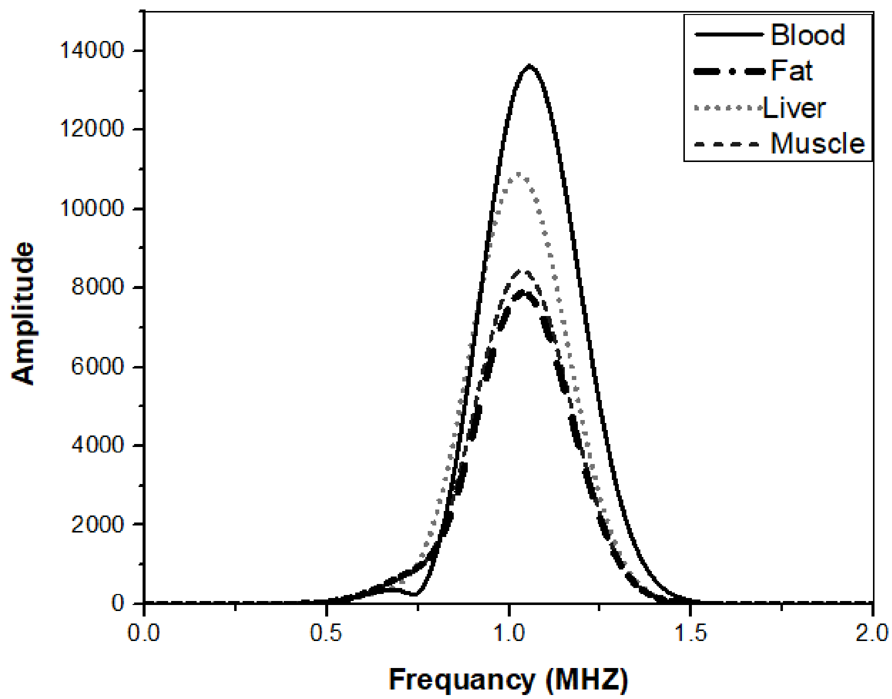

Figure 6 Frequency-domain results at different tissues at $\beta = 0$ with the effect of absorption.

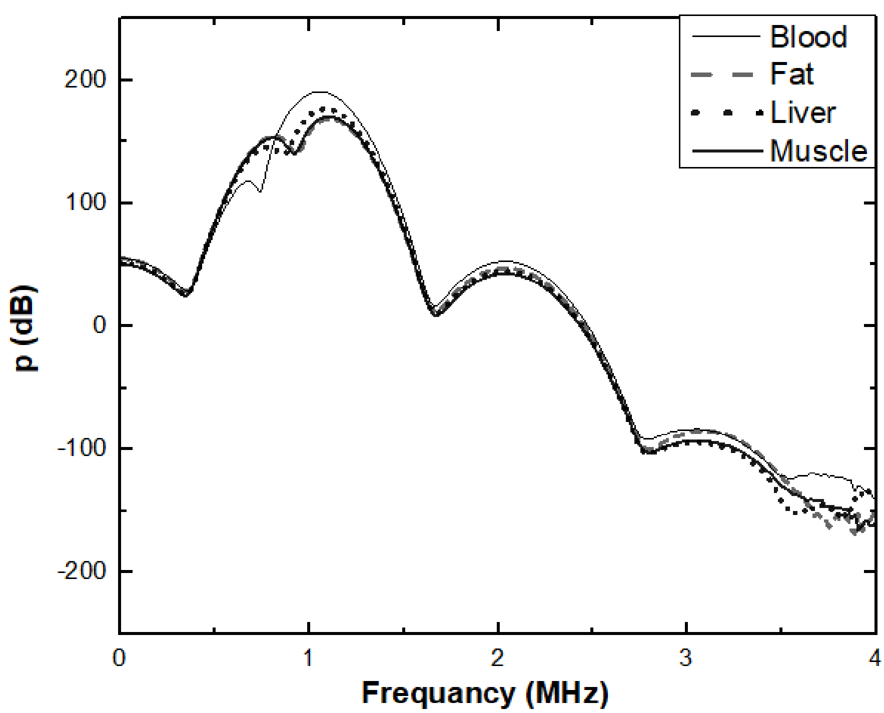

Figure 7 Frequency-domain results at different tissues with the effect of nonlinearity and absorption at distance 0.9 cm.

CONCLUSION

In this paper we presented a comprehensive investigation into the nonlinear propagation effects in biological materials using Westervelt equation was conducted. Finite-Difference Time-Domain (FDTD) scheme was successfully employed to solve Westervelt equation and analyze the interactions between acoustic waves and different biological tissues. Through its application, accurate numerical solutions were obtained for Westervelt equation. The successful application of the Finite Difference Method to Westervelt equation enhances our understanding of complex acoustic interactions and has the potential to improve medical the frequency response characteristic pattern provides deeper insights into how waves interact with biological tissues at different frequency ranges, which is essential for designing effective ultrasound imaging protocols and ultrasound applications. Our results show that the importance of considering nonlinear interactions in understanding the behavior of sound waves in biological tissues. Comparisons of pressure distribution and frequency response among different tissues (blood, fat, liver, muscle) revealed tissue-specific behaviors. These findings have implications for diagnostic accuracy and therapeutic efficacy as different tissues exhibit unique responses to acoustic waves.

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