Military Technical College
Kobry El-Kobbah,
Cairo, Egypt

7th International Conference
on Electrical Engineering
ICEENG 2010

Detection of PN sequence using Higher Order Statistics (HOS) Matched Filter Receiver

By

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Abstract:

This paper presents a Higher Order Statistic Matched Filter (HOSMF) receiver which use in signal detection of PN-sequence in the presence of the Gaussian and non Gaussian noise. We also propose a method for the realization of a correlator to compute the 3rd order cumulant. The PN-sequence is detected via multiple correlation and cumulants, respectively.

The detection algorithm is computationally simply and condrary to standard matched filter, it is insensitive to shift pn-sequence and does not required knowledge of the noise spectrum for pre whitening. The receiver can be viewed as like hood ratio test between sampled higher order statics. Simulation illustrated successful performance of the detection using correlation higher order statics matched filter (HOSMF) than the conventional MF under certain types of noise.

Keywords:

Matched Filter , PN-Sequence and Higher Order Statistics (HOS)

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1. Introduction:

Spread spectrum communication has been used in digital communication systems [1] due to its advantages: (a) security (b) resistivity to jamming (c) resistivity to multi path and resistivity to interference from other users. The main characteristic of the PN-sequence used is its low power level, and so the problem of PN-sequence detection has been always a hot area of research[3].
The conventional MF has been used as an optimum receiver for the detection of PN-sequence [2] if the noise is colored the MF is no longer optimum and we need a pre whitening filter, which is different in implementation.

Higher order statistics is insensitive to Gaussian noise and can be consider as a higher SNR domain for PN-sequence detection we especially use third order cumulants [as a base for our detection].This paper consists of three sections, the first one introduced a mathematical background about the different types of noise and the properties of HOS and their advantages rather than the convential MF. Section two analysis the case study MF+C3(0,0),MF+C3(t,t) and proposed HOS correlator to compute the triple correlation C3. The last section utilizes the conclusion of our works.

2- Mathematical Background:

There are several reasons of using higher order statics (HOS)[4] in signal processing application (detection). The first reason is based on the property that for Gaussian signals only, all cumulant spectra of the order greater than are identical zero. Thus when a non Gaussian signal is received along with additive Gaussian noise, a transform to HOS domain will eliminate the effect of noise.
Hence in these signal processing setting, there will be certain advantages to detection or/and estimating signal parameters from cumulant spectra of observed data[5]. As a result cumulant spectra become high SNR domain in which one may perform detection. The second reason for using HOS is based on the fact that polyspectra preserve (identify) both magnitude and non-minimum phase information, however the auto-correlation (2nd order spectra) suppresses phase information. An accurate phase reconstruction is the auto-correlation (or power spectrum) domain can only achieved if the system is minimum phase. The third reason, is based on observation that most of the world signals are non Gaussian and this have non zero HOS.

A- Definitions

If {x(k)}, k=0,1,…… is a real stationary random process and its moments up

$$m_n^{x}(\tau_1,\tau_2,\dots\dots,\ \tau_{n-1}) \cong E\left[x(k)x(k+\tau_1)\dots\dots\ \dots x(k+\tau_{n-1})\right] \tag{1}$$

To order n exist then Equation (1) represents the nth order moment function of x (k)
The nth order cumulant function of non Gaussian stationary random signal x (k) can be written as (for n>2 only);

$$c_n^{x}(\tau_1,\dots\dots,\ \tau_{n-1}) = m_n^{x}(\tau_1,\dots\dots,\ \tau_{n-1}) - m_n^{G}(\tau_1,\dots\dots,\ \tau_{n-1}) \tag{2}$$

Where mnG nth order moment function of an equivalent Gaussian signal that have the same mean value and auto correlation sequence as x(k). Thus if x(k) exist for order n=1,2,3 as
The following relationship between moment and cumulant of a sequence {x(k)} exist for order n=1,2,3 as :

1st order cumulants:

$$c_1^{x} = m_1^{x} = E\{x(k)\} \tag{3}$$

2nd order cumulants:

$$c_2(\tau_1) = m_2(\tau_1) - (m_1)^2 = m_2(-\tau_1) - (m_1)^2 = c_2(-\tau_1) \tag{4}$$

i.e. the 2nd order cumulant sequence is the covariance, while the 2nd order moment is the auto correlation.

3rd order cumulants:

$$c_3^{x}(\tau_1,\tau_2) = m_3^{x}(\tau_1,\tau_2) - m_1^{x}\left[m_2^{x}(\tau_1) + m_2^{x}(\tau_2) + m_2^{x}(\tau_1-\tau_2)\right] + 2\left(m_1^{x}\right)^2 \tag{5}$$

If the sequence {x (k)} is zero mean i.e. $m_1^x = 0$, then the second and the third order cumulant are identical to the second and third order moment.

$C_1 = m_1 = 0$,

$$c_2^x(0) = E\{x^2(k)\} = \gamma_2^x \equiv Variance \tag{6}$$

$$c_3^x(0,0) = E\{x^3(k)\} = \gamma_3^x \equiv Skewness \tag{7}$$

B- Detection Algorithm using HOSMF

The discussion focuses on Finite Impulse Response (FIR) signals and linear time invariant systems. Important relationships and properties are briefly reviewed because they are useful for the detection algorithm. The kth order correlation hk of deterministic signal $\{h(i)\}^N$ i=0 is defined as

$$h_k(I) \cong \sum_{i=0}^{N} h(i)h(i+I) \tag{8}$$

Where $I = i_1, i_2, \ldots\ldots i_{k-1}$.

Let y(i) be the output of an FIR h(i) , which is excited by a deterministic input s(i); i.e.

$$y(i) = \sum_{j=0}^{N} h(j)s(i-j) \tag{9-a}$$

Then the kth order output is equal to

$$y_k(I) = \sum_{J=-N}^{N} h_k(I)s(I-J) \tag{9-b}$$

Where y_k, h_k and s_k are defined as in (8).

A filter matched to a PN-sequence to a deterministic finite impulse response signal {s(i)}, i=1,…….N has impulse response h(i)=s(N-i). there for, if the matched filter MF h(i)=s(N-i) is excited by pn-sequence s(i) plus the noise n(i), the kth-order correlation of the output y(i) is given by

$$y_k(I) = \sum_{J=-N}^{N} x_k(J).s_k(J-I) \tag{9-c}$$

From equations (9-b), $h_k(j1,\ldots..jk-1) = s_k(-j1,\ldots..,-jk-1)$, then in case of noise free by mean x(i)=s(i) and the output of the MF is the auto correlation {s2(i)}, i=0, …. N. Hence, the kth order correlation of the MF output is given by

$$y_k(I) \cong \left(\frac{1}{2\pi}\right)^{k-1} \int_{-\pi}^{\pi} Y_k \, EXP\left[-jWI\right] dW \tag{9-d}$$

$$y_k(I) \cong \left(\frac{1}{2\pi}\right)^{k-1} \int_{-\pi}^{\pi} \left[S_k(W)\right]^2 EXP\left[-jWI\right] dW \tag{9-e}$$

$$y_k(I) \cong \left(\frac{1}{2\pi}\right)^{k-1} \int_{-\pi}^{\pi} S_2(W).S_2(-W) EXP\left[-jWI\right] dW \tag{9-f}$$

$$y_k(I) = \sum_{J=-N}^{N} s_k(J).s(J-I) \tag{9-g}$$

Applying the caushy-schwarz inequality to (9-e) it follows that:

$$\left|y_k(I)\right| \le \left(\frac{1}{2\pi}\right) \int_{-\pi}^{\pi} \left|y_k(W)\right| dW = y_k(0) \tag{9-h}$$

Hold because Y(w) equals the power spectral density s2(w)>0, and hence $[Y_k]=y_k>0$. Consequence of (9-h) is the following detection property, which for k=3 was originally shown in [5]. Kth order correlation of auto correlation sequences peaks at the origin by setting $i_1,\ldots..,i_{k-1}$ in (9-e)-(9-g) we obtain four equivalent ways for computing the zeroth lag of kth-correlation of the MF output. Specially, (9-g) yields the energy E_{ks} of the signal's kth-order correlation.

$$c_{ky}(0) \cong \sum_{J=-N}^{N} s_k^{\,2}(J) \cong E_{ks} \ \phi \ 0 \tag{10-a}$$

An alternative way to compute $C_{ky}(0)$, is to use the definition (8) which leads

$$c_{ky}(0) = \sum_{i=0}^{2N} y^k(i) \tag{10-b}$$

The zero lag of the kth-order correlation computed as in (10-b). To decide between H_0 and H_1, we use the detection statistic $[C_{3y}(0,0)]/T+N$, which equals the sum of the cubes of the MF output. Let $y_n(y_s)$ denote the MF output due to noise (signal) component of input. .
Under H_0, the MF output is

$$y(i) = y_n(i) = \sum_{J=0}^{T-1} n(j)h(j-i), \qquad i= 0, 1\ldots N+T-1 \qquad (10\text{-c})$$

And the detection statistic become

$$C_{3y}(0,0) = \sum_{i=0}^{T+N-1} y_n^{3}(i) = \sum_{J=-N}^{N} c_{n3}(J)c_{s3}(J) \qquad (10\text{-d})$$

Where :
C_{3y} the third order cumulant of the output,
C_{n3} the third order cumulant of the input noise,
C_{s3} the third order cumulant of the input PN-sequence, $J=[j_1,j_2]$.
Which leads as H_0:$C_{3y}(0,0)$ goes to zero as T tends to infinity under Gaussian noise. Under H_1, $y(i)=y_s(i)+y_n(i)$ where $y_n(i)$ is given by (10-c) and $y_s(i)= s_2(i)$

$$H1: \sum_{i=0}^{T+N-1} y^3(i) = \sum_{i=0}^{T+N-1} y_s^{\,3}(i) + \sum_{i=0}^{T+N-1} y_n^{\,3}(i) + 3\sum_{i=0}^{T+N-1} y_s^{\,2}(i)y_n(i) + 3\sum_{i=0}^{T+N-1} y_s(i)y_n^{\,2}(i)$$
$$= \frac{1}{T+N}\sum_{i=0}^{T+N-1} y_s^{\,3}(i) + \frac{3}{N+T}E\left\{y_n^{\,2}(i)\right\}s_2(0) \qquad (10\text{-e})$$

If the noise has high variance the peak of signal detection will enhance signal at the mean of s(i) is not equal zero. If the signal has zero mean, the 2nd term of (10-e) drops out, and the peaks of the detection statistic is proportional to the triple correlation energy.

The detection algorithm has the following steps:

1-design the matched filter h(i)=s(N-i), matched to the known signal $\{s(i)\}^{N}j=0$.

2-compute the matched filter output y (i).

3-compute the C3 (0, 0) or C3 (t,t).

4-compare the different cases at different locations (before the conventional MF, after the Mf and the output of the system proposed.

3- Cases Studies:

A- First case study (signal+noise+MF+C3(0,0))

The input of assumed that generated from the Pn-sequence (m-sequence)[6] which have five shift register generated the characteristic polynomial X=1+ x^2 + x^5 as shown in figure (1), and its

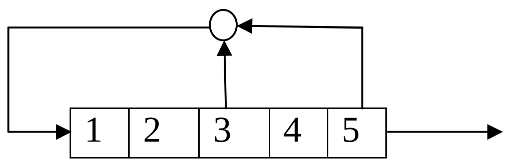

***Figure (1)** : Generation on PN-sequence*

Maximal length equal to 31. We added to the m-sequence a noise; may be Gaussian, uniform and exponential. This case is figured in figure (2) which consists of three sections. The first one add the signal s(i) (m-sequence) and the noise n(i). secondly conventional matched filter to the input h_{MF}=s(N-i). The triple correlation is evaluated at the third section at the location (0,0) and all the resulted are tabulated in the table 1-1 , in which we estimate the signal–to–ratio at the three stages at different types of noise.

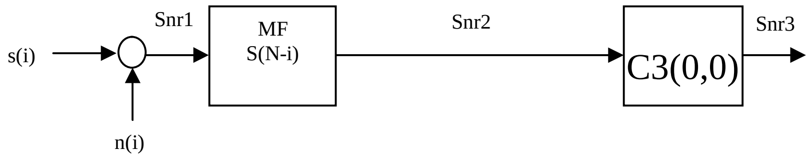

Figure (2): *First case study (signal+noise+MF+C3 (0, 0)*

Table (1-1):*(First case study (signal+noise+MF+C3 (0, 0))*

Noise	s/n1dB	s/n2 dB	IMF1	s/n3 dB	IMF2	IMFT
Gaussian	8.6866	17.9337	9.2471	32.5480	15.6143	24.8514
Exponential	11.2257	18.4790	7.2433	35.0165	16.5195	23.7628
Uniform	13.1319	28.5946	15.4625	65.5491	35.9545	51.5270

The good results are obtained in case of uniform noise which gives good improvement in the s/n ratio better than the Gaussian noise and exponential noise by amount 6 dB and 8 dB respectively after the output of the matched filter and by amount 33 dB and 31 dB respectively after the output of the C3(0,0). This due to the cancellation influence of the uniform and Gaussian distribution due to the triple correlation which in theory tends to zero all these results are figured in figure (5)

B-Second case study (signal+noise+MF+C3(t,t))

Another proposed system is introduced in figure (3). The system evaluated the triple correlation function not at zero lag but at (t,t) lags along the diagonal of C3(t1,t2). Generally the triple correlation function of the output of the MF can be evaluated as :

$$C_{YYY}(\tau_1,\tau_2) = E\{y(t)y(t+\tau_1)y(t+\tau_2)\} \tag{11}$$

$$C(p,q) = \frac{1}{L}\sum_{i=1}^{L} y(i)y_p(i)y_q(i) \tag{12}$$

In all position and take the best position gives the maximum improvement in the s/n better the other all these results are tabulated in 2-2

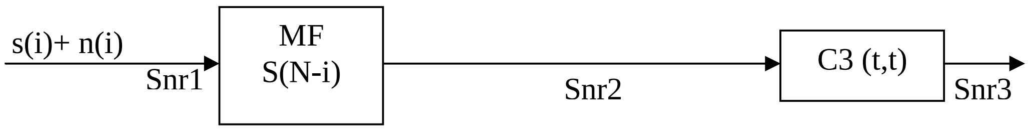

Figure (3) *: Second case study triple correction based detector HOSMF receiver*

We estimate the signal to the noise ratio at the output of the three stages at different types of the noise and tabulated at the table 2-2

Table 2-2:*(Second case study triple correction based detector HOSMF receiver)*

Noise	Snr1 db	Snr2 db	IMF1	Snr3 db	IMF2	IMFT
Gaussian	8.5163	18.0442	9.5279	33.5761	15.5319	25.0598
Exponential	10.4778	18.1486	7.6408	33.7433	15.5947	23.2355
Uniform	13.9517	28.6162	14.6645	64.9856	36.3694	51.0339

The results from table 2-2 shows that the improvement factor greater than the first case by amount 1 db with respect to Gaussian , uniform and may be equal to the exponential noise respectively as shown in figure(6).

C-Third case study (signal+ noise + (signal shifted and correlator) + MF)

In this case we proposed system in introduced in figure (4). The system evaluated the triple correlation function C3 (t,t) as a correlator followed by a matched filter receiver. We estimate the signal to noise ratio at the output of the three stages at different types of noise the result are tabulated in table 3-3 and figured in figure (7).

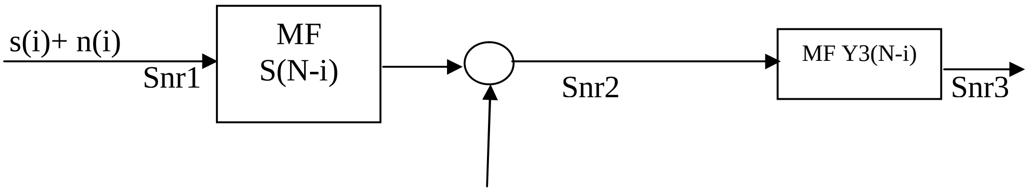

***Figure (4)**: Third case study (signal shifted and correlator + MF)*

***Table 3-3:** (Third case study (signal shifted and correlator + MF))*

Noise	Snr1 db	Snr2 db	IMF1	Snr3 db	IMF2	IMFT
Gaussian	8.4430	17.9663	9.5233	36.9919	19.0256	28.5489
Exponential	10.5329	17.8743	7.3414	36.8356	18.9613	26.3027
Uniform	13.9987	28.6945	14.6958	48.1017	19.4072	34.003

The result from table3-3shows that the improvement factor greater than the first case by amount 4db and 3 db with respect to Gaussian and exponential respectively the but the improvement factor less than the second case and the first case by amount 15.5 w.r.t the uniform noise but the cases more realizable due to the simplicity .

<u>4-Conclusions:</u>

This paper presented a new method for detection using Higher Order Statistics which insensitive to Gaussian noise and can be consider as a higher SNR domain for PN-sequence detection we especially use third order cumulants rather than the conventional match filter .As a result cumulant spectra become high SNR domain in which one may perform detection with different types of noise.

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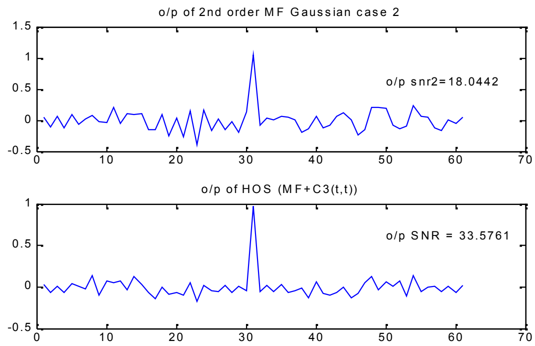
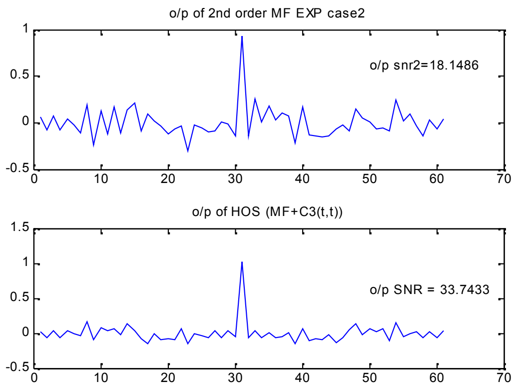
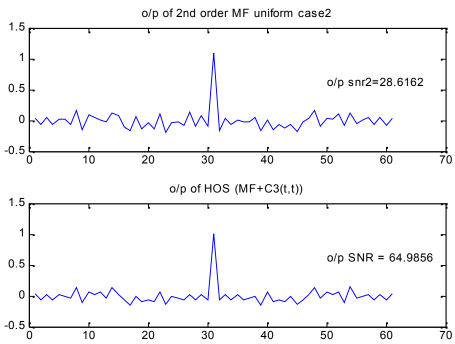


(a)

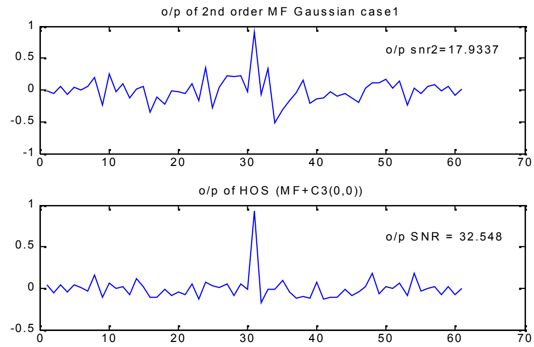


(b)

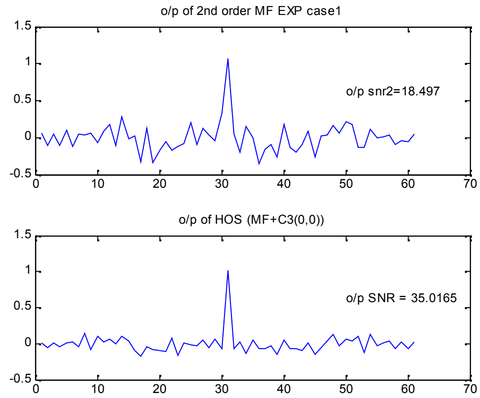


(c)

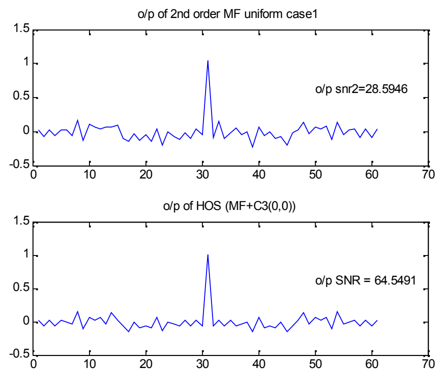
***Figure(6):** performance second case study (signal+noise+MF+C3(t,t)) (a)Gaussian (b)exponential (c)uniform noise*



(a)

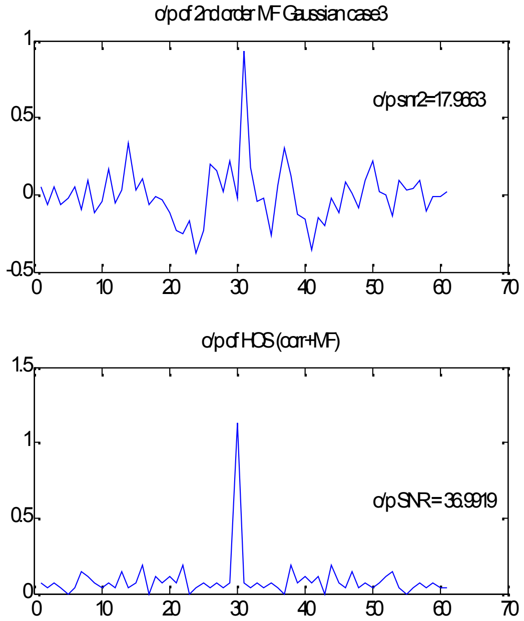


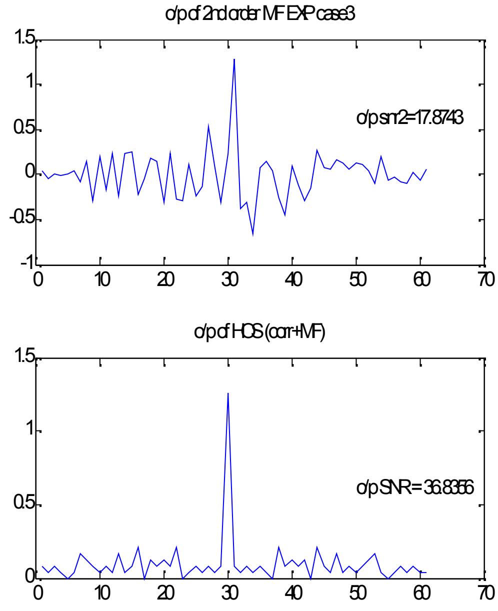
(b)



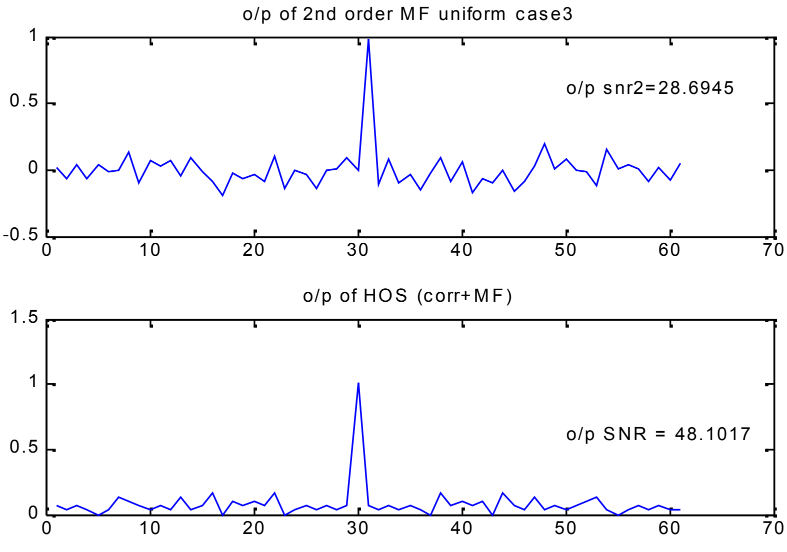
(c)

***Figure(5):** performance First case study (signal+noise+MF+C3(0,0)) (a)Gaussian (b) exponential (c) uniform noise*





(a) ***(b)***



(c)

Figure(7):*performance third case study*
(signal noise (signal shifted and correlator) +MF)
(a)Gaussian (b) exponential (c) uniform noise