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Review for Plastic Hinge Types Used for Modeling Reinforced Concrete Elements

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A B S T R A C T

Investigating the nonlinear behavior of reinforced concrete (RC) elements was the objective of many experimental and numerical studies. This work briefly presents the previous work done in support of modeling reinforced concrete (RC) elements to evaluate their structural behavior. Modeling RC elements requires reliable nonlinear models for their inelastic positions under high loading. These positions are conformed as concentrated plasticity. For simulating RC frames, the concentrated plasticity was used in many studies to provide simple modeling with low computation cost. The concentrated plasticity is modeled using lumped plastic hinges (PH). Four PH types (flexural, shear, torsion, and beam–column joint) are defined based on the loading type subjected to the frame element. Consequently, it reviews the previous studies that modeling procedures for the nonlinear behavior of four PH types. The flexural PH, Shear PH, torsional PH, and beam-column joints could be defined, respectively, using empirical length, the approach of Watanabe & Lee (1998), rigid–element, and bilinear torsion–rotation behavior.

1. Introduction

Reinforced Concrete (RC) Elements are commonly used in construction due to their strength and durability. In the field of structural engineering, accurately modeling the behavior of RC elements is crucial for evaluating their structural performance. This involves understanding the inelastic response of RC elements under high loading conditions. The nonlinearity in RC elements could be simulated using macro-model approach (concentrated plasticity or distributed plasticity) or micro-model approach (full finite element modeling) depending on the desired accuracy and computation cost.

Concentrated plasticity is commonly modeled using lumped plastic hinges (PH), which are defined based on the type of loading experienced by the frame element. There are four types of plastic hinges: flexural, shear, torsion, and beam-column joint, each corresponding to a specific loading condition.

To develop reliable nonlinear models for RC elements, it is essential to have a thorough understanding of the behavior of these plastic hinges. This requires a comprehensive review of previous studies that have focused on the modeling procedures for the nonlinear behavior of these four types of plastic hinges.

The purpose of this work is to present a brief overview of the previous research conducted in support of modeling RC elements and evaluating their structural behavior. Specifically, this review focuses on the development of empirical equations for estimating the length of the flexural plastic hinge (l_p) in RC frames under lateral load.

The plastic hinges (PH)used to model RC element located usually at the ends of its each clear length. Four PH types (flexural, shear, torsion, and beam–column joint) are defined based on the loading type (moment, shear or torsion) subjected to frame element.

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2. Flexural PH

The major role in determining the response of RC buildings is played by the nonlinear behavior of flexural PH. (**Sunil & Kamatchi 2022, Inel & Ozmen 2006**). Where, the behavior of beam and column members could be represented using the flexural PH. There are different types from flexural plastic hinges (fiber PHs and moment–rotation PHs). The modeling of flexural PH depends on the ultimate curvature of element section and plastic hinge length (l_p).

The length of the flexural plastic hinge (PH) is an essential factor to consider for reinforced concrete (RC) elements under lateral load. The equivalent flexural plastic hinge length could be estimated based on integration of the plastic curvature distribution for typical members. To make straightforward the calculations, an equivalent flexural plastic hinge length, l_P, could be defined over which the plastic curvature, $\emptyset_p$, is suggested to be equal to the difference between the maximum curvature ($\emptyset_u$) and the yield curvature ($\emptyset_y$). The PH length l_p is calculated so as to the plastic displacement, Δ_p, at the free end of the cantilever RC element obtained from an experiment or from a displacement design method is almost equal to the value obtained from the existent curvature spreading (**Firat 2010**). The lumped plastic rotation, θ_P, for along way the flexural PH length is subsequently calculated as **Eq. (1)**

$$\theta_P = \emptyset_P l_P = (\emptyset_u - \emptyset_y) l_P \tag{1}$$

The plastic rotation derived using **Eq. (1)** can be used to determine the displacement capacity of a section that experiences inelastic deformations. If the plastic rotation (flexural PH) is assumed to be concentrated at the beginning of plasticity, the plastic displacement (Δ_P) at the top of the cantilever column then becomes **Eq. (2)**:

$$\Delta_P = \theta_P H = (\emptyset_u - \emptyset_y) l_P H \tag{2}$$

where H is the column height. From the plastic displacement, Δ_p, the maximum nonlinear drift is obtained. Thus, the stander prediction of a flexural PH length is required to study the theoretical drift of multistory building. The PH length suggests the default length of concentric damage for the RC element (**Firat 2010**). It was shown that the equivalent l_p and the plastic region wherever reinforcing detailing is needed to be described separately because it has major effect on plastic rotation capacity. Several equations for estimating l_p were developed by various researchers for various types of RC elements.

2.1. Chan (1955)

Chan (1955) proposed **Eq. (3)** as a means to calculate the length of flexural plastic hinges (PH) after conducting tests on three different types of specimens. These included nine members with transverse steel reinforcements, seven members with spiral transverse steel reinforcements, and seven members with welded transverse steel reinforcements.

$$l_P = L_s(1 - M_y/M_u) \tag{3}$$

where, L_s is the shear length (moment to shear ratio), and M_y, M_u are the yield and ultimate moment, respectively.

2.2. Baker (1956)

Baker (1956) proposed **Eq. (4)** as a method to calculate the length of flexural plastic hinges (PH) after conducting tests on three different types of specimens. These specimens included 32 members with cold work steel, 30 members with mild steel, and 32 members subjected to both bending moment and axial load, consisting of mild steel and cold work steel.

$$l_P = g_1 g_2 g_3 L_s^{0.25} d^{0.75} \tag{4}$$

where, d is effective depth and the factors were defined using concrete compressive strength (f_c), the initial axial load (F_1), and the capacity axial load (F_o) as following:

g_1=0.9 for cold work steel or 0.7 for mild steel

g_2=1+0.5(F_1/F_o)

g_3 =0.9 –0.0128(f_c–11.7) if 11.7<f_c<32.2 MPa

2.3. Cohn & Petcu (1963)

Cohn & Petcu *(1963)* conducted tests on ten continuous RC beams with two spans, which were divided into two groups. Each group consisted of five beams, and they were loaded with a concentrated load at a specific distance from the central support until failure. One group had a distance of 40 cm, while the other group had a distance of 60 cm. The results obtained for these ten beams, which varied from 0.3D to 0.9D (where D represents the effective depth of the beam), were recorded almost the same lengths obtained from the equation proposed by **Chan (1955).**

2.4. Baker & Amarakone (1964)

Baker & Amarakone (1964) investigated the inelastic deformations of RC frames. It was indicated that the PH length between 0.5*h* to *h* is a safe estimation in columns, where *h* is the element depth.

2.5. Corley (1966)

Corley (1966) suggested **Eq. (5)** to calculate the flexural PH length. Corley conducted tests on 40 simply supported beams, which were loaded with a concentrated load at midspan. The results obtained for these 40 beams were analyzed using Eq. (5).

$$l_P = 0.032\ L_s/\sqrt{D} + 0.5\ D \tag{5}$$

2.6. Mattock (1967)

Mattock (1967) suggested **Eq. (6)** to determine flexural PH length based on his tested for three types of specimens: 32 members with cold work steel, 30 members with mild steel, and 32 members (with mild steel and with cold work steel) subjected to bending moment and axial load.

$$l_P = 0.032\ L_s/\sqrt{D} + 0.5\ D \tag{6}$$

2.7. Zahn et al. (1986)

Zahn et al. (1986) made tests for fourteen RC columns (with different cross-sections) subjected to combined bending moment and axial load. Based on their tests, **Eq. (7)** was suggested to determine flexural PH length. The 14 RC columns comprised four types of section shapes: four square sections with diagonal loading, two square sections with face loading, two octangular sections, and six circular sections with hollow.

$$l_P = 0.08L_s + 6\emptyset_b\left(0.5 + 1.67\frac{F_1}{f_c A_g}\right) \quad for \frac{F_1}{f_c A_g} < 0.3$$

$$l_P = 0.08L_s + 6\emptyset_b \quad for \frac{F_1}{f_c A_g} \geq 0.3 \qquad (7)$$

where, $\emptyset_b$, A_g and F_1 are diameter of longitudinal steel reinforcement, section gross area, and initial axial load, respectively.

Table 1 summarizes the studies previously done on flexural PH length presented in this section, including the type of members studied and the parameters that the authors considered to develop their models.

Table 1
Summary of mentioned previous research done on flexural PH length.

Researchers reference	Members studied	Parameters considered
Chan (1955)	columns	span and strain hardening
Baker (1956)	beams and columns	depth
Cohn & Petcu (1963)	beams	depth and strain hardening
Baker & Amarakone (1964)	beams and columns	depth, span, reinforcement properties and axial load ratio
Corley (1966)	beams	depth and span
Mattock (1967)	beams	depth and span
Zahn et al. (1986)	columns	span, reinforcement properties and axial load ratio

3. Shear PH

The determination of the shear resistance is more complicated compared with flexural PH. The behavior of RC element (column or beam) at shear failure is clearly different from its behavior in flexural failure. In shear failure, the RC element fails suddenly without sufficient advanced warning and the diagonal cracks are developed wider considerably than the flexural cracks. Therefore, adequate amount of shear reinforcement is generally provided to avoid an abrupt shear failure.

For modeling shear failure, it required to define shear PH hinge to RC elements with shear–deformation curve. Shear PH hinge is modeled to account for behavior of inelastic shear. Several capacity models have been developed for RC elements. One such model related the shear demand to the drift at shear failure were introduced by **Elwood & Moehle (2005**) based on the transverse reinforcement and axial load ratios. **Elwood & Moehle (2005**) conducted 50 specimens tests on RC columns with failure in flexural yielding before shear failure. The model introduced by **Elwood & Moehle (2005**) defines the drift at shear failure as the drift at which the shear capacity has degraded to 80% of the maximum measured shear. According to the model**,** the shear failure is determined by the intersection of an idealized bilinear force–deformation curve for the column where, the limit surface was defined by the drift capacity model. **Watanabe & Lee (1998**) introduced an incremental analytical approach to predict the shear force-deformation characteristics. Based on the truss mechanism, the strain of stirrup is gradually increased for each step with a very small increment. Besides, the demand shear is calculated at each increment in the analysis. This approach of **Watanabe & Lee (1998)** was explained step by step in **Sayed (2023)**.

4. Torsional PH

The torsional shear stresses are always present on the cross-section of a frame member subjected to a torsional moment. The determination of torsional stresses and their combination with stresses due to bending and axial load is very difficult to determine for frame element sections. The torsional effects are mostly obvious in the elastic range and early stages of plastic behaviour. The torsional effects decrease with an increase in the plastic deformations. A simple approach was developed based on space truss analogy to determine the torsional PH characteristics of the RC element section (**Park & Paulay 1975**). They suggested simple equation to determine the cracking torsion, the ultimate torsional resistance, and the cracked stiffness.

Bilinear torsion–rotation behavior was defined as adopted by **Sharma et al. (2013)**. The cracking torsion, ultimate torsion and stiffness after cracking are estimated as functions in the section geometry, concrete strength and transverse reinforcement properties to determine the torsion–rotation relationship (**Park & Paulay 1975**). The torsional PH characteristics of the section can be calculated based on space truss analogy as the following steps.

a) the cracking torsion, T_{cr} is calculated as

$$T_{cr} = 0.33\sqrt{f_c'(A_g^2/P_c)}$$

Where, f_c' is the compressive strength of concrete; A_g is gross area of concrete section (mm^2); P_c is perimeter of concrete section (mm)

b) the ultimate torsional resistance, T_u, is calculated as

$$T_u = 2A_0 A_{sv} f_{sv}/S_v$$

Where, A_0 0 represents the gross area within the shear flow path, which is assumed to be 85% of the zone enclosed by centerline of the outermost closed transverse reinforcement*;* A_{sv} is the area of one leg of transverse reinforcement*;* f_{sv} is yield/ultimate stress of transverse reinforcement(f_{ys}/f_{us}); S_v is the distance between center to center of transverse reinforcement.

c) the cracked stiffness of the section, $k_{t,cr}$ is determined as

$$k_{t,cr} = A_{Sv} E_S (B_0 D_0)^2 \sqrt{m_t}/((B_0 + D_0) S_v L)$$

Where, B_0 is the shorter dimension of transverse reinforcement; D_0 is longer dimension of transverse reinforcement; E_S is elasticity modulus of transverse reinforcing steel; m_t is ratio of yield stress of transverse reinforcement to that of longitudinal reinforcement $=f_{yl}/f_{ys}$, and L is clear length of the element

Though, these three equations are simplified and able to generally sufficient to model the torsional PH of RC element.

5. Beam–column joint PH

In RC moment frames, beam-column connections are designed for capacity and are not expected to be the main failure mode under seismic loading. However, the finite sizes of beam-column joints could impact the frame response and may be considered using rigid end offsets. Furthermore, bar slip at the beam-column joint interface plays a significant role in accurately capturing the stiffness behavior. The beam–column joint used in RC frames could be categorized according to detailing aspects (seismically detailed structures or non-seismically detailed joints; **Sharma 2013**). **Biddah & Ghobarah (1999)** used separate rotational springs to model the shear and bar slip deformations in the beam-column joint. The shear stress-strain relationship was simulated using a tri-linear idealization based on a truss model, while the bond-slip deformation was simulated using a bilinear model. This joint element was utilized in performing dynamic analyses of three- story and nine-story RC buildings which were designed under gravity load. The study compared the dynamic response of three and nine-story frames, which were modeled with joint elements, to the response of similar frames with rigid joints under the same motion records. These comparisons exposed that method of accounting for bar-slip deformations and joint shear in results with substantially larger drifts, especially for the higher frame. **Sharma (2013)** developed a practical model for non-seismically deigned RC frame structures to simulate the plastic behavior of beam-column joints. The joint element models follow lumped plasticity models with finite element simulations. **Sharma et al. (2011)** and **Sharma et al. (2013)** revealed the important of introducing shear PH (**Watanabe & Lee 1998**), torsional PH (**Park & Paulay 1975**), and beam–column joint PH (**Sharma et al. 2011, Pan et al. 2017,** and **Yu 2006**) for considering different types of deformation in special frames.

Sharma et al. (2011) tested a 3D RC frame structure with three-story three bays under monotonically increasing lateral pushover load till failure. It was found that the major failure modes were observed as flexural failure of beams and columns, torsional failure for transverse beams only, and shear failure for joints (beam–column connection). In additionally, it was modeled this structure with lumped plasticity using SAP2000 software (**CSI 2020**) considering all these failure modes. **Sharma et al. (2013)** made an experimental test for RC structure (four-story three bays) with a full-scale under monotonic lateral loading. Then, this structure were modeled and analyzed under pushover loading using SAP2000 software. The beam-column connection of the structure was constructed as non-seismic detailing. The foundation was built with rock anchors to constrain the base supports during the test. It was found that the major failure modes were observed as flexural failure of beams and columns, torsional failure for transverse beams only, and shear failure for joints (beam–column connection). Furthermore, it was modeled this non-seismically detailed RC structure with lumped plasticity.

6. Conclusion

The review of literature shows that significant experimental and analytical research related to RC elements in the last 50 years was done. Despite research efforts and nonlinear behavior of reinforced concrete elements in an earthquake still remains a major controversy among structural engineers and researchers today.

Numerical RC elements models were developed to gain an in-depth understanding of the behavior of concrete frame buildings subjected to extreme loading conditions. The nonlinearity in RC frames could be simulated using the concentrated plasticity method with acceptable accuracy and low computation cost.

The concentrated plasticity is modeled using lumped plastic hinges (PH) located usually at the ends of each clear length of frame element. Four PH types (flexural, shear, torsion, and beam–column joint) are defined based on the loading type subjected to frame element. The flexural PH often is set at ends of each frame elements with empirical length (Equations 3-7). sheer PH could be defined using straight forward approach of **Watanabe & Lee (1998).** The seismic detailed beam-column connection could be modeled by rigid–element. For torsion PH, bilinear torsion–rotation behavior was defined as adopted by **Sharma et al. (2013)**.

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