PORT SAID ENGINEERING RESEARCH JOURNAL
Port Said University - Faculty of Engineering
Volume 21 No. 1 March 2017 pp: 31:40

ECONOMIC DESIGN SOLUTION OF TRAPEZOIDAL OPEN CHANNEL SECTION INVOLVIONG PUMPING USING OPTIMUM DERIVATIVE OF WATER BENEFIT FUNCTION

Mohamed M.A. Somaida[1]

ABSRACT

In the present paper, an economic water benefit design mathematical solution has been derived for trapezoidal open channel section involving pumping which is based on hydraulic and economic considerations. The solution utilizes the derivative optimum of water benefit function and maximizes the benefit gained from the delivered water to reach the maximum water benefit MWB design solution. The solution is applied on a channel example and the results obtained are compared with those obtained by an earlier solution based on the minimum construction cost MC design of the channel only. The channel costs in both solutions are found to be of nearer magnitudes with little deviation. It is not necessary that the channel geometry in both cases must be identical, because the MWB solution depends on several factors such as; water benefit variables, pumping cost parameters and channel length, while the minimum cost MC solution depends on the channel cost parameters only. The MWB design solution is advantageous in that it achieves money gains even the delivered water is of cheap price, while this advantage is not satisfied in the earlier solution. However, the present study introduces an analytical solution for developing comprehensive water benefit and costing. The results of the given channel example involving pumping, demonstrate the simplicity and practicability of the MWB design solution as well as water benefit verification

KEYWODS: channel geometry, pumping costs, channel construction costs, optimization, maximum water benefit MWB function, optimal economic depth of flow, money gain

1. Introduction

The derivative method has been a classical example of application to the theory of pipe line size, / 1 /. However, it can be extended for optimal selection of irrigation channels involving pumping. In this view, the choice of main diameter as well as channel size can be easily made based on reasonable hydraulic and economic considerations.

The water supply scheme under study includes a trapezoidal open channel Fig. (1), fed from a canal by pumping due to the nature of the terrain, Fig. (2). The study depends basically on mathematical optimization of the water benefit function of the given scheme which is the difference between the water return and the total cost. This is achieved by using the derivative method and conducted basically on three concepts. Firstly, optimizing the diameter of the main feeding the channel. Secondly, optimizing the water benefit function to obtain the optimal economical channel section which achieves the maximum water benefit. Finally, evaluating the derived maximum benefit design solution. The present study will be aided by an illustrative example to demonstrate the reliability and practicability of the maximum benefit design solution as well as money gain from the used water.

[1] *Civil Engineering Department, Faculty of Engineering, Port Said University, Port Said, Egypt. E-mail: m.somaida@yahoo.com*

2. Cost Functions

The pumping system of the channel consists of three rising pipe mains, each one is equipped with a pump which lifts water from the main canal to the channel as shown in Fig (2)

2.1 Pumping System

For a pumping main, the most important cost elements are; pipe cost, pumping cost and pump cost. The pipe cost function is given by, /1/ :

$$C_{pipe} = a \; l \; D^x \qquad (1)$$

where D = diameter of pipe, l = length of pipe, a = pipe cost coefficient and x = pipe cost exponent

The pumping cost function is given by, /2,3,4/ :

$$C_{pumping} = \frac{w\,Q_p\,H\,p\,N\,Y}{1000\xi} = A_1 \; Q_p \; H \qquad (2)$$

where w = specific weight of water, Q_p = pumping rate per pump, H = operating head of pump, p = power cost/kwh, N = average hours pumping/annum, Y = life period of scheme, ξ = pump efficiency and A_1 = unit cost of pumping

. The pump cost function is given by, /2,3,4/ :

$$C_{pump} = \frac{w\;Q_p\,H\,c_p}{1000} = A_2 \; Q_p \; H \qquad (3)$$

where c_p = cost of pump/kw and A_2 = unit pump cost

However, the equation for total cost of pumping over the life of pumping mains is obtained by summing equations (1,2,3) :

$CT_{pumping} = (a \ l \ \ D^x \ + (A_1 \ + \ A_2)Q_pH) \ n$
(4)

where n = number of pipe mains.

2.2 Open Channel Cost Functions

The open channel cost functions include the construction cost for excavation and lining. The excavation cost function for trapezoidal open channel, Fig (1), can be written as follows,/5/ :

$$Ce = (c_e \ A_t \ + c_{ie} \ A_t \, h_c) \ L \qquad (5)$$

Where Ce = excavation cost, c_e = cost/unit volume of excavation at ground level, A_t = total excavation area including freeboard, c_{ie} = increase of unit excavation cost/unit depth, L = channel length and h_c = depth of centroid of the trapezoidal area from the ground level, $= \frac{h\,(3\,b\,+2zh))}{6(\,b\,+zh)}$, where h= excavated depth from the ground level, b = bottom width of the channel and z = channel side slope, Fig(1).

. The open channel lining cost C_L is given by / 5 / :

$$C_L = \ c_L \ P_t \ \ L \qquad (6)$$

where c_L =cost of lining/unit surface area of channel and P_t = total perimeter of the cross section including freeboard. However, the total construction cost of the channel can be obtained by summing the excavation and lining costs or $CT_{ch} = C_e + C_L$, then :

$$CT_{ch} \ = (c_e \ A_t + c_{ie} \ A_t \ h_c \ + c_L P_t \ ) \ L \qquad (7)$$

The equalities of A_t, h_c and P_t are interpreted from Fig (1) and substituted in equation (7) :

$$CT_{ch} = (c_e \ \ (bh + z \ h^2) + \ \frac{c_{ie}}{6} \ (3 \ b \ h^2 + 2 \ z \ h^3 \ ) + c_L \ (\ b \ +2 \ h\sqrt{1+z^2} \ \ ) \ ) \ L \qquad (8)$$

In equation (8), put h = (y + F), where y = water depth of flow and F depth of board from ground level, Fig (1). Also, put b = k y, where k is the bottom – width-to-depth ratio, then equation (8) will take the following form :

$$CTch \ = (\ c_e \ (ky \ (y+F) \ \ + z \ (y+ F \ )^2 \ ) + \frac{c_{ie}}{6} \ \ (3ky \ (y \ + F)^2 \ \ + 2z(\ y + F)^3 \ ) + c_L(ky + 2(y+F)\sqrt{1+z^2} \ \ ) \ ) \ L \qquad (9)$$

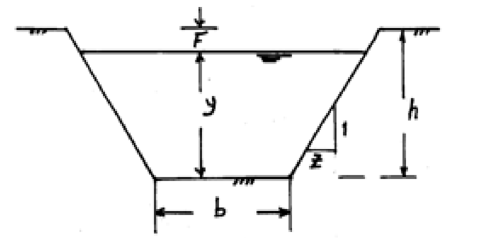

Fig (1). Sketch diagram for a trapezoidal open channel with freeboard

3. Hydraulic Considerations

3.1 Operating Head of Pump

Refer to the water supply scheme, Fig (2), by applying the energy equation, the operating head developed by each pump is given by :

$$H \ = (\ E + y +F +s \ ) + (\ h_f + h_l \ ) \qquad (10)$$

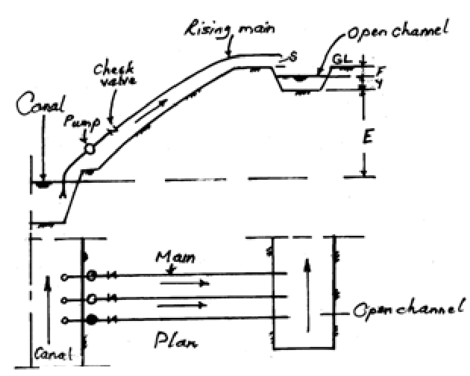

Fig (2). Sketch for the water-supply scheme of the channel illustrative example

where E = difference between water level in the main canal and the bottom of channel, s = difference between ground level and exit of pipe main, h_f = head lost by friction in main and h_l = minor head losses in main.

3.2 Main Friction Losses

In equation (10), h_f for any pipe main is given by:

$$h_f = K_1 \ \ Q_p^2 \qquad (11)$$

Where K_1 = resistance coefficient of friction, which in SI units would $\frac{32\,f\,l}{\pi^2 g D^5}$, where f = coefficient of friction and g = acceleration of gravity

3.3 Minor losses

The minor head loss is given by:

$$h_l = \ K_2 \ \ Q_p^2 \qquad (12)$$

where K_2= coefficient of minor losses, which in SI units would be C $\frac{8}{\pi^2 g D^4}$ where *C* is a coefficient specified for each fitting, for example in the present study, it will be 0.5 for entry loss and 0.4 for pipe bend (two bends) and 2.5 for check valve

3.4 Total Cost CT of Water Supply Scheme

In equation (10), the total head loss per main H_l, will be given by :

$$H_l = h_f + h_l = (K_1 + K_2)\ Q_p^2 \qquad (13)$$

However, the equation for operating head of pump will be :

$$H = (E + y + F + s) + (K_1 + K_2)\ Q_p^2 \qquad (14)$$

Also, in equation (4), The diameter D in the pipe cost function can be written in terms of Q_p using the Darcy's Weisbach equation, /2,3/ :

$$D = K\ \frac{Q_p^{0.4}}{S^{0.2}} \qquad (15)$$

where S = head lost by friction per unit length of pipe (HG) and K is a coefficient which in SI units would ($\frac{32\ f}{\pi^2 g}$). Then, the pipe cost function will have the new form :

$$C_{pipe} = a\ l\ (K\ \frac{Q_p^{0.4}}{S^{0.2}})^x \qquad (16)$$

Substitute C_{pipe} of equation (16) and operating head of pump H of equation (14) in equation (4) and add the resulting equation to equation (9) to get the total cost CT of the water supply scheme as given below :

$$CT = ((A_1 + A_2)\ Q_p\ (E + Y + F + s + (K_1 + K_2)\ Q_P^2) + (\frac{A\ l\ K^x}{S^{0.2x}})\ Q_p^{0.4x})\ n + (c_e((ky)(y+F) + z\ (y + F\ )^2 + \frac{c_{ie}}{6} (3\ k\ y(y + F)^2 + 2\ z\ (y + F)^3) + c_L\ (ky + 2\ (y + F)\sqrt{1 + z^2}\ ))\ L \qquad (17)$$

4. Economic Considerations

4.1 Formula for optimum main diameter

The optimum diameter is found by differential calculations. A formula has been readily derived / 1, 2,3 /, for the optimum diameter of main as :

$$D^{x+5} = \frac{5\ (A_1 + A_2\ )\ K\ Q_p^3}{a\ x\ l} \qquad (18)$$

The different parameters of equation (18) are previously defined and D is the optimum diameter of main

4.2 Best bottom-width-to-water depth ratio

Any of the open channel formulas show that, for a given bed slope and roughness, the velocity of flow increases with the hydraulic mean depth (R). For a given discharge, the cross-sectional area of flow will be minimum, when the dimensions are such that to make R maximum, i.e., perimeter P is minimum . This section is the best hydraulic section for the given discharge and can be reached in the following manner:

The hydraulic mean depth R = $\frac{(b + z\ y)y}{b + 2\ y\ \sqrt{1 + z^2}}$, or:

$$R\ (b + 2\ y\sqrt{1 + z^2}\ ) = (b + z\ y\ )\ y \qquad (19)$$

At a given z, and holding b constant, equation (19) can be differentiated with respect to y to relate the change of the hydraulic mean depth dR to the change of depth of flow dy, then :

$$(b + 2\ y\sqrt{1 + z^2}\ )\ dR = ((b + 2\ z\ y\ ) - (2R\sqrt{1 + z^2}\ )\ )\ dy \qquad (20)$$

According to /6/, an optimal trapezoidal cross section, will be at R = $\frac{y}{2}$ (dR = $\frac{dy}{2}$), hence, substituting for R and dR in equation (20), then :

$$\frac{b}{y} = 4\ (\sqrt{1 + z^2} - z\ ) = k \qquad (21)$$

where k is the bottom-width-to-water depth ratio. However, for trapezoidal section at specified z, equation (21), can be used to find the best ratio k. For example, if z = 1.5, equation (21) gives $\frac{b}{y}$ = 1.2 = k or b = 1.2 y, from which A = 2.7 y^2 and R = 0.5317 y (accepted). These parameters are helpful for computing the rate of flow in the trapezoidal channel using the Manning's formula

4.3 Water Benefit WB

The water benefit WB is defined as the difference between the water return WR and the total cost CT of the water supply scheme or

$$WB = WR - CT \qquad (22)$$

On the other hand, the water return WB is calculated by the following equation, /7,8/ :

$$WR = M\ (Y\ r * 365 *24 *3600)\ Q_{ch} \qquad (23)$$

In SI units Q_{ch} would be in m^3/sec. In equation (23). and for the present study, M = unit price of water (0.2 LE/m^3), Y = benefit time, which is considered to be the time during which the irrigation water is steadily available (15 Years), r = percent use of irrigation water/annum ($\frac{31}{365}$) = 0.08, /9/. However WR will be given by:

$$WR = 7568639\ Q_{ch} \qquad (24)$$

The rate of flow in the channel Q_{ch} can be determined by Manning's equation for the present study as follows:

$$Q = \frac{S_b^{.5}}{n} \ A \ \ R^{\frac{2}{3}} \qquad (25)$$

For $S_b = 0.00015$, *$n = 0.015$, $A = 2.7\,y^2$, and* $R^{\frac{2}{3}} = 0.656\ y^{\frac{2}{3}}$, then equation (25) gives:

$$Q_{ch} = 1.447\ y^{\frac{8}{3}} \qquad (26)$$

or $WR = 7568639 * 1.447\, y^{\frac{8}{3}} = 10951820\, y^{\frac{8}{3}}$ (27)

Substituting WR of (27) in (22), then :

$$WB = 10951820\, y^{\frac{8}{3}} - CT \qquad (28)$$

Which is the water benefit equation

5. Present Study illustrative Example

A canal feeds a horizontal open channel reach which runs parallel to it for 20 km. The bottom level of channel is 10 m above water level in the canal. The supply of water is used for irrigation and maintained by means of three pipe mains which run over a hill, where the discharge ends of pipes is 16 m above the water level in the canal and are elevated one meter from the ground level, Fig (2). Each main is 22 m long and fitted with a pump which delivers 10 m^3/sec. The channel is trapezoidal and concrete lined with *n* = 0.015 and bed side slope z = 1.5. The channel is designed to carry 30 m^3/sec (10 m^3/sec by each main) .The channel bed slope S_b = 0.00015 and passes through a stratum of ordinary soil for which the following unit prices are assumed: c_e = 10 LE/m^2/m', c_{ie} = 2 LE/m^3/m' and c_L = 30 LE/m/m', /5/.

5.1 Calculations of Coefficients

Cost coefficients of pumping system

According to / 3 , 4 /, for iron pipes :

. C_{pipe}= a l D^x = 2500 l $D^{1.03}$, where, a = 2500, l = 22 m and x = 1.03

. $C_{pumping}$= 41249 Q_p H = A_1 Q_p H, i.e., A_1 (unit cost of pumping) = 41249 LE/m^3/sec/m lift

. C_{pump} = 54656 Q_p H = A_2 Q_p H, i.e., A_2 (unit cost of pump) = 46565 LE/m^3/sec/m lift

Friction loss coefficient

Q_p = 10 m^3/sec, assume mean velocity of flow V = 1.75 m/sec, $D^2 = \frac{40}{1.75\pi}$, D = 2.7 m, R_N (Reynold's Number) = $\frac{V\,D}{\nu}$, ν = kinematic viscosity of water ($1*10^{-6}$ m^2/sec), then R_N= 1.75 * 2.7* 10^6 = 4715000 (rough turbulent flow), assume roughness height e = 0.3 mm, i.e., $\frac{e}{D}$ (relative roughness) = 0.0011. Use Swamee formula /10 /, to calculate the coefficient of friction f :

$$4\,f = \frac{1.325}{\left(ln\left(\frac{e}{3.7\,D}+\frac{5.74}{R_N^{0.9}}\right)\right)^2} \qquad (29)$$

Then, f is found 0.0051 and the resistance coefficient of friction K_1 = $\frac{32\,f\,l}{\pi^2 g\, D^5}$ = $\frac{32*0.0051*22}{\pi^2 * 9.81 * 2.7^5}$ = 0.00026. Use equation (18) to calculate the optimum diameter of the main : $D^{6.03} = \frac{5\ (41249+54656)*0.00026\ *10^3}{2500*22*1.03}$, from which D = 1.14 m , based on this V = 9.802 m/sec, R_N = 11174433 (rough turbulent flow), at a relative roughness ($\frac{e}{D}$) = 0.00111, Swamee formula gives f = 0.00505 (f does not change in the first three significant figures), /11 /, then K_1 (refined) = $\frac{32*0.00505*22}{\pi^2 *9.81*1.14^5}$ = 0.019

Coefficient of minor losses

The coefficient of minor losses of the pumping system K_2 is calculated as follows :

$$K_2 = \left(0.5\ \frac{8}{\pi^2\ x9.81x\ 1.14^4}\right) + \left(0.4\ \frac{8}{\pi^2 *9.81*1.14^4}\right) 2 + \left(2.5\frac{8}{\pi^2 *9.81*1.14^4}\right) = 0.1824$$

Coefficients in the pipe cost function

$K = \left(\frac{32\,f}{\pi^2\,g}\right)^{0.2}$, or $K^x = \left(\frac{32\,f}{\pi^2\,g}\right)^{0.2x} = \left(\frac{32*0.0051}{\pi^2*9.81}\right)^{0.206} = 0.27$

$S^{0.2\,x} = \left(\frac{h_f}{l}\right)^{0.206} = \left(\frac{32*0.0051*22*10^2}{\pi^2 *9.81*1.14^5*22}\right)^{0.206} = 0.61$

The pipe cost is for three pipe mains or n = 3

5.2 Total cost calculations of scheme

In equation (17), to express total cost of water supply scheme, substitute by the numerical values of the predetermined coefficients; A_1, A_2, , K_1, K_2, a , l, x ,K^x, $S^{0.2x}$, n, E, F, s,k, z, c_e, c_{ie}, and c_L, and simplifying, then :

$CT = ((41249 +54656)\,Q_p\,(10 + y +2 + (0.019 + 0.1824)Q_p^2 +\left(\frac{2500*22*0.27}{0.61}\right) Q_p^{0.4\,x})*3 +(2.2\,y^3 +31.2\,y^2 + 190.2\,y +124)\,L$ (30)

5.3 Water benefit WB function

Substitute by CT of equation (30) in equation (28), then:

$WB = (10951820\, y^{\frac{8}{3}}) - ((1177687\, Q_p + 95905\, Q_p\, y + 19315\ Q_p^3 + 24344\, Q_p^{0.412})* 3 - (2.2\,y^3 +31.2\,y^2 + 190.2\,y +124)\ L$ (31)

Put Q_{ch} =3 Q_p in equation (31) and simplifying :

$$WB = (10951820\, y^{\frac{8}{3}}) - (11777687\, Q_{ch} + 95905\, Q_{ch}\, y + 2146\, Q_{ch}^{3} + 46455\, Q_{ch}^{0.412}) - (2.2\, y^3 + 31.2\, y^2 + 190.2\, y + 124)\, L \quad (32)$$

In equation (32), put $Q_{ch} = 1.447\, y^{\frac{8}{3}}$ and simplifying, then :

$$WB = (9247707\, y^{\frac{8}{3}}) - (138775\, y^{\frac{11}{3}} + 6502\, y^8 + 54082\, y^{1.1}) - (2.2\, y^3 + 31.2\, y^2 + 190.2\, y + 124)\, L \quad (33)$$

This is the water benefit equation and is a function of depth of flow y at a given value of channel length L.

6. Derivative Optimum of Water Benefit Function

For maximum water benefit MWB, differentiate WB of equation (33) with respect to y, equate $\frac{dWB}{dy}$ to zero and rearrange, then :

$$(24660551\, y^{\frac{5}{3}}) = (508842\, y^{\frac{8}{3}}) + 52018\, y^7 + 59490\, y^{0.1}) + (6.6\, y^2 + 62.4\, y + 190.2)\, L \quad (34)$$

In the illustrative example, for a channel length = 20 km (20000m), equation (34) reduces to :

$$1233\, y^{\frac{5}{3}} = (25.44\, y^{\frac{8}{3}} + 2.6\, y^7 + 2.975\, y^{0.1}) + (6.6\, y^2 + 62.4\, y + 190.2) \quad (35)$$

Equation (35), may be called the maximum water benefit MWB design formula. The LHS of the equation represents the water return gradient ($\frac{dWR}{dy}$), while the RHS represents the total cost gradient ($\frac{dCT}{dy}$) and both must be equal for the equation to be satisfied.

However, in the present study illustrative example, where M unit price of water is taken previously as 0.2 LE/m^3, trial values of y are chosen and substituted in the MWB design equation(35), it is found that the equation is verified at ($\frac{dWR}{dy}$) = ($\frac{dCT}{dy}$) = 8121,when y = 3.1 m which should be the optimal economic depth of flow. Accordingly, under the set of conditions of the illustrative example, b = 1.2*3.1= 3.72 m and Q_{ch} = 1.447 * $3.1^{\frac{8}{3}}$ =29.564 m^3/sec, i.e.,Q_p = 9.855 m^3/sec in each main

7. Evaluation of The MWB design Solution

7.1 Comparison with a minimum cost design solution

The necessary equation for the design of the minimum cost irrigation canal section can be obtained by differential calculations. At a given bed slope, these equations are readily derived and the governing equation is given here below, / 5 / :

$$\left(\frac{\partial A_t}{\partial y} + k_{ie}\frac{\partial E_t}{\partial y} + k_L\frac{\partial P_t}{\partial y}\right)\left(\frac{5}{3}\left(\frac{A}{P}\right)^{\frac{2}{3}}\frac{\partial A}{\partial b} - \frac{2}{3}\left(\frac{A}{P}\right)^{\frac{5}{3}}\frac{\partial P}{\partial b}\right) - \left(\frac{\partial A_t}{\partial b} + k_{ie}\frac{\partial E_t}{\partial b} + k_L\frac{\partial P_t}{\partial b}\right)\left(\frac{5}{3}\left(\frac{A}{P}\right)^{\frac{2}{3}}\frac{\partial A}{\partial y} - \frac{2}{3}\left(\frac{A}{P}\right)^{\frac{5}{3}}\frac{\partial P}{\partial y}\right) = 0 \quad (36)$$

In which $k_e = \frac{c_{ie}}{c_e}$, $k_L = \frac{c_L}{c_e}$, $A_t = b\,(y + F) + z\,(y + F)^2$, $P_t = b + 2\,(y + F)\sqrt{1 + z^2}$ and $E_t = \frac{(y+F)^2}{2}$.

Equation(36), can be solved for y and b to give the minimum cost design channel section for the given illustrative example with the following data ; k_{ie} = 0.2, k_L = 3, z = 1.5,and F= 1.0 m. Substituting by these parameters and the necessary partials, in equation (36) :

$$(0.3\, y^2 + 3.6\, y + 0.2\, b\, y + 1.2\, b + 14.1)\, 0.844\, y^5 - (0.1\, y^2 + 1.2\, y + 4.1)(1.06\, b\, y^{\frac{2}{3}} + 2.4\, y^{\frac{5}{3}}) = 0 \quad (37)$$

Equation (37), has two unknowns y and b and can be solved in combination with the Manning's equation being put in the following form:

$$Q\, n\, S_b^{\frac{1}{2}} = A\, R^{\frac{2}{3}} \quad (38)$$

here, the LHS of equation (38) is known as the channel section factor, / 5 /. Putting R = $\frac{y}{2}$, /6/, Q_{ch} = 29.456 m^3/sec, n = 0.015 and S_b = 0.00015, then equation (38) reduces to :

$$b = \left(\frac{56.785}{y^{\frac{5}{3}}} - 1.5\, y\right) \quad (39)$$

Equations (37) and (39), are solved by trial and error and the exact values of y and b which satisfy both equations are found ; y = 3.436 m and b = 2.104 m. However, the results of channel geometry and construction costs obtained by the MWB design solution and the minimum cost design solution are given in Table (1). From this table the following conclusions are reached :**(A)** Both values of velocity of flow are of the same order of magnitude, nearly equal and are within the permissible limits of velocity, / 5 / **(B)** The channel construction costs are of the same order of magnitude. The MWB design solution shows a cost of 1090 LE/m' while the minimum cost solution gives a value of 1059 LE/m'. The deviation is 2.8 %. **(C)** Although the construction costs in both cases are in good agreement, but it is not necessary that these must show identical cross-sections, because the MWB design solution is dependent on several factors which affect the channel design such as; water benefit parameters, pumping cost variables and channel length. These factors are being absent in the minimum cost solution in /5/. **(D)** The present MWB design solution is considered advantageous in the selection of the optimal economic channel dimensions, because it verifies money gain from the delivered water as will be shown latter.

7.2 Check of the MWB design solution for optimal channel design

To check the rigidity of the results of the MWB design solution obtained previously at L = 20 km, M = 0.2 LE/m^3 and y = 3.1 m as given in Table (1), the channel geometry parameters, construction costs, and the annual water benefits are also calculated at y values relatively smaller and larger than 3.1 m being 3.0 and 3.2 m. The results are shown in Table (2). It is evident that the total construction costs increases with increase of depth of flow. For comparison purposes,Table (3) is constructed for water benefit details, where it is found that the maximum water benefit occurs at y = 3.1 m (345 LE/m'), because, it is the optimal economic depth of flow at M = 0.2 LE/m^3.However, the maximum WB is decreasing at y values smaller or larger than 3.1 m, since at y = 3.0 m, it is 337.24 LE/m' and 336 LE/m' at y = 3.2 m. This conclusion is being evident in Fig(3) and indicates that, there is a unique optimal solution for y (3.1 m) which under the given set of conditions of the illustrative example achieves a total water benefit of 6900000 LE/annum for the whole length of the channel.

Table (1) Section geometry and construction costs of the channel example using the MWB design solution and the minimum cost design solution at M = 0.2 LE/m³

Parameter	MB design solution	Minimum cost design solution
Channel discharge Q_{ch} (m^3/sec)	29.564	29.564
Bed slope z	1.5	1.5
Manning's coefficient *n*	0.015	0.015
Bed slope S_b	0.00015	0.00015
Channel length (km)	20.0	20.0
Depth of flow y (m)	3.1	3.346
Bottom width b (m)	3.72	2.104
Area of flow A (m^2)	25.28	24.94
Velocity of flow V (m/sec)	1.17	1.185
Total channel depth h = y+1 (m)	4.06	4.436
Total area A_t (m^2)	40.46	38.85
Total perimeter P_t (m)	18.48	18.074
Centroid depth of total area from GL h_c (m)	1.62	1.66
Channel construction cost (LE/m')	1090.0	1059.0

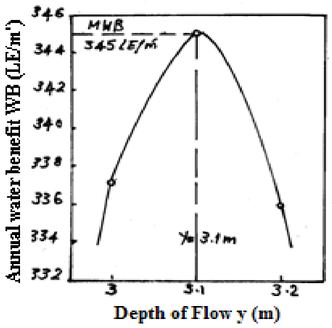

Fig (3). Plot of depth of flow y versus water benefit WB for the channel example at M = 0.2 LE/m³ (optimal economic depth of flow = 3.1 m)

Table (2) Section geometry and construction costs of the channel example at depth of flow y = 3, 3.1 and 3.2 m using MWB design solution at L = 20 km and M = 0.2 LE/m³

Parameter	y = 3 m	y = 3.1 m (optimal depth)	y = 3.2 m
Channel discharge Q_{ch} ($m^{3/}$/sec)	27.1	29.564	32.176
Bed slope z	1.5	1.5	1.5
Manning's coefficient	0.015	0.015	0.015
Bed slope S_b	0.00015	0.00015	0.00015
Channel length (20 km)	20.0	20.0	20.0
Depth of flow y (m)	3.0	3.1	3.2
Bottom width b (m)	3.6	3.72	3.84
Area of flow A (m^2)	24.3	25.94	27.648
Velocity of flow V (m/sec)	1.114	1.17	1.16
Total channel depth h = y+1 (m)	4.0	4.1	4.2
Total area A_t (m^2)	38.4	40.46	42.588
Total perimeter P_t	18.0	18.48	18.96
Centroid depth of total area from GL (m)	15.8	1.62	1.665
Channel construction costs (LE/m')	1045.0	1090.0	1137.5

7.3 Effect of unit price of water M

The effect of unit price of water on the optimal economic water depth y, at M = 0.16, 0.2 and 0.22 LE/m^3, and channel length = 20 km, (at the fixed given unit costs), is studied using the MWB design solution. The results are shown in Table (4).The table shows the following:

(A) the water return gradient ($\frac{dWR}{dy}$) increases with increase of unit price of water M, as well as by increase of water depth y **(B)** The total cost gradient ($\frac{dCT}{dy}$), increases with increase of water depth y, while being constant at each depth y, regardless of the change of the unit price of water since, the unit costs are fixed in the present study **(C)** Each value of unit price of water, has its own optimal economic water depth, indicating that the water depth of flow is sensitive to the variations in the unit price of water as shown in Fig (4).

7.2 Effect of channel length L

The effect of channel length on the optimal economic water depth y at L = 20, 22 and 25 km, M = 0.2 LE/m^3 and fixed unit costs is studied using the MWB design

Table (3) Comparison of water benefit details of the studied water supply scheme at y = 3.0 and 3.2 m with the optimal details at y = 3.1 m at L 20 km and M = 0.2 LE/m^3

Parameter	y = 3.0 m	y = 3.1 m (optimal depth)	y = 3.2 m
Channel discharge Q_{ch} (m^3/sec)	27.1	29.564	32.176
No of mains	3.0	3.0	3.0
Discharge/main Q_p (m^3/sec)	9.033	9.855	10.725
Depth of flow y (m)	3.0	3.1	3.2
Operating head of pump H (m)	31.435	34.66	38.366
Pipe costs (LE)	189694	189694	189694
Pumping costs (LE)	35138990	42267320	50920423
Pump costs (LE)	47709580	56005380	69136648
Channel construction costs (LE)	20900000	21820760	22750000
Total cost of scheme (LE)	103938264	120283214	142996765
Water return WR (LE)	205110144	223759273	243528561
Gross water benefit WB (LE)	101171880	103476051	100531796
Benefit time (year)	15	15	15
Channel length (km)	20	20	20
Annual water benefit (LE/m')	337.24	345	336

solution. The results are illustrated in Table (5). This table shows the following **(A)** The water return gradient ($\frac{dWR}{dy}$), increases with increase of water depth, while decreasing with increase of channel length.

(B) This trend is also true for the total cost gradient ($\frac{dCT}{dy}$) and both gradients intersect at an optimal economic depth of flow y about 3.1 m, which means that, the water depth is not sensitive to the variations in the channel length as long as these do not deviate far to abnormal channel lengths, Fig (4).

Table (4) Rate of change of WR and CT with water depth y at M = 0.16, 0.2 and 0.22 LE/m^3 and channel length = 20 km for the channel example

y (m)	$\frac{dWR}{dy}$ 0.16 LE/m^3	$\frac{dCT}{dy}$ 0.16 LE/m^3	$\frac{dWR}{dy}$ 0.20 LE/m^3	$\frac{dCT}{dy}$ 0.20 LE/m^3	$\frac{dWR}{dy}$ 0.22 LE/m^3	$\frac{dCT}{dy}$ 0.22 LE/m^3
1.0	941	290	1233	290	1379	290
2.0	2988	839	3915	839	4379	839
2.5	4329	2271	5672	2271	6426	2271
2.9	5581	5580	6646	5580	7433	5580
3.0	5778	6603	7694	6603	8605	6603
3.1	6201	8123	8126	8124	9088	8124
3.17	6436	9433	8434	9443	9441	9443
3.2	65211	9970	8568	9970	9557	9970

8. Economic Water Benefit Calculations

In developing countries, governments are faced with limited resources needed to provide funds for capital investments. For example, in water supply projects, it is necessary to cover all or a part of the investment and other expenses. However, water return WR and water benefit WB, should be implemented through the pricing of delivered water, / 12 /, but the water price should not exceed its value to the user and his willing to pay.

However, using the MWB design solution for trapezoidal channel section, it is possible to calculate the maximum water benefit gained from the delivered water at reasonable unit prices of water of 0.16, 0.2 and 0.22 LE/m^3. The results shown in Table (6), lead to the following : (A) Owing to the discharge and operating head data of the pumping system, large mixed flow pumps may be used, / 11 / **(B)** The cost of pumping and the channel construction costs form a major part of the total cost of the water supply scheme. However, these could be reduced if the channel is fed naturally from the main canal. This is being influenced by the nature of the terrain topography **(C)** A plot of unit price of water versus total costs , Table (6), in Fig(5) shows that, the total cost of scheme increases with increase of unit price of water and vice versa, (exhibited by large y). The same trends are found between unit price of water and both water return and water benefit.(exhibited by large Q_{ch}). **(D)** At lower unit price of water, the total costs are reduced because the optimal economic water depth y will do so. But this cost reduction will be on the expense of the water benefit WB. For example, at M = 0.16 LE/m^3, the total cost is reduced to 300.6 LE/m' and the WB is reduced to a lower value of 203 LE/m' **(E)** The plots of ($\frac{CT}{m'}$) and ($\frac{WB}{m'}$) shows that these parameters are converging with increase of unit price of water, Fig(5), and may reach nearer values at values of M larger than 0.22 LE/m^3 due to large costs exhibited by larger y values

Table (5) Optimal economic depth of flow y for the channel example at L = 20, 22 and 25 km and M = 0.2 LE/m^3

y (m)	$\frac{dWR}{dy}$ L=20 km	$\frac{dCT}{dy}$ L=20 km	$\frac{dWR}{dy}$ L=22 km	$\frac{dCT}{dy}$ L=22 km	$\frac{dWR}{dy}$ L=25 km	$\frac{dCT}{dy}$ L=25 km
1.0	1233	290	1121	288	986	284
2.0	3915	839	2559	794	3132	739
2.5	5672	2271	5162	2088	4542	1894
3.0	7694	6603	6995	6000	6155	5360
3.1	8126	8123	7398	7402	6509	6512
3.2	8568	9970	7790	9099	6855	8060

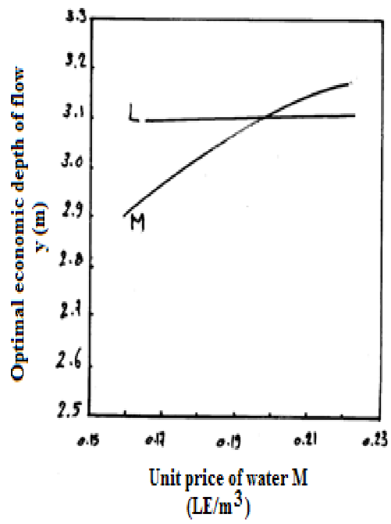

Fig (4). Effect of unit price of water M and channel length L on the optimal economic depth of flow y for the channel example, using the MB design solution

Table (6) Economic water benefit details of the water supply scheme at M = 0.16, 0.2 and 0.22 LE/m^3 using the MWB design solution

Parameter	M = 0.22 LE/m^3	M = 0.2 LE/m^3	M = 0.16 LE/m^3
Channel discharge Q_{ch} (m^3/sec)	31.378	29.564	24.976
No of mains	3.0	3.0	3.0
Discharge /main Q_p (m^3/sec)	10.46	9.855	8.325
Optimal economic depth of flow y (m)	3.17	3.1	2.91
Operating head of pump H (m)	37.2	34.66	28.87
Pipe costs (LE)	189694	189694	189694
Pumping costs (LE)	48148374	42267320	30463394
Pump costs (LE)	63797850	56005380	39410098
Channel construction costs (LE)	22460000	21820760	20120000
Total costs of the Scheme (LE)	134595918	120283214	90183186
Annual CT/m length of channel (LE/m')	449	401	300.5
Water return WR	265666833	223759273	151227482

(LE)			
Gross water benefit WB (LE)	131070915	103476059	61044296
Benefit time Y (years)	15	15	15
Channel length L (m)	20000	20000	20000
Annual water benefit (LE)	437	345	203

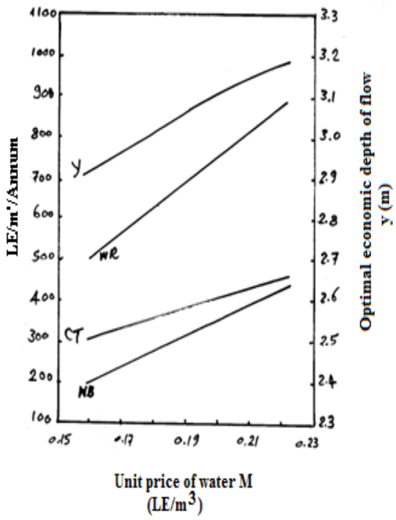

Fig (5). Plot of unit price of water M versus depth of flow y, water return WR, total cost of scheme CT and water benefit WB for the channel example at optimal economic conditions

9. Conclusions

In the present paper, a maximum water benefit MWB design mathematical solution for trapezoidal irrigation channel involving pumping has been reached. The objective non-linear function has been expressed as the cost of pumping and the channel construction costs for excavation and lining, The Manning's equation is used to determine the water return WR as a non-linear function equality. The derivative method is applied to the water benefit function to get the maximum benefit MWB solution for the channel design, which is applied on a trapezoidal channel example. The results of channel geometry and construction costs are compared with those obtained by an earlier solution which satisfies minimum cost conditions only. The results are in good agreement regardless of that, the channel geometry in both cases are not identical. However, in the MWB design solution several factors are bearing on the channel geometry such as; water return parameters, total cost variables, water benefit parameters and channel length. This condition is not satisfied in the minimum cost design solution The MWB design solution has been checked for optimizing the channel section dimensions and is found to give a unique optimal economic depth of flow and in turn channel section that maximizes the water benefit. The MWB design solution has been also evaluated and reveals that the depth of flow is sensitive to the variations of unit price of water, while being not sensitive to the variations in the channel length as long as the unit costs are fixed and the channel length does not deviate to abnormal values. It is evident that, the MWB design solution has the advantage of achieving money gains from the used water which should be delivered to the user as cheaply as possible. It is noted that, by reducing the unit price of water, the total costs are reduced, but this will be on the expense of the water return and water benefit. However, these considerations are not satisfied in the minimum cost design solution

The illustrative example given involving pumping, demonstrates the simplicity and practicability of the MWB design solution as well as water benefit verification and assures that, there are analytical solutions for developing comprehensive costing and benefit from any water supply scheme associated with pumping like that within the scope here.

It is recommended for **(A)** Applying the MWB design solution at variable unit costs for further investigation of this solution **(B)** The option of pricing water must be discussed with caution, since it is one of the measures to gain efficiency, productivity and benefits for the national economy (C) The water price should not exceed its value to the user and his willing to pay.

10. References

[1] Sharp, R.B. 1985, Economics of pumping and the utilization factor, Journal of hydraulic Engineering, ASCE, Vol.III, pp. 1386-1396
[2] Somaida, M.M., El-Zahar,M.M. and Sharran,M.S. (2011), computer simulation and mathematical solution for optimal design of pipe networks supplied from a groundwater source, Port-Said research Journal, Faculty of Engineering, Port-Said University, Vol. 16, No. 1, p. 106-(2)
[3] Somaida, M.M., 1994, Economical design of water-distribution networks supplied from groundwater source, Port-Said Scientific Engineering Bulletin, Vol.6, pp. 16-27
[4] Somaida, M.M., Elzahar, M.M., Yasser, A.H., and Sharran, M.S., 2011, A suggestion of optimization process for water pipe networks design, International Conference on Environment and Bioscience, IPCBEE, Vol. 21, IACSIT Press, Singapore
[5] Shamaa, M.T., and Abdel-Gawad, H.A., 2004, Minimum cost design irrigation canal sections, Mansoura, Eng. Journal (MEJ), Vol.4, No.1, pp. C134-C155
[6] Douglas, J.F., Gasiorek, J.M., and Swaffields, J.A., 1985, Fluid mechanics, longman-Scientific & Technical, 439p.
[7] Bouwer, H., 1978, Groundwater hydrology, McGraw-Hill Inc., , pp.188-194.
[8] Somaida, M.M., Elzahar, M.M., Yasser, A.H., and Sharran, M.S., 2011, The use of computer simulation and analytical solutions for optimizing pipe networks supplied from a pumped groundwater source, M.Sc.,

Thesis, Suez-Canal University, Faculty of Engineering, Civil Eng. Department, Port-Said
[9] Soliman, M.M., 1972, Irrigation and drainage engineering, Sherif Inc., 142p.
[10] Swamee, P.K., and Jain, A.K.,1976 , Explicit equations for pipe-flow problems, ASCE, Vol. 102, No. HY5, pp.65766(11)
[11]Streeter, V.L., and Wyle, E.M., 1975, Fluid mechanics, Mc Graw-Hill Inc., 301, 526 p
.[12] Darwich, M.R., 1982, Proposed methods for water pricing and charging in Egypt, M.Sc., Thesis, Instit. Center, for Advance Mediterranean Agronomic Studies, Bari, Italy.