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Non-Linear Local Optimal Control for Multivariable Systems

By

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Abstract:

To control non-linear systems along the whole range of operations, Non-linear control techniques are used. Neural network and generalized minimum variance controllers are two of the most common non-linear control techniques.
In this paper, Local optimal controller is generalized to deal with non-linear systems. This new non-linear controller is used to deal with multivariable systems as well as single-input single-output systems.
The effectiveness of the non-linear local optimal controller when dealing with multivariable systems is represented in this paper based on the results obtained from the high purity distillation column model.

Keywords:

Non-linear control, local optimal control, multivariable systems.

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1. Introduction:

Non-linear control is the branch of control engineering that deals with non-linear systems. As non-linear systems are difficult to be analyzed, linear approximations are used to analyze those systems. However, such approximations are only valid for certain ranges of operation. This approximation process is known as linearization. Along the whole range of operation of the non-linear system, certain operating points are defined. Based on the linearization technique, linear model is representing the non-linear system around these operating points. As such, the linear control techniques can be used to control the non-linear system for each region of operation [1, 2].
However, the controlled system performance degrades as the system subjected to rapid and large changes around its operating point. As a result, non-linear controllers are proposed to deal with the non-linear models [3]. Neural network and generalized minimum variance controllers are two of the most common non-linear control techniques [4–8].
Local Optimal Controller (LOC) is one of the advanced control techniques first introduced by Lyantsev et al. in 2004 [9]. However, this LOC approach is incapable of dealing with non-minimum phase systems. A modified LOC approach for multivariable systems is proposed By Ashry, et al. in 2008 [10]. The proposed method provides closed loop stability when dealing with non-minimum phase plant, which is a considerable advantage over the original LOC. As discussed in [9, 10], LOC is a model-based controller for multivariable systems as well as Single-Input Single-Output (SISO) systems.
The modified LOC is generalized to deal with non-linear SISO systems [11]. In this paper, LOC is used to control nonlinear multivariable systems as well as SISO systems. High purity distillation column model [12, 13] is used as a highly non-linear multivariable system to show the effectiveness of the proposed non-linear LOC.
An introduction to linear LOC is introduced in section 2. The proposed generalization of the linear LOC to become a non-linear controller is deduced in section 3. The non-linear LOC is designed in section 4 for a high purity distillation column as a highly non-linear multivariable system. The simulated results are introduced. Finally, the conclusion remarks are represented in section 5.

2. Linear local optimal control:

As mentioned, the local optimal control approach for linear model is described in [9, 10, 14]. This can be summarized as follows for a system model represented in discrete-time form. Consider the system model as in (1).

$$A(z^{-1})y(t) = B(z^{-1})u(t-1) \tag{1}$$

where z^{-1} is the back shift operator, and

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots\cdots + a_{n_a} z^{-n_a} \quad (2)$$

$$B(z^{-1}) = b_o + b_1 z^{-1} + b_2 z^{-2} + \cdots\cdots + b_{n_b} z^{-n_b} \quad (3)$$

where n_a and n_b represent the order of the system model polynomials with $n_a > n_b$. A model of the system in (1) can be presented in compact form as:

$$y(t) = \varphi^T (t-1)\theta \quad (4)$$

where *y(t)* is the reference trajectory at sample *t*, θ is a vector of the identified system parameters:

$$\theta^T = \left[a_1, \cdots\cdots, a_{n_a}, b_0, b_1, \cdots\cdots, b_{n_b} \right] \quad (5)$$

and φ is a data vector includes the past measured values of inputs and outputs

$$\varphi^T (t-1) = \left[-y(t-1), \cdots\cdots, -y(t-n_a), u(t-1), \cdots\cdots, u(t-n_b-1) \right] \quad (6)$$

A modified form for the output deviation is derived using (4) as:

$$y(t+1) - y(t) = \Delta\varphi^T (t)\theta \quad (7)$$

where $\Delta\varphi(t)$ is the measured input and output data deviations between successive samples at sample *t* and can be defined as:

$$\Delta\varphi^T (t) = \left[-\Delta y(t), \cdots\cdots, -\Delta y(t-n_a+1), \Delta u(t), \cdots\cdots, \Delta u(t-n_b) \right] \quad (8)$$

where

$$\Delta y(t) = y(t) - y(t-1), \quad (9)$$

and

$$\Delta u(t) = u(t) - u(t-1) \quad (10)$$

The model given in (4) presents an accurate description of the system. However, in this expression the vector of the system parameters θ is approximately identified. This leads to a residual error between the targeted trajectory, which can be defined as $y^*(t+1)$ and the actual related measured output *y(t)*. A weighting coefficient *h* is used to derive the reference trajectory deviation from (5), (6) and (7) as:

$$\Delta y^*(t+1) = y^*(t+1) - y(t) = h \left[-\sum_{i=1}^{n_a} a_i \Delta y(t-i+1) + \sum_{j=0}^{n_b} b_j \Delta u(t-j) \right] \quad (11)$$

The controller output rate of change Δu(t) can be derived from (11) as:

$$\Delta u(t) = \frac{1}{b_0} \left[\frac{\Delta y^*(t+1)}{h} + \sum_{i=1}^{n_a} a_i \Delta y(t-i+1) - \sum_{j=1}^{n_b} b_j \Delta u(t-j) \right] \quad (12)$$

It's clear from (12) that controller output deviation is function of system parameters, past measured input and output data, reference trajectory, and the weighting coefficient *h* (the only tunable parameter). For digital implementation, the controller output *u(t)* is derived from (12) using backward Euler discrete integration [10, 14].

3. Non-linear local optimal control:

The non-linear LOC design procedure is deduced based on the general NARMAX model structure represented in (13) [12, 15].

$$\begin{aligned} y(t) = [\theta_0 &+ \sum_{i=1}^{n_y} \theta_i^1 y(t-i) + \sum_{i=d}^{n_u} \theta_i^2 u(t-i) + \sum_{i=1}^{n_y}\sum_{j=1}^{n_y} \theta_{i,j}^3 y(t-i) y(t-j) \\ &+ \sum_{i=d}^{n_u}\sum_{j=d}^{n_u} \theta_{i,j}^4 u(t-i) u(t-j) + \sum_{i=1}^{n_y}\sum_{j=d}^{n_u} \theta_{i,j}^5 y(t-i) u(t-j) \\ &+ higher\ order\ terms\ up\ to\ \deg ree\ \ l] + [\sum_{i=1}^{n_y}\sum_{j=1}^{n_e} \theta_{i,j}^6 y(t-i) e(t-j) \\ &+ \sum_{i=d}^{n_u}\sum_{j=1}^{n_e} \theta_{i,j}^7 u(t-i) e(t-j) + \sum_{i=1}^{n_y}\sum_{j=d}^{n_u}\sum_{k=1}^{n_e} \theta_{i,j,k}^8 y(t-i) u(t-j) e(t-k) \\ &+ all\ possible\ combinations\ of\ \ y(t), u(t), e(t)\ up\ to\ \deg ree\ \ l] \\ &+ [\sum_{i=1}^{n_e} \theta_i^9 e(t-i) + \sum_{i=1}^{n_e}\sum_{j=1}^{n_e} \theta_{i,j}^{10} e(t-i) e(t-j) \\ &+ higher\ order\ terms\ up\ to\ \deg ree\ \ l] \end{aligned} \qquad (13)$$

where:

n_y is the output dynamic order,
n_u is the input dynamic order,
n_e is the error dynamic order,
l is the degree of non-linearity,
d is the input time delay.

This model structure can be presented in a compact form as in the following equation.

$$y(t) = \theta^0 + \phi^T (t-1)\theta, \qquad (14)$$

where θ is the vector of the identified model parameters and ϕ is the data vector includes the regressors. These two vectors are given in (15).

$$\theta = \begin{bmatrix} \vdots \\ \theta_i^1 \\ \vdots \\ \theta_i^2 \\ \vdots \\ \theta_{i,j}^3 \\ \vdots \\ \theta_{i,j}^4 \\ \vdots \\ \theta_{i,j}^5 \\ \vdots \\ \theta_{i,j}^6 \\ \vdots \\ \theta_{i,j}^7 \\ \vdots \\ \theta_{i,j,k}^8 \\ \vdots \\ \theta_i^9 \\ \vdots \\ \theta_{i,j}^{10} \\ \vdots \end{bmatrix}, \qquad \phi(t-1) = \begin{bmatrix} \vdots \\ y(t-i) \\ \vdots \\ u(t-i) \\ \vdots \\ y(t-i)y(t-j) \\ \vdots \\ u(t-i)u(t-j) \\ \vdots \\ y(t-i)u(t-j) \\ \vdots \\ y(t-i)e(t-j) \\ \vdots \\ u(t-i)e(t-j) \\ \vdots \\ y(t-i)u(t-j)e(t-k) \\ \vdots \\ e(t-i) \\ \vdots \\ e(t-i)e(t-j) \\ \vdots \end{bmatrix} \tag{15}$$

A modified form for the output deviation is derived using (14) as:

$$y(t+1) - y(t) = \Delta\phi^T(t)\theta, \tag{16}$$

where $\Delta\phi(t)$ is the measured input and output data deviations between successive samples at sample *t* and can be defined as:

$$\Delta\phi(t) = \begin{bmatrix} \vdots \\ \Delta y(t-i+1) \\ \vdots \\ \Delta u(t-i+1) \\ \vdots \\ \Delta[y(t-i+1)y(t-j+1)] \\ \vdots \\ \Delta[u(t-i+1)u(t-j+1)] \\ \vdots \\ \Delta[y(t-i+1)u(t-j+1)] \\ \vdots \\ \Delta[y(t-i+1)e(t-j+1)] \\ \vdots \\ \Delta[u(t-i+1)e(t-j+1)] \\ \vdots \\ \Delta[y(t-i+1)u(t-j+1)e(t-k+1)] \\ \vdots \\ \Delta e(t-i+1) \\ \vdots \\ \Delta[e(t-i+1)e(t-j+1)] \\ \vdots \end{bmatrix} \qquad (17)$$

and:

$$\begin{aligned}
&\Delta y(t-i+1) = y(t-i+1) - y(t-i), \qquad \Delta u(t-i+1) = u(t-i+1) - u(t-i), \\
&\Delta[y(t-i+1)y(t-j+1)] = y(t-i+1)y(t-j+1) - y(t-i)y(t-j), \\
&\Delta[u(t-i+1)u(t-j+1)] = u(t-i+1)u(t-j+1) - u(t-i)u(t-j), \\
&\Delta[y(t-i+1)u(t-j+1)] = y(t-i+1)u(t-j+1) - y(t-i)u(t-j), \\
&\Delta[y(t-i+1)e(t-j+1)] = y(t-i+1)e(t-j+1) - y(t-i)e(t-j), \\
&\Delta[u(t-i+1)e(t-j+1)] = u(t-i+1)e(t-j+1) - u(t-i)e(t-j), \\
&\Delta[y(t-i+1)u(t-j+1)e(t-k+1)] = y(t-i+1)u(t-j+1)e(t-k+1) - y(t-i)u(t-j)e(t-k), \\
&\Delta e(t-i+1) = e(t-i+1) - e(t-i), \\
&\Delta[e(t-i+1)e(t-j+1)] = e(t-i+1)e(t-j+1) - e(t-i)e(t-j).
\end{aligned} \qquad (18)$$

The model given in (14) presents an accurate description of the system. However, in this expression the vector of the system parameters is identified (estimated). This leads to a residual error between the targeted trajectory, which can be defined as $y^*(t + 1)$ and the actual related measured output $y(t)$. A weighting coefficient h is used to derive the reference trajectory deviation from (15) and (16) as:

$$\Delta y^*(t+1) = y^*(t+1) - y(t) = h(\Delta\phi^T(t)\theta) \tag{19}$$

The controller output $u(t)$ that minimizes $\Delta y^*(t + 1)$ in (19) can be obtained as a function of the system parameters, the past measured input and output data, the reference trajectory, and the weighting coefficient h, which is the only tuneable parameter.

4. Non-linear LOC for multivariable systems:

Identification and control of highly non-linear processes pose a challenging problem to the process industry. In the absence of a reasonably accurate model, these processes are fairly difficult to be controlled. High-purity distillation column is considered as an example of non-linear multivariable system in this section [12, 13]. The NARMAX is the model structure chosen for this case.
The non-linear multivariable LOC is designed in similar method to that for the linear multivariable LOC but based on the relations deduced in Section 3 for NARMAX models.

4.1 Mathematical model:

The process that is considered in this section is a methanol-ethanol distillation column simulation model with 27 trays. The temperature on tray 21 (near the top) and tray 7 (near the bottom) are to be controlled by manipulating the reflux flow rate and the reboiler vapor rate. The process variables are given in the following dimensionless forms [12, 13].

$$\begin{aligned} y_1 &= \frac{T_{21} - T_{21ss}}{30.0}, & y_2 &= \frac{T_7 - T_{7ss}}{30.0}, \\ u_1 &= \frac{L - L_{ss}}{6.0}, & u_2 &= \frac{V - V_{ss}}{6.0}, \end{aligned} \tag{20}$$

where:
T_{21}, T_7 are temperature on tray 21 and tray 7 (^oR), respectively;
T_{21ss} is the steady state temperature on tray 21 (614.10^oR);
T_{7ss} is the steady state temperature on tray 7 (636.77^oR);
L is the reflux flow rate (*mol/min*);
L_{ss} is the steady state reflux flow rate (3.384 *mol/min*);
V is the reboiler flow rate (*mol/min*);
V_{ss} is the steady state reboiler flow rate (3.856 *mol/min*).

The inputs used for model identification in this case are random variables taken from a uniform distribution with appropriate range [16]. This range, which cover the operating interval of interest for the given column has higher and lower, limits of 0.05 and -0.05 respectively [12,13]. Each input is allowed to change at each sampling time (every minute in this case) with a predetermined probability (switching probability). I/O data are generated and the model has been identified using Multi-Input Single-Output (MISO) algorithm in which, each output is modeled separately. The identified model given in [12, 13] is represented in (21).

$$\begin{aligned} y_1(k+1) &= 0.0012 + 0.98 y_1(k) - 0.18 u_1(k) \\ &\quad + 1.1 y_1(k-2) u_2(k) - 1.8 y_2(k) u_1(k) \\ y_2(k+1) &= 0.0018 + 0.92 y_2(k) - 0.22 u_1(k) \\ &\quad + 30.4 y_2^2(k) u_2(k) - 1.7 u_2^2(k) \end{aligned} \tag{21}$$

4.2 Non-linear controller design:

The non-linear LOC is designed for the high purity distillation column using the non-linear model given in (21). Similar designing technique to that of the linear LOC is used but based on the non-linear model obtained. The non-linear LOC outputs *($u_1(k)$, $u_2(k)$)* are deduced in (22) and (23).

$$\begin{aligned} u_1(k) = -\frac{1}{0.18}[&\frac{1}{h_1}(r_1(k) - y_1(k)) - 0.98 y_1(k) + 0.98 y_1(k-1) \\ &- 0.18 u_1(k-1) - 1.1 y_1(k-2) u_2(k) + 1.1 y_1(k-3) u_2(k-1) \\ &+ 1.8 y_2(k) u_1(k) - 1.8 y_2(k-1) u_1(k-1)] \end{aligned} \tag{22}$$

$$
\begin{aligned}
u_2(k) = (30.4y_2^2(k) - 1.7u_2(k))^{-1}[\frac{1}{h_2}(r_2(k) - y_2(k)) - 0.92y_2(k) \\
+ 0.92y_2(k-1) + 0.22u_1(k) - 0.22u_1(k-1) \\
+ 30.4y_2^2(k-1)u_2(k-1) - 1.7u_2^2(k-1)]
\end{aligned} \tag{23}
$$

The system controlled by non-linear LOC is simulated using Simulink [17]. Two reference inputs are applied to the controlled system, one at 16.67 *min* and the other at 50 *min*. The reference inputs and the simulated output responses of the high purity distillation column model controlled using non-linear LOC are shown in Figure (1). Also, the non-linear LOC outputs (plant input) are shown in Figure (2).

From these figures, the reference input trajectories are achieved. It should be noted that the two outputs are completely decoupled.

Comparing these results with the results presented in [13] using non-linear model predictive controller described in [18], faster output responses and better decoupling between the outputs are achieved using the non-linear LOC.

Figure (1): *The reference inputs and the simulated output responses of the controlled system.*

Figure (2): *The simulated controller outputs (input to the plant).*

5. Conclusions:

In this paper, a generalized version of the LOC is introduced as a non-linear controller that deals with non-linear models presented in NARMAX model structure.
The non-linear LOC is designed to control non-linear multivariable systems. The results obtained based on the high purity distillation column model show the effectiveness of the non-linear LOC when dealing with multivariable systems. Faster response and high degree of decoupling are the advantages of the new non-linear LOC.
As a model-based controller, the non-linear LOC can be used in automatic adaptation mode based on the on-line system identification.

References:

[1] C. F. Lin, *Advanced control systems design*. Printice-Hall Inc., 1994.

[2] P. A. Cook, *Nonlinear dynamical systems*. Printice-Hall Inc., 2nd ed., 1994.

[3] D. Sbarbaro and R. Murray-Smith, *Self-tuning control of non-linear systems using Gaussian process prior models*. Springer Berlin, 2005.

[4] M. Agarwal and D. E. Seborg, “Self tuning controllers for nonlinear systems,” *Automatica*, vol. 23, no. 2, pp. 209–214, 1987.

[5] J. I. Canelon, L. S. Shieh, and N. B. Karayiannis, "A new approach for neural control of nonlinear discrete dynamic systems," *Information Sciences*, vol. 174, pp. 177–196, 2005.

[6] J. Sarangapani, Neural *network control of nonlinear discrete time systems*. CRC Press, 2006.

[7] M. M. Bayoumi and L. Mo, "Adaptive minimum variance control for SISO non-minimum phase systems," in *The 27th conference on decision and control*, pp. 1930–1933, 1988.

[8] L. Mo and M. M. Bayoumi, "Adaptive minimum variance and model reference control for non-minimum phase systems," *International Journal of control*, vol. 50, no. 6, pp. 2125–2139, 1989.

[9] O. Lyantsev, T. Breikin, G. Kulikov, and V. Arkov, "Optimal multi-variable control of gas turbine engines," *International journal of systems science*, vol. 35, pp. 79–86, 2004.

[10] M. Ashry, U. Abou-Zayed, and T. Breikin, "Control of multivariable systems using modified local optimal controller," in *The 17th IFAC world congress*, (Seoul, Korea), pp. 8767–8772, 2008.

[11] Z. Kamalova, M. Ashry, and T. Breikin, "Study of reduced order and non-linear local optimal control application to aero gas turbines," in *UKACC international conference on control*, 2008.

[12] M. A. Henson and D. E. Seborg, *Nonlinear process control*. Printice-Hall, Inc., 1997.

[13] G. R. Sriniwas, Y. Arkun, I. L. Chien, and B. A. Ogunnaike, "Nonlinear identification and control of a high-purity distillation column: a case study," *Process control*, vol. 5, pp. 149–162, 1995.

[14] M. Ashry, Z. Kamalova, and T. Breikin, "Tuning of digital PID controller parameters using local optimal control," in *The 16th Mediterranean conference on control and automation*, (Ajaccio, France), pp. 587–592, 2008.

[15] G. G. Kulikov and H. A. Thompson, *Dynamic modelling of gas turbines: identification, simulation, condition monitoring, and optimal control*. Springer-Verlag, 2004.

[16] I. Leontartis and S. Billings, “Experimental design and identifiability for nonlinear systems,” *International Journal of Systems Science*, vol. 18, pp. 189–202, 1987.

[17] Matlab, *Simulink 7.1*. Matlab, The MathWorks, 2008.

[18] E. Hernandez and Y. Arkun, “Control of nonlinear systems using polynomial ARMA models,” *AIChE*, vol. 39, pp. 446–460, 1993.