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Flexural Plastic Hinge Length for Reinforced Concrete Frames

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Abstract

The flexural plastic hinge (PH) length is a critical parameter for reinforced concrete (RC) frames subjected to lateral load. In this work, a review of past studies for introducing empirical equations for estimating the flexural PH length (l_p) of RC frames was introduced. Another key aspect explored in this paper is to study the effect of different available l_p equations on the overall nonlinear structural response of RC frames. Therefore, ten l_p equations were selected from this review for simulating the nonlinear behavior of two RC frames with available experimental data. Each frame was analyzed ten times with alternative l_p values under pushover analysis using SAP2000 software. The two modeled frames displayed an average error ranging from 12.3% to 23% in the ultimate lateral load, and a deviation ranging from -30.4% to 20.1% in the initial lateral stiffness. These errors indicated that the accuracy of predicting the behavior of RC frames is highly dependent on the chosen l_p equation. Finally, the proper l_p equation which could represent the nonlinear behavior of RC frames accurately was recommended.

Keywords

RC frame; Plastic hinge; Plastic hinge length; Pushover analysis; SAP2000

1. Introduction

Reinforced Concrete (RC) Frame structures are commonly used in construction due to their strength and durability. These frames are modeled and designed to resist lateral loads, such as wind and earthquake loads.

Two main general approaches are used to model the RC frame structures accounting for material nonlinearity: lumped (concentrated) inelasticity and distributed inelasticity (which includes fiber models). Lumped plasticity (plastic hinge; PH) approach had been used for modeling RC frames under lateral loads since the 1960s.

The nonlinear behavior of the flexural PH plays a dominant role in determining the RC frame response (Sunil & Kamatchi 2022, Inel & Ozmen 2006).The PHs exist at maximum bending moments that sections were associated with yielding of steel reinforcement or compression failure of concrete. In order to simplify PH modeling, researchers had attempted to represent the PH zone with a constant length known as the flexural PH length (l_p). To accurately model these PHs, the curvature and strain are integrated over the l_p. The behavior of PHs and consequently the global nonlinear behavior of RC frames are highly influenced by l_p value.

2. Review for PH Length (l_p) Equations

Several empirical equations for estimating l_p were introduced. There are important parameters that influence the l_p value such as the section depth, the steel reinforcement yield stress, the concrete compressive strength, the shear strength, and the bar diameter of longitudinal reinforcement. Various studies had presented empirical equations to estimate l_p for different types of RC elements.

2.1. Chan (1955)

Chan (1955) suggested Equation (1) to determine l_p based on experimental tests for three types of specimens: nine members with transverse ties steel reinforcement, seven members with transverse spiral steel reinforcement, and seven members with transverse welded ties steel reinforcement.

$$l_P = Z\left(1 - M_y/M_u\right) \tag{1}$$

Where, Z is the shear length (moment to shear ratio), and $\boldsymbol{M_y}$ **and** $\boldsymbol{M_u}$ are the yield and ultimate moment, respectively.

2.2. Baker (1956)

Baker presented Equation (2) to determine the PH length l_P based on testing three types of specimens under bending moment and axial load: 32 members reinforced with cold work steel, 30 members reinforced with mild steel, and 32 members reinforced with both mild steel and cold work steel.

$$l_P = k_1 k_2 k_3 (Z/d)^{0.25} d \tag{2}$$

where the factors k_1, k_2, and k_3 were defined based on the concrete compressive strength (f_c'), the initial axial load (P_1), and the capacity axial load (P_o); as per ACI 318-05 (2005)

k_1 =0.7 for mild steel or 0.9 for cold work steel

$k_2 = 1+0.5(P_1/P_o)$

$k_3 = 0.9 - 0.01277(f_c' - 11.7)$ if $11.7 < f_c' < 32.2$ MPa

2.3. Cohn & Petcu (1963)

Ten continuous RC beams with two spans were tested categorized into two groups in which the beams were loaded with a concentrated load at a specified distance from the central support, and were monotonically loaded until failure. The load distance for the first group was 40 cm whereas it was 60 cm for the other group. They recorded the results of l_p obtained for 10 beams varied from 0.3d to 0.9d where d is the effective depth of the beam.

$$l_P = 0.3d \sim 0.9d \tag{3}$$

2.4. Sawyer (1965)

The inelastic deformation of RC frames was investigated based on a bilinear moment-curvature relationship with assumption that the ratio M_y/M_u is equal to 0.85, and the maximum moment at any section is equal to the ultimate moment).

$$l_P = 0.075\, Z + 0.25\, d \tag{4}$$

2.5. Corley (1966)

The PH length l_P was determined based on a test for 40 simply supported beams. These beams were subjected to a concentrated load at the midspan. The results obtained for the 40 beams were fitted using the introduced Equation (5). Similar to Sawyer (1965), the l_P was a function in the effective section depth and the shear length of the beam.

$$l_P = 0.2\ Z/\sqrt{d} + 0.5\, d \tag{5}$$

2.6. Mattock (1967)

Mattock (1965) made a study to determine l_p based on 37 beam tests with various parameters (effective depth, shear length, concrete strength, and yield stress of tension reinforcement). Mattock in 1967 presented a simplification to his previous study by introducing Equation (6). The l_P was related to the section depth and the member shear length similar as Corley (1966) and Sawyer (1965).

$$l_P = 0.05\,Z + 0.5\,d \tag{6}$$

2.7. Zahn et al. (1986)

Zahn et al. suggested Equation (7) to determine the PH length l_P based on the tests of 14 RC columns (with different cross-sections) subjected to combined bending moment and axial load. The 14 RC columns comprised three types of section shapes: six square sections, two octangular sections, and six circular hollow sections. Consequently, Equation (7) was divided s into three sub-equations (7-1, 7-2, and 7-3), as following:

$$l_P = 0.08Z + 6d_b\left(0.5 + 1.67\frac{P_1}{f_c' A_g}\right) \quad \text{for } \frac{P_1}{f_c' A_g} < 0.3 \quad (7\text{-}1)$$

$$l_P = 0.08Z + 6d_b \quad \text{for } \frac{P_1}{f_c' A_g} \geq 0.3 \quad (7\text{-}2)$$

$$l_P = 0.06Z + 4.5d_b \quad \text{for } \textit{circular hollow sections} \quad (7\text{-}3)$$

where d_b, A_g and P_1 are the diameter of longitudinal steel reinforcement, section gross area, and initial axial load, respectively.

2.8. Priestly & Park (1987)

Priestly & Park provided Equation (8) to determine the PH length l_P based on two tests for short columns and two tests for slender columns (square and octangular sections). Equation (8) was completely identical to Equation (7-2) of Zahn et al. (1986).

$$l_P = 0.08Z + 6d_b \tag{8}$$

2.9. Paulay & Priestly (1992)

In 1992, Paulay & Priestly suggested to add the yield stress of longitudinal reinforcement f_y^l (MPa) to Equation (8), and presented by Equation (9) based on several tests on beams and columns. Also, they indicated that l_P for typical beams and columns in the typical floors are approximately $0.5h$.

$$l_P = 0.08Z + 0.022d_b f_y^l \tag{9}$$

2.10. Sheikh & Khoury (1993)

Sheikh & Khoury introduced Equation (10) to determine PH length l_P based on several tests on the beams and columns. They simply assumed that l_P for all RC elements was equal to the section depth, h.

$$l_P = h \tag{10}$$

2.11. Panagiotakos & Fardis (2001)

Panagiotakos & Fardis proposed Equation (11) for estimating PH length l_P by testing over than 1000 specimens. These specimens consisted of RC members subjected to uniaxial bending, with and without axial loads. These members represented the different characteristics of the beams, columns, and shear walls. There were 266 beam specimens with unsymmetrical steel reinforcement under uniaxial moment, 682 column specimens (rectangular and square cross-sections) with symmetrical reinforcement under axial loads, 23 column specimens with diagonal reinforcement, and 61 shear wall specimens with rectangular or T cross sections.

$$l_P = 0.18Z + 0.021a_{sl} d_b f_y^l \tag{11}$$

where a_{sl} is a longitudinal bar pullout factor; zero–one variable. If slippage of the longitudinal reinforcement is possible, a_{sl} is equal to a value of zero, whereas if slippage is not possible, a_{sl} is equal to a value of one.

2.12. EN 1998-3:2005 Eurocode8 (2005)

Eurocode8 provided Equation (12) to determine the PH length for members with earthquake reinforcement details and for those without lapping of longitudinal bars in the section where yielding is expected.

$$l_P = 0.1Z + 0.17h + 0.24d_b f_y^l / \sqrt{f_c'} \tag{12}$$

2.13. Bae & Bayrak (2008)

Bae & Bayrak (2008) suggested Equation (13) to determine the PH length l_P based on an experimental and analytical research focusing on the seismic behavior of RC columns. Four RC columns were tested under axial load values ranging from moderate to high relative to their capacities.

$$l_P = Z\left(0.3\frac{P_1}{P_o} + 3\frac{A_t}{A_g} - 0.1\right) + 0.25h \geq 0.25h \tag{13}$$

where A_t is the tension steel reinforcement area.

2.14. Berry et al. (2008)

Berry et al. presented Equation (14) to determine the PH length l_P based on the data of 37 tests of large-scale circular bridge columns. Their equation was a function in Z, d_b, f_y^l, and f_c' similar to Equation (12) with excluding h.

$$l_P = 0.05Z + 0.1 d_b f_y^l / \sqrt{f_c'} \tag{14}$$

3. Case of Study

Two single bay one-story frame structure specimens (Figure 1) tested by Dautaj & Kabashi (2019) were non-linearly analyzed under pushover load. The PH length l_P was estimated ten alternative equations selected from the review (section 2). Table 1 presents the ten l_P studied equations. For concrete properties, the compressive strength (f_c') and the compressive strain (ε_c) were 20 MPa and 0.19%, respectively. The yield stress of the longitudinal steel reinforcement (f_y^l) was 620 MPa and 590 MPa for, respectively, for model 1 and model 2. The general geometry of the two RC frames are shown in Figure 1. For the frame dimensions, the span frame (L) and height (H) for model 1 were 2550 mm and 2075 mm, respectively, whereas they were 2500 mm and 2070 mm, respectively, for model 2. For the two models, 20 kN constant vertical force was applied at the top of each column as shown in Figure 1. For frame 1, the cross section for both the beam and the column was 150×250 mm with 6 bars of 10 mm diameter as a longitudinal steel reinforcement and 6 mm diameter ties spaced every 75 mm as a transverse reinforcement. For frame 2, the cross section for both the beam and the column was 150×300 mm with 8 bars of 10 mm diameter as a longitudinal steel reinforcement and 6 mm diameter ties spaced every 75 mm as a transverse steel reinforcement. The two frames were simulated under pushover analysis using SAP2000 software (CSI 2020).

The RC frame models were simulated using bar element with two nodes for the columns and the beam. They were defined as elastic elements. The cross-section was discretized into fibers using the section designer tool (Figure 2). The PH was defined and the l_P was set according to Table 1 as shown in Figure 3. The PHs were assigned to the frame members at the maximum moment positions (Figure 4). The PH ID was *element type*-PH-*plastic hinge position*. For example, CPH1 refers to PH at the start (1) of a column member (C).

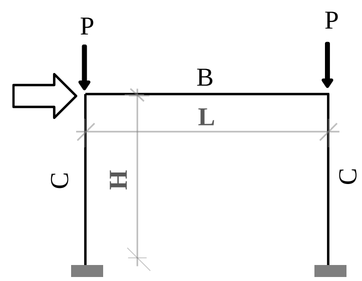

Figure 1. Details of frame 1 and frame 2 (Dautaj & Kabashi, 2019)

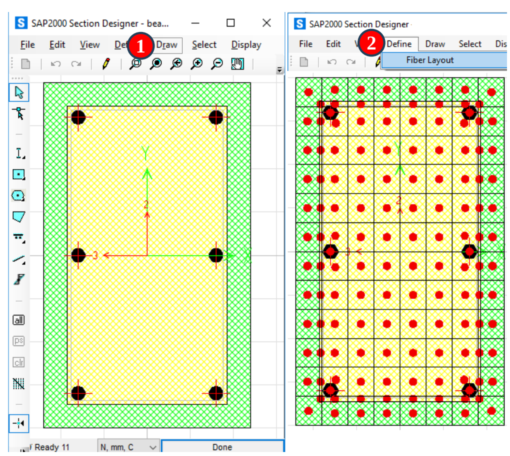

Figure 2. Section designer tool in SAP2000 to desctize the RC cross-section into fibers (CSI 2020)

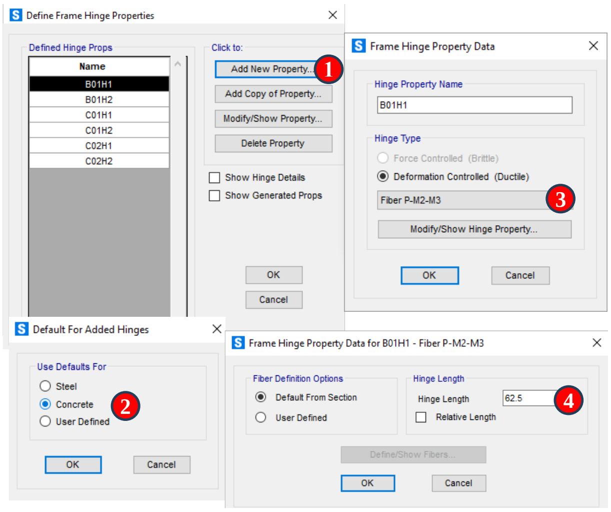

Figure 3. Definition of the PH and setting l_p length (CSI 2020)

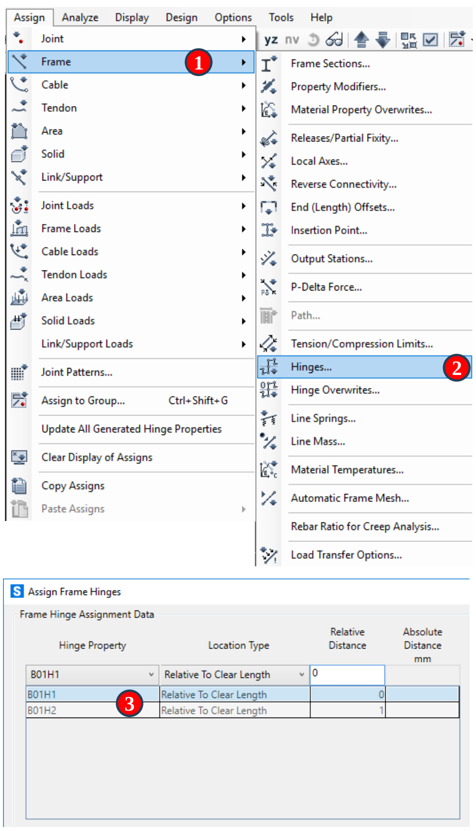

Figure 4. Assigning the PH to the frame members (CSI 2020)

4. Results

The ten equations presented in Table 1 were investigated for the two frames models presented in Figure 1. PH lengths l_p values determined from these equations were presented in Table 2. Using SAP2000, the lateral response curve for each trial was determined (Figure 5). Based on these results, the ten equations were categorized into two groups according to the average error in the initial stiffness (ΔK) of each trial compared with the reference experiment (Table 2). The trials, which resulted in higher initial stiffness K than the experimental test, were gathered in group 1, whereas group 2 contained the trials which produced initial stiffness K smaller than the reference experiment. It is worth mentioning that the initial stiffness was used to compare the results because the ultimate lateral load was always overestimated for the twenty trials and its average error was 18.3% and 15.8% for the ten investigations of frame 1 and frame 2, respectively.

Figure 6 and Figure 7 show the lateral load–displacement curves resulted using these equations compared with reference experiment for the studied frame 1 and frame 2, respectively. It was observed that Equation (13) exhibited the maximum positive deviation in the initial stiffness with average values of 20.1%, whereas Equation (12) showed the maximum negative deviation of -30.4%. This could be attributed to the value of the PH length where Equation (13) provided the smallest plastic hinge length (62.5 mm for model 1 and 75 mm for model 2), whereas Equation (12) produced the largest PH length (average 478 mm for the two models). This implies that increasing the PH length l_p decreases the lateral initial stiffness and vice versa. Definitely, a higher l_p means more deformation and weaker frame with lower lateral initial stiffness. Conversely, Equations (2) and (9) resulted in the minimum absolute deviations in the initial stiffness with average values of only 0.4% and -0.7%, respectively. This could be attributed to the value of the PH length l_p where the average l_p of Equations (2) and (9) for the two models was 210 mm which is approximately equal to the average l_p of the extreme Equations (13) and (12).

Finally, it was found that Equations (2), (9), and (14) exhibited the best nonlinear behavior curve with the least absolute deviation in the initial stiffness; 0.4%, -0.7%, and 2.9%, respectively.

These findings highlighted the importance of carefully selecting a proper equation to represent the PH length in order to ensure accurate nonlinear predictions of the RC frames behavior. Further research and analysis are needed to improve the accuracy of these equations.

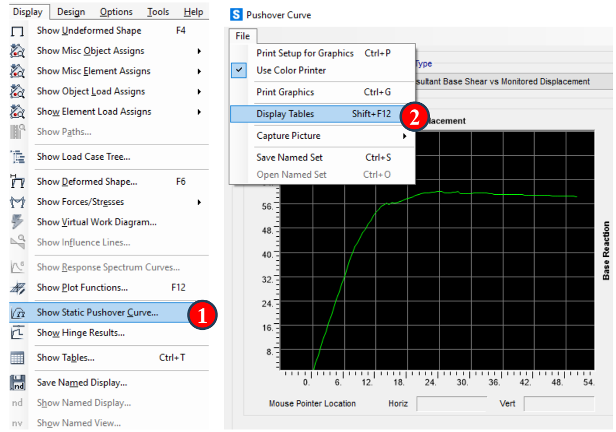

Figure 5. The lateral response cuvre using SAP2000 (CSI 2020)

Table 1. The selected ten PH length l_p equations for the case study

N°	Equation	Reference
Equation (2)	$l_P = k_1k_2k_3(Z/d)^{0.25}d$	Baker (1956)
Equation (5)	$l_P = 0.2\ Z/\sqrt{d} + 0.5\ d$	Corley (1966)
Equation (6)	$l_P = 0.05\ Z + 0.5\ d$	Mattock (1967)
Equation (8)	$l_P = 0.08Z + 6d_b$	Priestley & Park (1987)
Equation (9)	$l_P = 0.08Z + 0.022d_bf_y^l$	Paulay & Priestly (1992)
Equation (10)	$l_P = h$	Sheikh & Khoury (1993)
Equation (11)	$l_P = 0.18Z + 0.021a_{sl}d_bf_y^l$	Panagiotakos & Fardis (2001)
Equation (12)	$l_P = 0.1Z + 0.17h + 0.24d_bf_y^l/\sqrt{f_c'}$	EN 1998-3:2005 Eurocode8 (2005)
Equation (13)	$l_P = Z\left(0.3\frac{P_1}{P_o} + 3\frac{A_t}{A_g} - 0.1\right) + 0.25h \geq 0.25h$	Bae & Bayrak (2008)
Equation (14)	$l_P = 0.05Z + 0.1d_bf_y^l/\sqrt{f_c'}$	Berry et al. (2008)

Table 2. PH lenght l_p for the case study categroized in two groups according to the deviation in the initial stiffness

Group number	Model ID	l_P for the PHs of Model 1 (mm)				l_P for the PHs of Model 2 (mm)				ΔK %
	PH Label	BPH1	BPH2	CPH1	CPH2	BPH1	BPH2	CPH1	CPH2	
Group 1	Equation (13)	62.5	62.5	62.5	62.5	75	75	75	75	20.1
	Equation (8)	147.2	157.1	134.7	143.7	139.2	156.1	130.7	138.9	8.2
	Equation (5)	129.4	131	127.3	128.8	151.8	154.4	150.6	151.8	6.3
	Equation (14)	194	200	186	191	183	194	178	183	2.9
	Equation (9)	224	234	212	221	210	226	201	209	0.4
Group 2	Equation (2)	188.6	193.8	183.5	188.7	214.8	225.4	210.8	216.6	-0.7
	Equation (11)	321	343	293	313	297	335	278	296	-11.1
	Equation (10)	250	250	250	250	300	300	300	300	-13
	Equation (6)	136.9	139.3	133.7	136	159.9	164.1	157.7	159.8	-16.4
	Equation (12)	485	497	469	480	470	491	460	470	-30.4

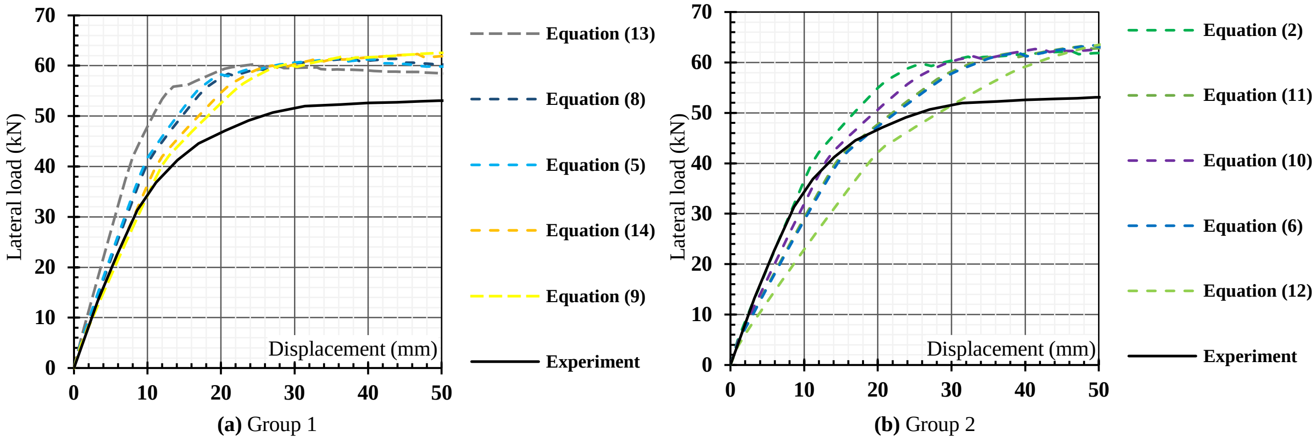

Figure 6. Lateral response of model 1 using the ten *lp* equations versus experiment (Dautaj & Kabashi, 2019)

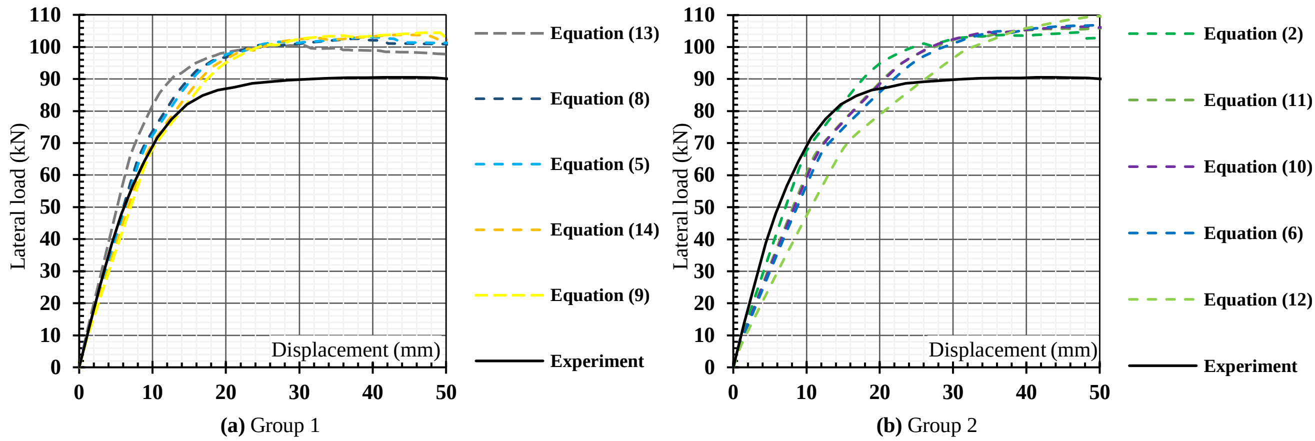

Figure 7. Lateral response of model 2 using the ten *lp* equations versus experiment (Dautaj & Kabashi, 2019)

4. Conclusion

This work was motivated to study the effective of using various equations of PH length *lp* in modeling the RC frames. A review of past studies for flexural plastic hinge length was introduced. Furthermore, a case study of the nonlinear behavior of two one-story RC frames under pushover analysis incorporating ten *lp* equations from the literature was conducted using SAP2000. The following conclusions can be drawn:

1- The average error in ultimate lateral load was 17.05% for the all the twenty investigations of both the studied frames. The error value is acceptable for such simple analysis. Better accuracy could be achieved by finding more reliable estimation for the *lp* based an intensive experimental plan and by considering new parameters such as the frame type whether strong or weak and the slenderness of the columns. Definitely, using a more advanced model such the distributed plasticity or full finite element analysis will improve the accuracy.

2- The average deviation in the initial lateral stiffness ranged from +20.1% to -30.4%.

3- The equation of Pauley & Priestly (1992) and the equation of Baker (1956) were the proper expressions to determine the PH length *lp* to match the real initial stiffness of RC frame; minimum deviations of +0.4% and

-0.7% was recorded for the two equations, respectively.

4- The Bae & Bayrak (2008) equation led to the largest overestimation in the initial lateral stiffness of the RC frames (+20.1%).

5- The Eurocode8 (2005) equation led to the largest underestimation in the initial lateral stiffness of the RC frames (-30.4%).

6- Although similar parameters were used for the highlighted four equations in the previous three items, the results of these equations were different. Therefore, not only the utilized parameters but also the coefficients of these parameters govern the accuracy of a certain empirical equation. In other words, the sample used to derive the equation has direct influence on its accuracy.

7-Increasing the plastic hinge length led to decreasing the lateral stiffness of the RC frame and vice versa.

8- Significant variation in the nonlinear response of the RC frames was noticeable for different *lp* lengths based on the utilized *lp* equation.

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