"Analytical and numerical study of some problems in nonlinear mechanics" **Ibtihal Mohammed ALSalem** 

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# Abstract:

This study deals with the topic of "Analytical and numerical study of some problems in nonlinear mechanics." The study aimed to analyze a problem in nonlinear mechanics, which is "wave function collapse." Which is represented by the laboratory experiment "Schrödinger's cat", the interpretations of Heisenberg and Copenhagen, and an application of the method of Lagrange's equations to explain the phenomenon of wave function collapse and the EPR paradox, and Einstein's interpretation in pure algebraic theory and his interpretation of wave collapse, in addition to the Schrödinger equation. The study concluded with a set of results, the most important of which are: that the problem Wave function collapse represents one of the measurement problems in quantum mechanics. It has been proven that it can be solved based on quantum mechanics and does not require any additional assumptions or new theories. The processes of particle creation and annihilation described based on quantum field theory play a major role in measurement processes. Decoherence theory also refers to the creation and annihilation of particles and these processes are a nonlinear consequence of quantum mechanics, in which case the wave function collapse term becomes a consequence of other statements of quantum mechanics rather than a separate assumption of quantum mechanics.

**Keywords:** some problems in nonlinear mechanics, nonlinear equations, Wave function collapse.

# **Introduction:**

In order to apply mathematical methods to a physical problem, We must formulate the problem in mathematical terms that is, We must construct a mathematical model for the problem. Many physical problems concern relationships between changing Quantities. since rates of change are represented mathematically by derivatives.

Mathematical models often involve equations relating an unknown. Function and one or more of its derivatives such equation are differential equations.

Nonlinear wave phenomena appears in various scientific and engineering fields such as fluid mechanics, plasma physics, optical fibers, biophysics, geochemistry, electricity, propagation of shallow water waves, high-energy physics, condensed matter physics, quantum mechanics, elastic media, biology, solid state physics, chemical kinematics, chemical physics and so on. This is also noticed to arise in engineering, chemical and biological applications. The application of nonlinear traveling waves has been brought prosperity in the field of applied science. In order to understand better the nonlinear phenomena as well as further application in the practical life, it is important to seek their more exact travelling wave solutions. Essentially all the fundamental equations in physical sciences are nonlinear and, in general.

Historically, little was known about the extraordinary range of behavior exhibited by the solutions to nonlinear partial differential equations. Many of the most fundamental phenomena that now drive modern-day research, including solitons, chaos, stability, blow-up and singularity formation, asymptotic properties, etc., remained undetected or at best dimly perceived in the pre-computer era. The last sixty years has witnessed a remarkable blossoming in our understanding, due in large part to the insight offered by the availability of high performance computers coupled with great advances in the understanding and development of suitable numerical approximation schemes. In light of this, the current research addresses the topic of "Analytical and numerical study of some problems in nonlinear mechanics." Wave function collapse model.

### **Research questions:**

- What are the problems of nonlinear mechanics?
- What are the most important theories that address the problems of nonlinear mechanics and how are they analyzed?
- What is a wave function?
- What are the types of wave functions and their collapse?
- What is the normalization condition of the wave function?
- What is the wave function and its statistical meaning?
- Why the wave function?
- How does the wave function collapse?

### **Research objectives:**

- Knowledge of some nonlinear mechanics problems.
- Identify the most important theories that addressed the problems of nonlinear mechanics and how they were analyzed.
- Address the problem of wave function collapse analytically and numerically.
- Knowing the wave function, its types, and how it collapses.

# Wave Function:

It is called Psi and is symbolized by  $\Psi$  - it represents the state of a physical system, i.e. a physical system. It expresses everything we can know about the system before we make any observations with measuring devices, that is, there are no hidden variables. In the following lines, we will understand the meaning of this group of complex sentences. (Planck-Einstein, 2008, p. 1)

### **Illustrative equation:**

$$i\hbar \frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V(x,y,z)\psi$$

The wave function appears inside the famous Schrödinger equation, which reads the evolution of the system over time. The wave function is not the Schrödinger equation, but rather a solution to one of the Schrödinger equations to describe the system to be studied.

Do not let the complex form of the equation or the difficulty of the terms surrounding it, such as it being a partial differential linear wave equation or the word "function" itself, deter you from understanding more about it. Putting all that aside for now, it is, like other equations in the world of applied mathematics - nonlinear, with a clear goal, which is to determine where we can find the electron in - say - five minutes. It also gives us information about some other parameters, such as its potential energy and kinetic energy. It simply describes the state of the system in front of us, and here it is very similar to Newton's second law. (P.W. Atkins, 1974, p. 16).

$$F = m\ddot{a}$$

Let's understand some complex sentences. Let's assume that there are 10 balls on a billiard table. You will choose to hit the cue ball, according to Newton's law. All we need to learn in order to predict the future state of this system is some little preliminary information, such as the force and direction of the hit. Then we will let the equations It tells us about the future of this system - the billiard table and the movement of all the balls through successive collisions - and its development over time, as well as our wave equation. (Pieter-Kok, 2018, p. 11)

• A dance to the tunes of de Broglie

Schrödinger used Louis Victor de Broglie's idea as a basis for describing particles as waves. De Broglie says that when he read Einstein's research on the movement of light waves in the form of photons, he asked himself: Why not generalize that idea to everything in the universe? If those photons have a particle nature because they actually collided with metal plates, causing the photoelectric effect - and a wave nature - Because they cause doubleslit interference patterns – why wouldn't the whole world? Here DeBroglie formulated his famous equation 1924 years as follows:

$$\lambda = \frac{h}{p}$$

Where  $\lambda$  is the wavelength, h is Planck's constant, and p. is the momentum.

#### Mathematical equations:

Mathematical equations do not call for fear, on the contrary, it is a clearer picture of the world. We have avoided language errors, insults, nicknames and their multiple interpretations, all that happens is multiplication, division, addition and subtraction sometimes, as we can see.



Wavelength is an attribute of a wave, that is, a wave, it is simply the distance between two peaks of a wave that you make with your own hands in a dish of water. As for the quantity of motion, it is a quality of particles, it is the speed multiplied by the mass, both of which are qualities of particles. Don't you find that strange!, The equation of one end of which is wave and the other is particle. But if everything in the universe is dancing like a wave, Why don't we notice it (Rudolph, 2012, p. 476).

Simply put, because your mass is very large, while Planck's constant is an extremely small number, so the quotient of the previous equation – the wavelength – will be a very, very, very small number that cannot be observed, whereas if we were talking about the mass of an electron it can be clearly observed. Let's compare:

Planck constant:  $6.62607004 \times 10-34$  m2 kg /s, that is, a decimal point, then 34 zeros, then 6!

Electron mass: 9.10938356 × 10-31 Kg

Your mass: 60-80 kg

So, what distinguishes the wave function from seeing Newton's equations besides being a probabilistic description (Griffiths, 2008, p. 162).

As an illustrative example:

# **About Schrodinger's cat:**

According to the Copenhagen Interpretation, the wave function expresses the state of a physical system, complete, all possible possibilities together. Let's return to the double slit experiment, the Copenhagen interpretation holds that if we want to put one interpretation that allows the electron to pass through the two slits together, we must consider that the wave function is a real entity in some way, that is, it is not only a wave of mathematical probabilities that we use to describe the atomic world, but it can be represented inside the experiment when we do not observe. But this interpretation distances itself from the fact that it speaks of a "real world that actually exists inside the atom in which all the states of the particle exist together, " rather, it is a state that we can call quantum superposition (Dolling, Lisa M.; Gianelli, Arthur F.; Statile, Glenn N., eds., 2003, pp. 359-370).

Schrodinger did a thought experiment in an attempt to criticize the idea of generalizing the Copenhagen interpretation to the world as we see it, and said that this hypothesis represents a real contradiction because it will enter the real world even if it speaks of a strange, unreal entity that is difficult to understand, it is Schrodinger's cat that will help us understand the idea of the "complete system state".



We will put a cat in a closed box with a small amount of radioactive material, for example, it is 50% likely that an atom of it will decompose within a specified period, if it decomposes it will cause the Geiger counter to work, here the counter will release the hammer to fall on the poison bottle, it will spread in the place and the cat will die. And now you and I are standing in front of that box before we open it to ask: is the cat alive or dead.

Here the Copenhagen interpretation intervenes to answer: both states are alive and dead, because this atom has a wave function that describes it in all cases, that is, as decaying and non-decaying at the same time, that is what the concept of wave superposition means in the quantum state. Theoretically, it is assumed that an electron – or any other particle – exists in several quantum states at the same time. That's what allowed the electron to pass through both holes together in our first experiment.

What is that quantum state How to identify her, Is it recognizable, The explanation here answers that this is not possible at all; because attempts to observe it will cause the wave function to collapse, we will explain this a little later. Therefore, the Copenhagen interpretation is an epistemological – cognitive-interpretation of quantum mechanics, that is, it gives us some "knowledge" of a phenomenon, but does not necessarily speak of a "real" entity, as the word "real actually exists" – ontological – in our physical world. However, recent developments in quantum mechanics are taking a path that already shows that there may be a real entity. The difference between epistemology and ontology here is the difference between knowledge and what actually exists. Niels Bohr says:

There is no quantum world, there is only an abstract physical description. It is a mistake to think that the task of physics is to find out how the truth is. Physics is interested in what we can say about nature

- Measurement problem: when we open the box, we will not find any surprises, the cat is either alive or dead. This situation can be called the "wave function collapse", which is exactly what happened when we tried to introduce measuring devices for the double slit experiment where electrons were treated as particles and not as waves. The particle loses its quantum state-the state in which all states exist together – to an eginstate self – State-one specific state. That sudden jump from the quantum world to our natural world is not yet understood.

- Trying to understand why the wave function collapses pushes us to try to understand the meaning of the word "measurement". The Copenhagen interpretation states that during the observation, the physical system interacts with the measuring instruments, and here the wave function collapses to the observable state, and the results presented by classical measuring instruments are classical results, that is, they are described in ordinary language and we cannot do otherwise. As for the nature of the inner world of the atom, it cannot be recognized or monitored, because if we try, we will influence it with our tools.

- The subject of measurement presented several incomprehensible problems related to the nature of the universe in which we live. If we live in a world that is determined by our breaking a quantum state, does that mean that we are part of the composition of the world, It seems a simple but very cruel question on what words like "science" and "physics" mean. Physics treats the world objectively, so that this world is separate from our own, but the entry of the element of observation here threatens that perception with more subjective thoughts! (faye, 2008, p. 57).

#### As Heisenberg says:

"What we are seeing is not nature itself, but nature exposed to our way of exploring it, " he said.

Historians of science disagreed about the way it developed; but they all agreed that quantum mechanics is a real revolution in our understanding of the world. He has asked questions that are not accepted by us: how is an electron a wave and a particle together Can we imagine that situation with our current brains and cognitive abilities, Can the quantum interpretation be generalized to the entire universe, How?: What do we mean by " uncertainty?", And what is quantum entanglement (werner, 2000, p. 176).

Let's figure out what the wave function breakdown process is:

# - Wawacan Sulanjana wave function breakdown

The process by which a quantum system assumes a single final state due to interaction with the global exterior. This interaction is called " observation.(Werner, 1949, p.142)

In quantum mechanics, wave function collapse occurs when a wave function-initially in a superposition superposition of several eigenstates - reduces to a single eigenstate state due to interaction with the global exterior. This interaction is called "observation". It is the essence of a measurement in quantum mechanics that relates the wave function to classical scenes such as position and momentum. Collapse is one of two processes by which quantum systems evolve over time; the other is continuous evolution via the Schrodinger equation. Folding is a black square of irreversible thermodynamic interaction with a classical environment. Quantum decoherence calculations show that when a quantum system interacts with the environment, the superposition seems to shrink to a mixture of classical alternatives. Remarkably, the combined wave function of the system and the environment continues to obey the Schrodinger equation. And most importantly, this is not enough to explain the collapse of the wave function, since decoherence does not reduce it to a single eigenstate (Bombeli, 2018, p. 26).

In 1927, Werner Heisenberg used the idea of minimizing the wave function to explain it. Quantum measurement. However, if the collapse was a fundamental physical phenomenon, rather than just the apparent phenomenon of some other process, it means that nature was essentially random, that is, non-deterministic, an undesirable property of the theory

# Mathematical description:

The observables represent classical dynamical variables, and when one of them is measured by a classical Observer, the wave function falls on the random Eigen-state state of that observation. The Observer simultaneously measures the classical value of that which can be observed to be the eigenvalue of the final state.

# His mathematical background:

The quantum state of a physical system is described by a wave function (in turn - an element of the target of Hilbert space). This can be expressed as a vector using Dirac or bra - ket notation:

 $| \psi \rangle = \sum i c i | \phi I \rangle$ . {\displaystyle | \ psi \ rangle = \ sum \_ {i} c\_ {i} | \ phi \_ {i} \ rangle. } |\ psi \ rangle = \ sum\_i c\_i | \ phi\_i \ rangle.

Groups  $|\phi1\rangle$ ,  $|\phi2\rangle$ ,  $|\phi3\rangle \cdots$  {\displaystyle |\ PHI \_ {1} \ Rangle, |\ Phi \_ {2} \ Rangle, |\ PHI \_ {3} \ Rangle \ cdots} {\ displaystyle |\ PHI \_ {1} \ Rangle, |\ Phi \_ {2} \ Rangle, |\ PHI \_ {3} \ Rangle \ cdots}, select different quantum "alternatives" are available - the state of a certain quantity. They form the orthonormal basis of elementary vectors, formally

Where  $\delta$  ij {\ displaystyle \ delta \_ {ij}} \ delta \_ {ij} represents the kronker Delta.

An observable parameter (that is, a measurable parameter of the system) is associated with each subjective norm, with each quantitative alternative having a specific value or an eigenvalue, e i, of the observable. A "measurable system parameter" can be the usual position r and Momentum p. (for example) of a particle, but also its energy E and Z components of rotation (s z), orbital (L z) and total angular momentum (J z) etc. In the basic representation, These are, respectively,  $|P, T\rangle = |x, t\rangle + |y, t\rangle + |z, t\rangle$ , for  $|P, T\rangle = |p, X, t\rangle + |z, t\rangle$ RR, T $\rangle$  + | p. z, t $\rangle$  and | E $\rangle$ , and | s z $\rangle$ , L | z $\rangle$  and | J z $\rangle$ , the  $\cdots$  { displaystyle | mathbf {r}, t rangle = | x, t rangle + | y, t rangle +  $|z, t \setminus rangle, a | \setminus mathbf \{p\}, t \setminus rangle = |p_{x}, t \setminus rangle + |p_{x}|$  $\{y\}, t \setminus rangle + | p_{\{z\}}, t \setminus rangle, | E \setminus rangle and | s_{\{z\}} \setminus rangle, a$  $|L_{z} \setminus \text{rangle and } |J_{z} \setminus \text{rangle and } (\text{displaystyle })$ mathbf {r}, t \ rangle =  $| x, t \setminus rangle + | y, t \setminus rangle + | z, t \setminus rangle, a$ | mathbf {p}, t \ rangle =  $| p_{x}, t$  \ rangle +  $| p_{y}, t$  \ rangle + |p\_ {z} 't \ rangle '  $E \ rangle ' s_ {z} \ rangle ' L_ {z} \ rangle ' J_$  $\{z\} \setminus rangle \cdot \setminus cdots\}.$ 

Transactions c 1, c 2, c 3... Is the probability amplitude corresponding to each basis  $|\varphi_1\rangle$ ,  $|\varphi_2\rangle$ ,  $|\varphi_3\rangle$  ... {\displaystyle | PHI \_ {1} \ Rangle, | \ Phi \_ {2} \ Rangle, | \ PHI \_ {3} \ Rangle \ cdots} {\ displaystyle | \ PHI \_ {1} \ Rangle, | \ phi \_ {2} \ Rangle, | \ PHI \_ {3} \ Rangle cdots}. These are composite numbers. The standard square of c i, that is, | c i / = c i\*ci (\*denotes a complex conjugation), is the probability of measuring the system to be in the case  $|\varphi_i\rangle$  {\ displaystyle | \ phi \_ {i} \ rangle} | \ phi \_ {1} \ rangle. To simplify in the following, all wave functions are assumed to be normal ; the total probability of measuring all possible States is one:  $\psi \psi / \psi \psi = \sum i / c i / 2 = 1$ . {\ displaystyle \ langle \ psi | \ psi \ rangle = \ sum \_ {i} | c \_ {i} | ^ {2} = 1.}

### The process of collapse:

By these definitions, it is easy to characterize the breakdown process. For any observable function, the wave function is initially a linear combination of eigenbasis {/ $\phi$  i}} {\displaystyle \{/ phi\_{i} {i} \\rangle \}} \{/ phi\_{i} {I} \\rangle \} from this observable. When an external agency (Observer, experimenter) measures what can be observed associated with eigenbasis {|  $\phi$  I} {\displaystyle \{| phi\_{i} {i} \\rangle \}} \{| phi\_{i} {i} \\rangle {I}

 $| \psi \rangle \rightarrow | \phi I \rangle$ . {\ displaystyle | \ psi \ rangle \ rightarrow | \ phi \_ {i} \ rangle. } |\ psi \ rangle \ rightarrow | \ phi\_i \ rangle.

The probability of collapse to a certain eigenstate /  $\varphi$  k) {displaystyle / phi \_ {k} rangle} / \ phi \_ {k} \ rangle is the probability of birth, p. K = / C K | 2 {\ displaystyle P\_ {k} = / c\_ {K} / ^ {2}} P\_k = / c\_k / ^ 2. Immediately after the measurement, the other elements of the vector of the wave function, c i  $\in$  k {\ displaystyle c\_ {i \ neq k}} c\_ {I \ neq k}, "collapsed" to zero, and / c i / 2 = 1 {\ displaystyle | c\_ {i} | ^ {2} = 1} {\ displaystyle | c\_ {i} | ^ {2} = 1}.(Francisco, 2014, p156). In general, the collapse is defined for the factor Q ^ {\ displaystyle {\ hat {Q}}} {\ hat {Q}} with eigenbasis {/ $\varphi$  I} {displaystyle {/phi\_{I} rangle}} \ {| \ phi\_{I} \ rangle \}. If the system is in the state |  $\psi$  {\ displaystyle | \ psi \ rangle} | \ psi \ rangle and Q ^ {\ displaystyle {\ hat {Q}}} {\ hat {Q}} the probability of system collapse is measured to eigenstate |  $\varphi$  I} {displaystyle | phi\_{I} \ rangle (and measure the value of eigenvalue Qi {\ displaystyle q\_{I}} q\_{I} of |  $\varphi$  I} {\ displaystyle |\ Phi\_{I} \ Rangle {\ lambda e of Q ^ {\ displaystyle q\_{I}} q\_{I} of |  $\varphi$  I} {\ displaystyle {\ hat {Q}}} {\ hat {Q}}} the probability of system collapse is measured to eigenstate |  $\varphi$  I} {\ displaystyle |\ Phi\_{I} \ Rangle {\ lambda e of Q ^ {\ displaystyle q\_{I}} q\_{I} of |  $\varphi$  I} {\ displaystyle |\ Phi\_{I} \ Rangle {\ lambda e of Q ^ {\ displaystyle {\ hat {Q}}} {\ hat {Q}} {\ hat {Q}}} {\ hat {Q}} {\ hat {Q}} {\ lambda e of Poi \ {\ displaystyle |\ Phi\_{I} \ Rangle |^ {2} {\ lambda e of Poi |\ Phi\_{I} \ Rangle |^ {2} {\ lambda e of Poi |\ Phi\_{I} \ rangle; it is in the case of |  $\varphi$  I} {\ displaystyle |\ pi\_{I} \ rangle {\ lambda e of Poi \ {\ displaystyle |\ psi \ rangle {\ lambda e of Poi \ {\ displaystyle |\ Phi\_{I} \ Rangle {\ lambda e of Poi \ {\ lambda e of Poi \ lambda e of Poi \ {\ lambda e of Poi \ lambda e of Poi \ {\ lambda e of Poi \ lambda e of Poi \ {\ lambda e of Poi \ lambda e of Poi \ lambda e of Poi \ {\ lambda e of Poi \ lambda e of Poi \ {\ lambda e of Poi \ lamb

However, we never observe the collapse to the singular eigenstate of the continuous spectrum operator (e.g. position, momentum, or Hamiltonian dispersion), because such eigenfunctions are not normalizable. In these cases, the wave function will partially collapse into a linear set of "nearby" eigenstates (necessarily including a spread in the values of the eigenvalues) embodying the inaccuracy of the measuring device. The more accurate the measurement, the narrower the range. The probability calculation proceeds identically, except for integration on the expansion coefficient c (q, t) d q {\ displaystyle c (q, t) dq} c (q, t) dq. This phenomenon is not related to the uncertainty principle, although increasingly accurate measurements of one operator (such as position) will naturally homogenize the expansion coefficient of the wave function with respect to another incompatible factor (for example momentum), reducing the probability of measuring any given value of the latter.

#### History and context:

The concept of wave function collapse was introduced by Werner Heisenberg in his 1927 paper on the uncertainty principle, "Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik", and incorporated into the mathematical formula of quantum mechanics by John von Neumann, in his 1932 dissertation Mathematicche Grundlagen der quantenmechanik. Heisenberg did not try to determine what exactly the collapse of the wave function means. However, he stressed that it should not be understood as a physical process. Niels Bohr also repeatedly warned that we should abandon the "pictorial representation". The founders of the Copenhagen interpretation preferred to emphasize the mathematical formalism of what was happening.

Consistent with Heisenberg, von Neumann assumed that there are two processes for changing the wave function:

- Probabilistic, non - unitary, non-local, discontinuous change caused by observation and measurement, as described above.

- Deterministic, unitary, continuous time evolution of an isolated system subject to the Schrodinger equation (or relativistic equation, that is, the Dirac equation).

In general, quantum systems exist in a superposition of those rules that are prescribed to closely correspond to classical descriptions and, in the absence of measurement, develop according to the Schrodinger equation. However, when performing the measurement, the wave function collapses - from the Observer's perspective-to only one of the ground states, the property being measured uniquely acquires the eigenvalue of that particular state,  $\lambda I$  {\ displaystyle \ lambda \_ {i}} \ lambda \_ {I}. After the collapse, the system develops again according to the Schrodinger equation.

By explicitly dealing with the interaction of an object and a measuring instrument, von Neumann tried to create consistency between the two processes of changing the wave function.

He was able to prove the possibility of a quantum-mechanical measurement scheme corresponding to the collapse of the wave function. But he did not prove the necessity of such a breakdown. Although von Neumann's postulates are often presented as a standard description of quantification, they were conceived by taking into account the experimental evidence available during the Thirties of the last century (in particular, the Compton-Simon experiment was typical), but many important current measurement procedures do not fulfill them (the so-called Type II measurements) (Bombel, 2010, p. 47).

# The Copenhagen interpretation:

The set of phenomena descr ibed by the expression of the wave collapse function represents a fundamental problem in the interpretation of quantum mechanics, it is known as the measurement problem. This problem has been deviated by the Copenhagen interpretation, which assumes that this is a special feature of the "measurement" process. Everett's interpretation of the many worlds deals with it by getting rid of the process of collapse, thereby reformulating the relationship between the measuring device and the system in such a way that the linear laws of quantum mechanics are universally valid, that is, the only process according to which a quantum system develops is governed by the Schrodinger equation or its relativistic equivalent.

It originated from the de Broglie-Bohm theory, but is no longer associated with it, the physical process of decoherence, which causes a pronounced breakdown. Decoupling is also important for interpreting fixed dates. A general description of the evolution of quantum mechanical systems can be given using density operators and quantum operations. In this formalism (which is closely related to the C \* -algebraic formalism), the collapse of the wave function corresponds to a non-unitary quantum process.

The importance attributed to the wave function varies from interpretation to interpretation, and even varies within an interpretation (such as the Copenhagen interpretation). If the wave function only encodes the Observer's knowledge of the universe, then the collapse of the wave function corresponds to the receipt of new information. This is somewhat similar to the situation in classical physics, except that the classical "wave function" is not necessarily subject to the wave equation. If the wave function is physically real, in some sense and to some extent, then the collapse of the wave function is also perceived as a real process, to the same degree (Wimmel, 2011, p. 45).

# Form the LaGrange equations to explain the wave function collapse phenomenon:

Let's consider a single particle with mass m and a ray of position R. A Force F is applied to it, then we can express this system with a particle moving in a potential well, so it has kinetic energy and also a potential energy. We assume that the voltage acting on the particle (v, r, t) is a function that depends on time t and Space r (like the voltage of the nucleus of an atom acting on an electron orbiting around it):

 $\mathbf{F} = abla V.$ 

Such a force is independent of the third derivative or higher-order derivatives of the ray of position r, so Newton's second law forms a set of three systemic differential equations of the second order.

Accordingly, the motion of this particle can be described in terms of independent variables or so-called "degrees of freedom". These degrees of freedom are a combination of six variables:

{ rj, r'j | j = 1, 2, 3}

The Cartesian compounds of the position Vector r and its time derivatives (derivatives with respect to time), at a given moment in time that is, the position (x, y, z) and the velocity with their three Cartesian components:

(vx, vy, vz). (penrose, 2007, p. 474)

More generally, we can work within a generalized set of coordinates, qj, with their time derivatives, or so-called generalized velocities, Q'j.

The position Vector r is associated with the generalized coordinates by means of a set of transformation equations

.  $\mathbf{r} = \mathbf{r}(q_i, q_j, q_k, t)$ 

For example, when dealing with a simple NOAs of length l, the logical choice of generalized coordinates is the angle that the NOAs makes with its vertical line,  $\theta$ .

The conversion equations are:

 $\mathbf{r}(\theta, \theta', t) = (l\sin\theta, l\cos\theta)$ 

The term generalized coordinates is one of the remnants of the period of using Cartesian coordinates as a virtual coordinate system. Let's consider the nominal displacement of the body  $\delta r$ , so the work done by The Force F is:

# $\delta W = F \bullet \delta r.$

# Using Newton's second law we can write:

$$\mathbf{F} \cdot \mathbf{\delta} \mathbf{r} = \mathbf{m} \mathbf{r} \, " \cdot \mathbf{\delta} \mathbf{r}$$

Since work is a standard physical quantity (a quantity, not a vector), we can rewrite these equations in terms of generalized coordinates and velocities on the left side (Hann-hubert, 2016, p. 540).

$$\mathbf{F}\cdot\delta\mathbf{r} ~=~ -
abla V\cdot\sum_irac{\partial\mathbf{r}}{\partial q_i}\delta q_i$$

$$= -\sum_{i,j}rac{\partial V}{\partial r_j}rac{\partial r_j}{\partial q_i}\delta q_i$$

$$= -\sum_i \frac{\partial V}{\partial q_i} \delta q_i.$$

The process of formatting the right side is more difficult, but after arranging and switching:

$$m \mathbf{r}'' \cdot \delta \mathbf{r} = \sum_i \left[ rac{d}{dt} rac{\partial T}{\partial q'_i} - rac{\partial T}{\partial q_i} 
ight] \delta q_i$$

Where is the kinetic energy of the particle T= 1/2 m r2 and the equation of the work done becomes:

$$\sum_{i} \left[ \frac{d}{dt} \frac{\partial T}{\partial q'_{i}} - \frac{\partial (T-V)}{\partial q_{i}} \right] \delta q_{i} = 0$$

In any case, this must be true for any set of generalized oqi displacements, so we have:

$$\left[rac{d}{dt}rac{\partial T}{\partial q_i'}-rac{\partial (T-V)}{\partial q_i}
ight]=0$$

For any of the generalized Oqi coordinates

We can simplify this equation by observing V that it is a dependency of r and T and the position Vector, R, is also a dependency of the generalized coordinates and Time t, so the potential energy v is independent of the generalized velocities

$$\frac{d}{dt}\frac{\partial V}{\partial q'_i} = 0.$$

By entering this previous equation and substituting v-t=L you get the LaGrange equations

$$\frac{\partial L}{\partial q_i} = \frac{d}{dt} \frac{\partial L}{\partial q'_i}.$$

There is always a single LaGrange equation for every generalized coordinate qi. And when qi = ri (that is, the generalized coordinates are simply Cartesian coordinates), then we can easily reduce LaGrange's equation to Newton's second law (feather stone-roy, 2014, p. 387).

The above derivation can be generalized to a system (sentence) composed of N particles. Then there are 6N generalized coordinates that are related to the position coordinates by means of the 3N triple transformation equations. In LaGrange's 3N equations, T is always the total kinetic energy of the whole, and V is the total potential energy.

In practice it is easier to solve the problem withusing the Euler-LaGrange equation instead of Newton's laws. That's because the generalized qi coordinates can be chosen to fit the symmetries of the system (Castell, 2003, p.14).

# Let's get into this famous paradox:

The EPR paradox: the Einstein-Podolsky-Rosen paradox (EPR paradox) is a thought experiment proposed by physicists Albert Einstein, Boris Podolsky and Nathan Rosen and interpreted as indicating that the interpretation of physical reality provided by quantum mechanics was incomplete in an article published in 1935 entitled "Can quantum mechanics' description of physical reality be considered complete?"", And they tried to prove mathematically that the wave function does not include complete information about unwelcome physical reality, as well as the Copenhagen interpretation. The work was completed at the Institute for Advanced

Study at Princeton University in 1934, which Einstein had joined the previous year after fleeing Nazi Germany. The essence of the paradox is that particles can interact in such a way that both their location and moment can be determined more accurately than by Heisenberg's uncertainty principle, unless the determination of one particle immediately affects the other to prevent such accuracy and which would ensure the transmission of information faster than light. This result had not been observed previously and seems to have been implausible at the time as the phenomenon is now known as quantum entanglement.

About the history of the developments of the paradox (EPR): the article that first put forward the developments of this paradox was published in 1935 entitled "Can the quantum mechanical description of physical reality be considered complete". This article was published in the newspaper by Bohr and then a discussion ensued between Bohr and Einstein about the fundamental nature of reality. Einstein had his own doubts about Heisenberg's uncertainty principle and the role of probability in quantum theory. The essence of this discussion was not about probability (chance), but something deeper, where Einstein's opinion was as follows: "is there an objective physical reality that every observer sees from his own point of view" Bohr's opinion was manifested by responding to Einstein where he said: "Does the Observer participate in the creation of actual reality through the questions he asks with experiments". (yuan -ping feng, 2005, p.164).

Einstein struggled until the end of his life for a theory that could better comply with his idea of causality, protesting against the view that there is no objective physical reality other than what is revealed by measurement interpreted in terms of quantum mechanical formalism. However, experiments similar to those described in the text of the paradox have been conducted since Einstein's death starting in 1976 by French scientists at the Saclay Nuclear Research Center. These experiments seem to show that the idea of local realism is wrong. About quantum mechanics and its interpretation: quantum theory since the early twentieth century has successfully demonstrated the description of the actual accuracy of the physical reality of the microscopic and microscopic world in multiple reproducible physical experiments. The development of quantum mechanics was aimed at describing atoms and explaining the spectral lines observed in the measuring device. Philosophical explanations of the quantum phenomenon seem to be another matter, questions about how the mathematical formulation of quantum mechanics is interpreted have been asked a variety of different answers from different people in the philosophical point of view.

Einstein's opposition: Einstein was the most prominent opponent of the Copenhagen interpretation. In his opinion, quantum mechanics was incomplete. Commenting on this, other writers such as John von Neumann and David Bohm hypothesized that there must be hidden variables responsible for the results of random measurement, something that was not explicitly claimed in the original text. The EPR paradox has enabled the philosophical debate to turn towards a materialistic argument. The authors claim that in the light of a specific experiment in which the measurement result is known before the measurement is made, and there must be something in the real world "element of reality" that determines the measurement result; since they assume that these elements of reality are local in the sense that each element belongs to a certain point in space-time. Each element may be affected only by events that fall into the backlight cone of the point in spacetime (i.e., the past). These claims are based on assumptions about nature and constitute what is now known as local realism (Panavotis, 1996, p. 45).

The original EPR paradox challenges the prediction of quantum mechanics that it is impossible to know both the position and the moment of a quantum particle. This challenge can be extended to other pairs of physical properties.

The original text is intended to describe what should happen to the first and second systems, with which we allow interaction, and after some time we assume that there is no longer any interaction between the two parts. Manjit Kumar in 2009 gave examples that include the description of the paradox where he said "there are two particles (A and B) that interact briefly and then move in opposite directions". According to the principle of indeterminism, which says that it is impossible to measure the exact moment and position of particle B. However, it is possible to measure the exact position of particle A through calculation and thus with the exact position of particle A known, the exact position of particle B can be known. Alternatively, the exact moment of particle A can be measured so that the exact moment of particle B can be derived. "The text of the paradox argues that it may prove that particle B can have exact values with respect to position and momentum, and that particle B has a real position and a real moment", Kumar wrote.

We perform measurements on an entangled state: suppose we have a source emitting electron-positron pairs with the electron being sent to destination A where there is an observer named Alice, and the positron is sent to destination B where there is an observer named Bob. According to quantum mechanics, we can arrange our source so that each pair of sources occupies a quantum state called Spin Singlet, and therefore the two particles are said to be entangled. This can be considered as a quantitative correspondence of two states, and we call them: the first case and the second case. In the first case, the electron rotates upward along the axis of rotation (-z). In the second case, the electron rotates on the axis (-z) and the positron is on the axis (+z). Due to the entanglement state of the two particles, it is impossible to know the specific spin state of any particle in a single spin without measurement (Bohr, 1961, p. 43).

The method of solving the paradox: there are several ways to solve the EPR paradox. The model proposed by the text is that, despite the success of quantum mechanics in a wide range of experimental scenarios, it is actually an incomplete theory. In other words, she did not discover the whole theory of nature where quantum mechanics acts as a kind of statistical approximation (albeit a very, very successful one). Unlike quantum mechanics, the most complete theory contains variables corresponding to all elements of reality. There must be an unknown mechanism acting on these variables to trigger the observed effects of non-predictive quantum observations" i.e. Heisenberg's indeterminism principle. This theory is called the theory of the hidden variable.

The meaning of Einstein's explanation is in a purely algebraic theory: Bohm's interpretation of quantum mechanics assumes that the state of the universe evolves smoothly through time without quantum sound waves breaking down. One of the problems of the Copenhagen interpretation is to accurately determine the wave breakdown. Einstein asserted that quantum mechanics is physically incomplete and logically unsatisfactory. In Einstein's book " The Meaning of relativity "he wrote:" one can give good reasons for a reality that absolutely cannot be represented by a continuous context. It would seem that starting from the quantum phenomenon can be traced the inevitability of the existence of a finite system of finite energy that can be perfectly described by a finite set of numbers (quantum numbers), and this does not seem to happen according to the theory of communication and should lead to an attempt to find a purely algebraic theory in order to represent reality. No one knows how to find a basis for this theory" (wolchover, 2018, p. 123).

If time, space and energy are the secondary attributes derived from the Planck scale it means that Einstein's hypothetical algebraic system has solved the EPR paradox. If physical reality is strictly limited, then the Copenhagen Interpretation may be an approximation of the information processing system of the Planck scale (H. P. Stapp, 1981, p18).

The field of laboratory experiments: in 1964, "John Stuart Bell" produced a theory that allows to determine the contents of the EPR paradox, opening the way for experimentation: thus the solution of the EPR paradox can become an experimental question, and not an cognitive option. The technology at that time did not allow the possibility of conducting an experiment to test the "Bell" inequality, but another laboratory, Alain Aspet, was able to implement it in 1981, and then in 1982, at the Institute of optics in Orsay,

confirming the correctness of the predictions of quantum mechanics in the case of the EPR paradox.

This experiment was initiated by "aspeh" from the idea of its publication as early as 1976 but no one has adopted it since then. For this he was awarded the Nobel Prize in Physics in 2022.

The 2022 Nobel Prize in Physics was awarded to three scientists-Alain Aspet, France, John Klauser, USA, and Anton Zeilinger, Austria; for their success "for conducting experiments on entangled photons, proving the violation of Bell's inequality and pioneering in Quantum Information Science" (Roland, 2011, p. 16).

In 1988-1989, other experiments allowed (Maryland, Rochester) more complicated, by testing quantum entanglement at very long distances - and avoiding the small experimental flaws that Orsay's experiments left open.

However, if these experiments indicate that we renounce one of the three hypotheses (we decided that quantum mechanics becomes a non-local physical theory, quantum mechanics becomes a "non-local physical theory"), then in no case does it allow faster-than-light signal transmission (otherwise "causality" or "relativity" would be violated.)



Diagram of the EPR half-paradox experiment: two photons are emitted by the atom before passing through the polarizer, the polarization states of the two photons are indeterminate: these two states are characterized at best by the entanglement of two polarization states horizontally and vertically. However, assuming that the total angular momentum of the system is zero, the conservation of the latter requires that the two photons have opposite polarizations.in the EPR paradox, this may mean that the polarization of the second photon is known even without measuring it. the quantum entanglement of the two photons may explain this phenomenon.

- 1. any system at the atomic level is described by the wave function  $\theta$ .
- 2. the nature of the system at the atomic level is based entirely on statistics and probabilities.
- 3. the principle of suspiciousness provides for the inability to know all the properties of a system at once. Even if we know information about a particular variable, it is probabilistic.
- 4. the principle of completeness States the dual nature of particles, an atomic particle can either appear as a particle or a wave, but it cannot appear on both bodies simultaneously.
- 5. the corresponding principle states that there is a general correspondence between the predictions of quantum mechanics and classical mechanics in cases that are described by large quantum numbers, that is, classical mechanics is a special case of quantum mechanics.(jean-Marcus,

Based on this, we can interpret the twofold experiment as follows: when a single electron emits from its source, we can imagine it and it came out in the form of a wave, when it enters from both slits it interferes with itself, and if all the single electrons interfere with themselves, the figure is produced (4-5). But if we put a detector at the two holes, the photon will hit the electron, which leads to changing its first (wave) state to the second (particle) state, and the electron hits the screen as a particle, hence figure (2-5) shows.



Two-pronged barrier



The summary of what the Copenhagen School tells us, is that there is no particular case before the measurement process. Measurement is everything in experience. If we imagine that we have a box in which there is an electron (a system), and this system is described by the function  $\theta$ , then this electron does not have a specific  $\theta$  before the measurement process, and therefore it does not have a position, Momentum, Energy, or any other variable before the measurement process. Rather, all that can be imagined is the existence of a commutative state of the function  $\theta$ , and only this commutativity disappears when measured or observed.

In fact, the Copenhagen interpretation introduces us to many problems on a philosophical level. If there is a direct relationship between the Observer and the observed, this will introduce us into the problem of the relationship of consciousness with existence. Reality is created only when we feel it and conjure it up in our mind although this statement is not valid for large bodies, as quantum effects are neglected, in the end large bodies are nothing but electrons, protons and quarks.

# The normalization state of the wave function: the wave function and its statistical meaning:

The diffraction pattern observed for microparticles is characterized by an uneven distribution of microparticle fluxes in different directions - there is a minimum and a maximum in other directions. The presence of a maximum in the diffraction pattern means that the de Broglie waves of higher intensity are distributed in these directions. The intensity will reach a maximum if the maximum number of particles propagates in this direction. Those. The diffraction pattern of microparticles is a manifestation of the statistical (probabilistic) pattern in the distribution of particles: where the intensity of the de Broglie wave is at a maximum, there are more particles (G-Mosca, 2008, p. 89).

De Broglie waves in quantum mechanics are considered as probability wavesthose. The probability of detecting a particle at various points in space changes according to the wave law. But for some points in space, this probability will be negative (that is, the particle does not fall into this region). M. proposed. Born (German physicist) stated that it is not the probability itself that changes according to the wave law, but the amplitude of the probability, which is also called the wave function.

The wave function is a function of coordinates and time.

The square of the coefficient of the psi function determines the probability that the particle will be detected inside the volumedv - the physical meaning is not the PSI function itself, but the square of its coefficient.

 $\Theta$  \* is a complex conjugate function of  $\theta$ 

(Z = A + B, D \* = A - B, D \* - cocci are complex)

If the particle is in a finite volume V, then the ability to detect it in this volume is 1, (a reliable event)

D=1

In quantum mechanics, it is assumed that  $\Psi$  and A, where A = a constant quantity, describe the same state of a particle. So the normalization condition, means that it is calculated on an infinite volume (space).

# - The function should be

- 1- final (since Sala can be more than 1).
- 2- unambiguous (it is impossible to detect a particle under constant conditions with a probability of 0.01 and 0.9, since the probability must be unambiguous).
- 3- continuous (follows from the continuity of space. There is always the probability of finding a particle at different points in space, but at different points it will be different).

The wave function satisfies the principle superposition: if a system can be in different states described by wave functions 1, 2..., then it can be in the state described by linear combinations of these functions:

With N (N=1, 2...) no numbers.

The wave function is used to calculate the average values for any physical quantity of a particle.

#### The Schrodinger equation:

like other fundamental equations of physics (Newton's, Maxwell's equations), it is not derived, but assumed. It should be considered the initial fundamental assumption, the correctness of which is proved by the fact that all the consequences of it fully correspond to the experimental data.

The problem of the temporary Schrodinger equation (dart-LaPlace operator)

The potential function of a particle in a force field is the desired function:

If the force field in which the particle moves is constant (that is, it does not change over time), then the function Yola depends on time and has the meaning of potential energy. In this case, the solution of the Schrodinger equation (for example,  $\theta$  is a function) can be represented as the product of two factors - one depends only on the coordinates, the other depends only on time —

Form the Schrodinger equation for stationary States: there are an infinite number of solutions. By imposing boundary conditions, solutions are selected that have a material meaning (towler, 2009, p. 145).

Boundary conditions: the wave functions must be normal, in other words.

1- Final.

2- it is unambiguous.

3- continuous.

Solutions that satisfy the Schrodinger equation are called kingfunctions, and the corresponding energy values are called eigenvalues of eigenvalues is called the range of quantities. If E we take discrete values, then the spectrum - separateif it is continuous - solid or continuous (David-Griffiths, 2011, p. 47).

The wave function, or the function (psi  $\theta$  displaystyle psi) is a complex-valued function used in quantum mechanics to describe the pure state of a system. It is the coefficient of expansion of the state vector in the basis (usually coordinate):

 $\langle \theta(t) \rangle = \int \theta(X, T) | X \rangle$  D x (displaystyle left | psi (t) right range = int Psi (x, t) left | x right range dx)

Where  $|x\rangle = |X1, X2, ..., xn\rangle$  (displaystyle left | x right rangle = left | x\_ (1), x\_ (2), ldots, x\_ (n) right rangle) is the coordinate - based Vector, and  $\psi$  (X, T) =  $\langle x | \theta$  (t) $\rangle$  (displaystyle Psi (x, t) = lang x left | psi (t) right rangle) is the wave function in representation of coordinates.

A normalization of the wave function: the wave function  $\theta$  (displaystyle Psi) in its meaning, must satisfy the so-called normalization condition, for example, in the coordinate representation containing the form:

 $\int V \theta * \theta D K = 1 \text{ (display style (int limits _ (V) (Psi ^ (ast) Psi) dV)} = 1)$ 

This condition is expressed by the fact that the probability of finding a particle with a certain wave function anywhere in space is equal to unity. In the general case, integration should be carried out on all the variables on which the wave function depends in this representation.

The principle of superposition of quantum states

For wave functions, the superposition principle is valid, which states that if a system can be in states described by wave functions  $\theta 1$  (\ displaystyle \ Psi \_ (1)) and  $\theta 2$  (\ displaystyle \ Psi \_ (2)), then it can also be in a state described by a wave function

 $\Theta \Sigma = P1 \ \theta 1 + P2 \ \theta 2$  (displaystyle Psi \_ (Sigma) = c\_ (1) Psi \_ (1) + c\_ (2) Psi \_ (2)) for any complex P1 (displaystyle c\_ (1)) and P2 (displaystyle c\_ (2)) (Wimmel-Hermann, 2020, p. 47).

Obviously, we can talk about the superposition (addition) of any number of quantum states, that is, about the existence of a quantum state of the system, which is described by the wave function  $\theta \Sigma = c$ 

 $1 \ \theta \ 1 + c \ 2 \ \theta \ 2 + ... + c \ N \ \Psi \ N = \sum n = 1 \ N \ cn \ \Psi \ n \ (displaystyle \ Psi \ (Sigma) = c \ (1) \ Psi \ (1) + c \ (2) \ V \ Psi \ (2) + \ ldots + (c) \ (N) \ (V \ Psi) \ (N) = \ sum \ (n = 1)^{(N)} \ (N) \ (c) \ (n) \ (V \ Psi) \ (n)).$ 

In this case, the square of the coefficient of the coefficient C n (displaystyle (c) \_ (n))determines the probability of detection of the system in the state described by the wave function during measurement  $\theta$  n (displaystyle (Psi) \_ (n)).

So, for the measured wave functions  $\sum N = 1$  n | c n | 2 = 1 (displaystyle sum \_ (n = 1) ^ (N) left | c\_ (n) right | ^ (2) = 1). Regularity conditions of the wave function

The probabilistic meaning of the wave function imposes certain restrictions or conditions on wave functions in quantum mechanical problems. These standard conditions are often referred to as wave function regularity conditions (Dipankar-Home, 1997, p. 169).

The wave function in different representations uses States in different representations - it will match the expression of the same vector in different coordinate systems. The rest of the operations with wave functions will also have analogues in the vector language. Wave mechanics uses a representation in which the arguments of the PSI function are full-system continuous transpositions of observations, and The Matrix uses a representation in which the arguments of the PSI function are full-system discrete transpositions that can be observed. Therefore, it is obvious that the functional (wave) and Matrix formulations are mathematically equivalent.

Particle-wave duality in quantum physics describes the state of a particle using the wave function

 $(\$ \ psi(\ overrightarrow(r), t)\$ - psi-function).$ 

# **Conclusion:**

Through this study, it turned out that the problem of wave function collapse is one of the measurement problems in quantum mechanics, and we have proved that it can be solved based on quantum mechanics and does not require any additional assumptions or new theories, and the processes of particle creation and annihilation described based on quantum field theory play a major role in measurement processes.

The principle of superposition is invalid for the system of equations of quantum field theory of particles and fields; this is because this system is nonlinear, and as a result of the formation of the annihilation of the particle an additional uncertainty arises, which stains the interference pattern, and the imposition of such a large number of uncertainties in repeated measurements leads to the classical behavior of particles.

Decoherence theory also refers to the creation and annihilation of particles and these processes are a nonlinear consequence of quantum mechanics, in this case the term wave function collapse becomes a consequence of other terms of quantum mechanics rather than a separate assumption of quantum mechanics.

Moreover the implicit wave function of nonlinear quantum mechanics can be determined by solving a nonlinear wave function of a nonlinear partial differential equation, which is also realized by nonlinear quantum mechanics.

#### Sources:

- 1. Bohr, Niels, Atomic theory and description of nature, Cambridge University Press, 1961.
- 2. Bombel, Wave-Function Collapse in Quantum Mechanics, 2010.
- 3. Bombeli, wave function collapse in quantum mechanics, springer, 2018.
- 4. Castell- Lutz, Time Quantum and Information, Springer, 2003.
- 5. David- Griffiths, Introduction to elementary particles. Wiley-VCH, 2011.
- 6. Detlef -Dürr, understanding Quantum Mechanics: The World According to Modern Quantum, springer, 2020.
- 7. Dipankar- Home, Conceptual Foundations of Quantum Physics: An Overview from Modern Perspectives, Springer US, 1997.
- 8. Dolling, Lisa M., Gianelli, Arthur F., Statile, Glenn N., eds., "Introduction". The Tests of Time: Readings in the Development of Physical Theory. Princeton University Press, 2003.
- 9. Faye, Copenhagen Interpretation of Quantum Mechanics, Stanford university, 2008.
- 10.Featherstone-Roy, ridig body dynamics Algorithms, springer, 2014.
- 11.Francesco, on unitary evolution and collapse in quantum mechanics, 2014.
- 12.G- Mosca, Physics for Scientists and Engineers with Modern Physics, Freeman, 2008.
- 13.Griffiths- David, Introduction to elementary particles, California university, 2008.
- 14.H. P. Stapp, Mind Matter. and Quantum mechanics, Henry P. Stapp, 1981.
- 15.Hann-hubert, ridig body dynamics of mechanics, springer, 2016.
- 16.Jean-Marcus Shwindt, Conceptual Basis of Quantum Mechanics, Springer, 1671.
- 17.P.W. Atkins, A handbook of concepts, Oxford University Press, 1974.
- 18. Panavotis- G. kevrekidis, Emeraina frontiers in nonlinear science, University of Athens, 1996.

- 19. Penrose, vintage books, springer, 2007.
- 20.Pieter-Kok, A First Introduction to Quantum Physics, Springer, 2018.
- 21.Planck-Einstein, Representations and Wave Functions, California university, 2008.
- 22. Roland- Amis, Quantum Philosophy, Jeremy Butterfield, 2011.
- 23.Rudolph, on the reality of quantum state, 2012.
- 24.Towler, the foundation of quantum mechanics and wave thory, University of Cambridge, 2009.
- 25.Werner Heisenberg, Physics and Philosophy, Penguin Classics, 2000.
- 26.Werner- Heisenberg, Physical Principles of Quantum Theory, university of Chicago, 1949.
- 27.Wimmel- Hermann, Quantum physics & observed reality, World Scientific, 2011.
- 28.wimmel-hermann, critical interpretation of quantum mechanics, World scientific, 2020.
- 29. Wolchover, famous experiment dooms, quanta magazine, 2018.
- 30. Yuan-ping Feng, Quantum mechanics in nonlinear systems, 2005.