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## **Performance of phase Coded Radar for Detection of Stealth Targets**

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### **Abstract:**

The stealth technology has been made to reduce the radar cross section (RCS) of the target to a level where the echo reflected from it can not be detected by the radar receiver. There are many techniques to reduce the RCS, the most useful ones are shaping of the target and coating it with absorbing materials. Coats of absorbing material over metallic surfaces, can substantially reduce the energy of a returned signal. In this paper we evaluate the performance of two classes of signals to deal with stealth point targets, namely the nonsinusoidal radar signals and pulse compression signals. The target response depends on, time delay of absorbing material, waveform of radar transmitted signal and coding pulses. Matched filter are used for detection of various target responses. The autocorrelation function, represents a delayed version of of the matched filter output, is determined analytically for various types of the transmitted signals. It is shown that the proper choice of the transmitted signal duration relative to the absorbing material time delay can improve the stealth target detection process. In case of fast rate phase coded signals the autocorrelation functions for stealth targets are similar to those of conventional point targets.

### **I- Introduction:**

The challenge mission in radar system design has been the detection of target with small radar cross section known as stealth target [1]. The aim of stealth technology is to decrease the capability of the radar receiver to detect reflected signal by reduction the RCS of the target. The power of the reflected signal from a target is linearly proportional to its RCS when the other parameter are kept constant [2]. There are many techniques to reduce the power of the reflected signal from the target, but the most useful ones are the shaping and coating of the target with absorbing material [3]. The aim of stealth target shaping is to reflect the incident waveform toward-away the line-of-sight while the absorbing material is to convert the incident energy into heat to reduce the reflected energy [4]. The required thickness of the absorbing material depends on the operating frequency band. As illustration for radars operating on 35 GHz, there are absorbers of thickness 1 cm to attenuate the reflected power of the incident wave to about 1%, while at 94 GHz the thickness is only 3 mm. For power reflection of 0.1%, the thickness becomes 5 cm at 35 GHz and 1.6 cm at 94 GHz [5]. Therefore radars should not be designed to work at these high frequencies if the radar deals with targets covered with absorbing materials. In this paper we evaluate the performance of two classes of signals to deal with stealth point targets, namely the nonsinusoidal radar signals and pulse compression signals. The results show that short pulses could significantly reduce the effect of stealth coating materials on the detection performance. This is conditioned by the pulse width is smaller than the delay time of the coating materials. However these too short pulse width reduces the velocity resolution and accuracy of the radar system. For this reason phase coded radar with fast phase variation is an optimization for both detection of stealth targets and acceptable resolution and accuracy of velocity measurements.

## II - Effect of Coating Material on the Radar Received Signal.

In this paper, it is considered that the stealth point target consists of absorbing material over the metal surface as shown in Fig. 1. The energy of the backscatter signal determined as a function of the time duration of the transmitted pulse and the absorbing material time delay. The returned signal is passed through a matched filter for detection and target parameters measurement. The reflected signal to the radar consists of the superposition of the reflected one from the exposed surface of the absorbing material and that one from the metal surface. The electric field strength at the surface of absorbing coat remains unchanged with amplitude,  $A$ . The electric field strength reflected by the metal surface is polarity reversed and it has the amplitude  $B$  [6]. In this paper the effect of radar waveform selection is analyzed, to avoid the effect of coating material on the detection of stealth targets. It is shown that short radar pulses could avoid the effect of stealth target. However short pulses reduces the capability of velocity resolution in radar system. Fast rate phase coded radars could provide both the capabilities of stealth target detection in presence of coating materials and high velocity resolution.



Fig.1. A stealth target consisting of a metal surface covered with absorbing material at distance  $R$  from the radar.

First, assume that a radar signal  $f(t)$  incident on a point target coated by an absorbing material, at distance  $R$  from the radar. Further assume that neither the reflection nor the attenuation in

the absorbing material causes distortions of the transmitted waveform [7]. The reflected signal can be represented in terms of the transmitted signal as:

$$\begin{aligned} h(t) &= Af(t - t_0) - Bf(t - t_0 - T_a) \\ h(t) &= Af(t') - Bf(t' - T_a), \quad t' = t - t_0 \end{aligned} \quad (1)$$

where  $t_0 = 2R/c$  is the propagation delay,  $T_a$  is the wave propagation time delay in the absorbing material,  $c$  is the speed of light. The coefficient  $A$  depends on the reflection properties and impedances of the interface between air and absorbing material, the coefficient  $B$  depends on the thickness, impedance, attenuation factor of absorbing material [8]. Second it is assumed that the power returning from both the absorbing material and metal surface are approximately equal, that  $A$  and  $B$  are essentially equal [9]. Hence, the returned signal of (1), usually defined as the target response, becomes:

$$h(t') = f(t') - f(t' - T_a) \quad (2)$$

The energy of the target response of equation (2) can be written as a function of  $T_a$  as

$$E(T_a) = \int_0^{T_a + T_p} h^2(t') dt' = \int_0^{T_a + T_p} [f(t') - f(t' - T_a)]^2 dt' \quad (3)$$

$$E(T_a) = \int_0^{T_a + T_p} f^2(t') dt' + \int_0^{T_a + T_p} f^2(t' - T_a) dt' - 2 \int_0^{T_a + T_p} f(t') f(t' - T_a) dt' \quad (4)$$

The first and second terms represent the energy of the transmitted pulse  $E_f = \int_0^{T_a} f^2(t') dt'$

(5)

while the third term is the cross correlation coefficient between the two pulses  $f(t')$ ,  $f(t' - T)$ , it is

determined by:

$$k(T_a) = \int_0^{T_a + T_p} f(t') f(t' - T_a) dt' \quad (6)$$

From (7) and (8) the energy of (6) can be rewritten:  $E(T_a) = 2(E_f - k(T_a))$  (7)

The normalized target response energy  $E_N(T_a)$  can be obtained by normalizing (7) by  $2E_f$ :

$$E_N(T_a) = \frac{k(T_a)}{2E_f} = 1 - \rho(T_a) \quad (8)$$

where  $\rho(T_a)$  is the normalized cross correlation coefficient defined by:  $\rho(T_a) = \frac{k(T_a)}{E_f}$  (9)

A matched filter is used for detection and selection the target response. The autocorrelation function at the output of a matched filter set for the input response can be obtained according to the definition:

$$k_a(t) = \int_{-\infty}^{+\infty} f(u) h(u - t) du \quad (10)$$

This definition different from the one introduced by [9], because the thickness of absorbing material is unknown. A radar signal consists of  $N$  pulses and each pulse has the time variation of  $f(t)$  can be expressed in the form

$$s_t(t) = \sum_{i=0}^{N-1} f(t - iT_r) \quad (11)$$

where  $T_r$  is the time interval between the radiated pulses. The reflected signal from a stationary stealth point target can be written:

$$s_r(t) = \sum_{i=0}^{N-1} ah(t - t_0 - iT_r) \quad (12)$$

A stealth point target moving away from the radar with a velocity  $v$  will produce a Doppler delay,  $\tau$  in the reflected signal, then the returned signal in (14) can be rewritten as:

$$s_r(t) = \sum_{i=0}^{N-1} h[t - t_1 - i(T_r \pm \tau)] \quad (13)$$

where  $t_1$  is the arrival time of the first response received from the moving target. The returned signal of (13) passed through matched filter to produce at its output the sequence of correlation functions

$$k_s(t) = \sum_{i=0}^{N-1} k_a[t - t_1 - T_d - i(T_r + \tau)] \quad (14)$$

A processor for the range-velocity resolution function can be realized by connecting the matched filter for detection of particular target to a bank of Doppler processors. Each Doppler processor has an input and an output hybrid coupler HYC, a feedback delay line circuit DEL with Doppler time  $\tau = \tau \mp iT_r$  to resolve the target velocity  $v = c\tau_0 / 2T_r$ . The output of the Doppler processors have been obtained in terms of the input correlation function as:

$$k_D = \sum_{j=1}^N k_a[t'' - (j-1)\Delta\tau] \quad \text{where,} \quad t'' = t - [(N-1)(T_r + \tau) + T_d] \quad (15)$$

The range-velocity resolution function is obtained by first normalizing (15) by  $N$  and then replacing  $t''$  by  $t/T_p$  and  $\Delta\tau$  by  $\tau_0/T_p$  so we get

$$k_{DN} = \frac{1}{N} \left| \sum_{j=0}^N k_a \left( \frac{t}{T_p} - j \frac{\tau_0}{T_p} \right) \right| \quad (16)$$

The absolute value has been used in (16) since the normalized input correlation function  $k_a(t)$  has negative part. In the next two subsections, the performance of two types of radar signals are evaluated and compared in presence of Stealth targets. The two signals are the nonsinusoidal signal with Beta time variation and the phase coded one.

### III-1 Nonsinusoidal Transmitted Signal

Consider a nonsinusoidal transmitted signal consisting of a single pulse represented by a function  $f(t)$  with Beta time variation and duration  $T_p$  as shown in Fig.2.

$$f(t) = \begin{cases} 4\left(\frac{t}{T_p}\right)\left(1 - \frac{t}{T_p}\right) & 0 \leq t \leq T_p \\ 0 & \text{otherwise} \end{cases} \quad (17)$$

The target response obtained through computer simulation and it is shown in Fig.3, for different time delays in the absorbing material. It is clear that the target response depends on the time delay of the absorbing material  $T_a$  and the time variation of the pulse,  $f(t)$ . The peak amplitude increases from the small value of 0.5 at  $T_a = 0.3T_p$  to the maximum value of 1 at  $T_a = T_p$  and remains constant at value 1 as  $T_a$  increases beyond  $T_p$ . As evident it is clear that the time variations of the target response are produced by the interaction between the two pulses  $f(t)$ ,  $f(t-T)$ , particularly, when  $T_a$  is less than  $T_p$ , but when  $T_a$  becomes larger than  $T_p$  the two previous pulses do not overlap. The variation of the correlation coefficient  $\rho(T_a)$  and the normalized energy  $E_N(T_a)$  with the normalized delay in the absorbing material ( $T_a/T_p$ ) are computed during the simulation and shown in Fig.4 a,b respectively. It is clear that the correlation coefficient  $\rho(T_a)$  decreases from the maximum value of 1 at  $T_a=0$ , to the minimum value of zero for  $T_a \geq T_p$ . The normalized energy  $E_N(T_a)$  increases from the minimum value of zero at  $T_a=0$ , to the maximum value of 1 for  $T_a \geq T_p$ . As a result, the returned energy  $E_N(T_a)$  becomes very small when the time delay  $T_a$  is very small. When  $T_a$  increases and becomes equal or greater than  $T_p$ , the two pulses  $f(t)$ ,  $f(t-T)$ , do not overlap and both  $k(T_a)$  and  $\rho(T_a)$  becomes zero as shown in Fig. 4 (a), in this case, the normalized energy of  $E_N(T_a)$  increases to maximum value of 1 and remains constant at value 1 when  $T_a$  increases beyond  $T_p$  as shown in Fig. 4 (b).

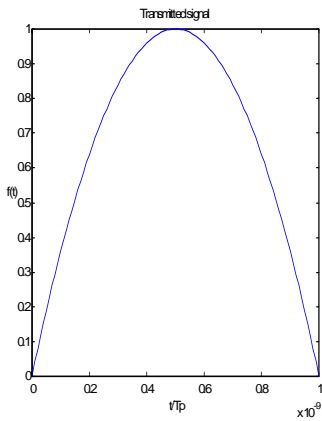


Fig. 2. Time variation of transmitted signal  $f(t)$

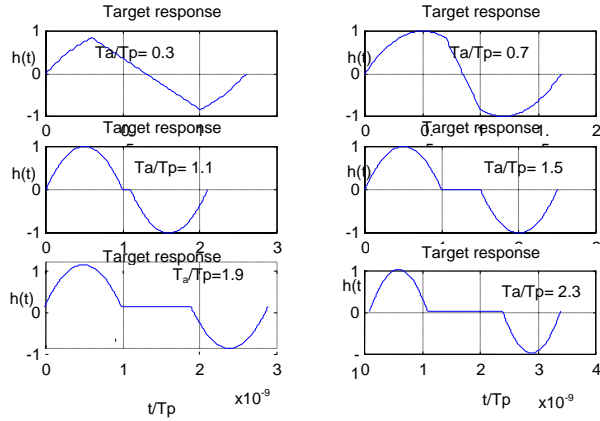
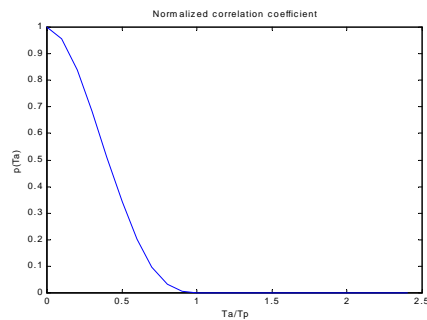
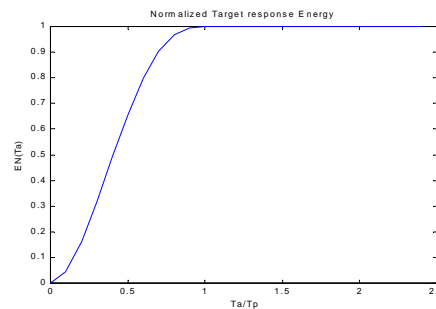


Fig. 3. Target response for  $T_a = 0.3, 0.7, 1.1, 1.5, 1.9, 2.3 T_p$



(a)



(b)

Fig. 4. (a) Normalized correlation function coefficient  $\rho(T_a)$ ,

(b) Normalized target response energy  $E_N(T_a)$  as a function of normalized time delay  $T_a/T_p$ .

The variation of the correlation functions of (12) for the input signals  $h(t')$  are illustrated in Fig.5. The correlation functions are characterized by two equal parts with positive and negative magnitudes. The magnitude of two parts increase with  $T_a$  and reaches the maximum value of 1 as  $T_a$  increases to  $T_a \approx T_p$ , and remains constant at 1 for  $T_a \geq T_p$ .

The correlation function  $k_a(t)$  of the target response shown in Fig.5(b) can be transformed to the correlation function of the transmitted signal shown in Fig. 5(a) in two steps. First, the time delay of the absorbing material should be at least  $T_a \geq 2T_p$ , then, the correlation function has two equal parts with positive and negative magnitudes with absolute value of 1. Second, the negative part can be deleted from the correlation function  $k_a(t)$  by rectification. As a result, the duration of the transmitted pulse can be determined for maximum returned signal energy for stealth point target by using the relation  $T_a/T_p > 2$ .

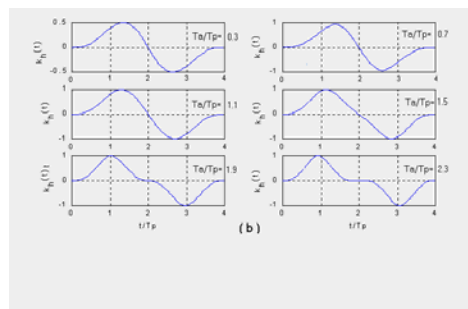
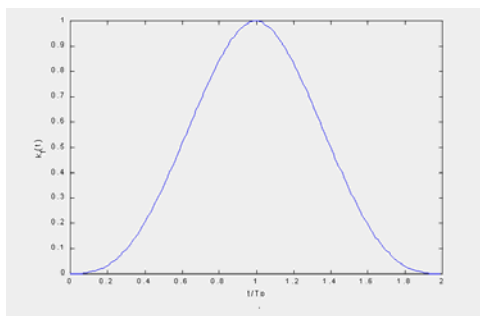


Fig.5 . Matched filter output for: (a) transmitted signal,  $f(t)$  & (b) stealth target response,  $h(t)$ .

The range-velocity resolution function of (16) as a function of range represented by the time delay  $t/T_p$  and velocity represented by the Doppler time  $\tau_0/T_p$  for five pulses ( $N=5$ ), is evaluated for the correlation function  $k_a(t)$  with  $T_a = T_p$  and shown by three-dimensional surface in Fig. 6. Clearly the shorter the pulse width,  $T_p$  the lower the velocity accuracy and resolution.

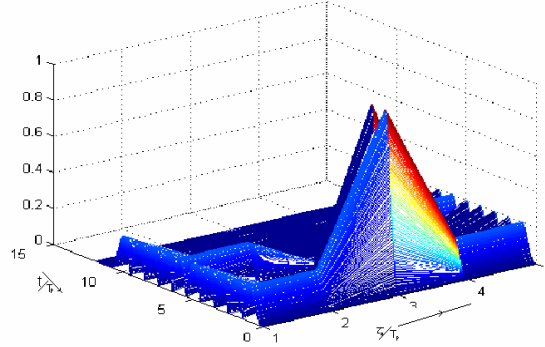


Fig. 6 Rang-velocity resolution function for a transmitted pulse duration  $T_p = T_a$  and  $N=5$ .

### III-2 Performance of Pulse compression for Stealth Target

Pulse compression allows a radar to utilize a long pulse to achieve large radiated energy, but simultaneously to obtain the range resolution of a short pulse [10]. It accomplishes this by employing frequency or phase modulation to widen the signal bandwidth. In this paper we discuss phase-coded pulse compression radars. In this form of pulse compression, a long pulse of duration  $T$  is divided into  $N$  subpulses each of width  $T_p$ . The phase of each subpulse is chosen to be either 0 or  $\pi$  radians. The binary phase-code sequence of 0,  $\pi$  values that result in equal sidelobes after passage through the matched filter is called a Barker code [11]. Barker codes of length  $L$  have the required attributes for a signal character since their autocorrelation function has a main lobe with peak value  $L$  and a number of time sidelobes with maximum magnitude of one. The longest of Barker code is of length 13. A coded signal character using Barker codes of length  $L$  has the form:

$$b(t) = \sum_{l=0}^{L-1} c_l f(t - lT_p) \tag{19}$$

where  $c_l$  is  $l^{th}$  element of the Barker code  $\{c_0, c_1, \dots, c_{L-1}\}$ ;  $c_l = \pm 1$  and  $f(t)$  is the time variation of individual code elements with nominal time duration  $T_p$ . The coded waveform of (19) can be presented by using (3) as

$$b(t) = \sum_{l=0}^{L-1} 4c_l \left( \frac{t - lT_p}{T_p} \right) \left( 1 - \frac{t - lT_p}{T_p} \right) \tag{20}$$

where  $LT_p$  is the duration of the coded character. The time variation of the character  $b(t)$  in (20) is shown in Fig. 7(a) for the Barker code of length  $L=5$  and a character time duration of  $5T_p$ . The autocorrelation function at the output of matched filter can be obtained by using the definition of (8)

$$k_b(t) = \int_{-\infty}^{+\infty} b(u)b(u-t)du = \int_{-\infty}^{+\infty} \sum_{p=0}^{L-1} \sum_{q=0}^{L-1} c_p c_q f(u - sT_p) f(u - t - qT_p) du = \sum_{s=0}^{L-1} \sum_{q=0}^{L-1} c_s c_q \int_{-\infty}^{+\infty} f(u') f\{u' - [t - (s - q)T_p]\} du' \tag{21}$$

The autocorrelation function can be rewritten as: 
$$k_b(t) = \sum_{s=0}^{L-1} \sum_{q=0}^{L-1} c_s c_q k[t - (s - q)T_p] \tag{22}$$

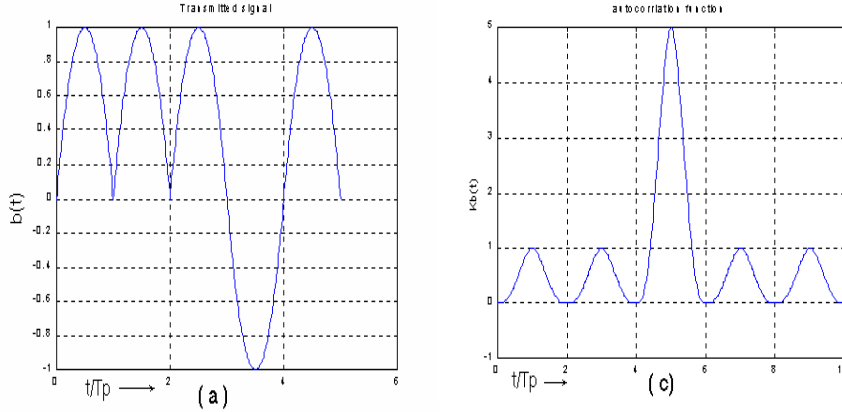


Fig.7. (a) Time variation of Barker code  $b(t)$ . (b) The output of matched filter.

The autocorrelation function of (22) for character of Fig.7 (a) is shown in Fig.7 (b). The peak amplitude of the main lobe is  $L=5$  and the maximum magnitude of the sidelobe is 1, yielding pulse compression ratio equals 5. Higher pulse compression ratio can be obtained by using another length of Barker code for example ( $L=7, 11, 13$ ).

The returned signal from the stealth point target of Fig.1 using Barker code as transmitted signal is the target response which can be presented by

$$h_b(t) = b(t') - b(t' - T_a) = \sum_{l=0}^{L-1} c_l [f(t' - lT_p) - f(t' - lT_p - T_a)] \quad (23)$$

The time variation of the target response of (23) is shown in Fig.8 for the absorbing material time delays  $T_a = T_p, 2T_p, \dots, 5T_p$ . The duration of the target response according to Fig.8. can be obtained as:  $T_h = LT_p + T_a$ . When the time delay  $T_a$  assumes integer multiple of  $T_p$ , the duration of (23) becomes:

$$T_h = LT_p + mT_p = (L+m)T_p, \quad m = 1, 2, \dots \quad (24)$$

The target response of (23) can be rewritten in terms of the character element function  $f(t)$ :

$$h_b(t) = \sum_{l=0}^{L+m-1} d_l f(t - lT_p) \quad m = 1, 2, \dots \quad (25)$$

where  $d_l$  is the difference between the magnitude of the pulse  $f(t + LT_p)$  and shifted pulse  $f(t - (L-m)T_p)$ .

The stealth point target response of (25) to Barker code of length  $L = 5$  is a non-binary coded waveform with multilevel code structure  $d_l = \{d_0, d_1, d_2, \dots, d_{L+m-1}\}$  and a code length  $L+m$  as shown in Fig.8 for  $T_a = mT_p, m = 1, 2, \dots, 5$ . The magnitude of the multilevel code element  $d_l$  depends on the absorbing material time delay  $mT_p$  and can take on any value from the set of integers  $\{-2, -1, 0, 1, 2\}$ . When  $m=1$  then  $d_l = \{1, 0, -2, 2, -1, 0\}$  with the length  $L+m = 6$  as shown in Fig. 8 for  $T_a = T_p$ . In general, when  $m < L$ , the time variation of the target response of (25) depend on the Barker code structure  $c_l$  and the time delay of the absorbing material [11].

The magnitude of the target response of (25) will have a binary structure when the time delay of absorbing material  $T_a$  equal to or greater than  $LT_p$ , corresponding to  $m \geq L$ . When as shown in Fig.8, the target response consists of the transmitted signal followed by amplitude reversed signal. As the time delay  $T_a$  becomes larger than  $LT_p$ , the time separation between the positive and negative signal characters increase as shown in Fig. 8 for  $T_a \geq 5T_p$ . The autocorrelation function at the output of the matched filter for a target response can be represented in a similar way of (12):

$$k_h(t) = \int_{-\infty}^{\infty} b(u)h_b(u-t) \quad (26)$$

The autocorrelation functions in (26) are shown in Fig. 9, and this different from exposed by [12]. Each autocorrelation function has a main narrow lobe, a number of time sidelobes with

positive and negative magnitude, and large negative sidelobe equals to the magnitude of the main lobe. From Fig. 9 it is clear that, the time variation of the autocorrelation function depends on  $T_a$ , when the time delay  $T_a$  increases to  $T_a = 2LT_p$  and beyond, the correlation function of (26) characterizes by two characteristics. First, two symmetrical autocorrelation functions with the same Barker code, the first ones positive magnitude and the last ones negative magnitude. Second, the free interval between the positive and negative autocorrelation functions increases with  $T_a$ , when  $T_a \geq 2LT_p$ .

The range resolution based on the correlation function of Fig.9, for  $T_a \geq 2LT_p$  can be improved by two methods, increasing the sidelobe free interval and eliminating the negative time sidelobes [12].

In the first method, the sidelobes free interval can be increased by decreasing the subpulse duration  $T_p$  according to:  $T_p \leq T_a/2L$ . In the second method, the correlation function is passed through a rectifier to eliminate the negative time sidelobes of the correlation function of Fig. 9 for  $T_a \geq 2LT_p$ . After rectification, the autocorrelation function becomes the autocorrelation function  $k_b(t)$  of Fig.8(b). In this case, the stealth point target has similar characteristics to that of the conventional point target.

**Conclusions**

The stealth point target is designed to reduce the returned signal energy by the proper choice of the absorbing material time delay. Two principles should be considered in the design of radar signals for detection of stealth point targets. First, by using the technology of carrier free radar signals with 10GHz band-width, one would need absorbing coatings with a thickness 3 cm at 10 GHz and 30 cm at 1GHz for reduction of power reflection to 1%. However these short radar pulses of so extended bandwidth reduce the resolution and accuracy of velocity measurement. Therefore the second principle in the design of radar signal is to utilizes a fast phase varying signal such as the Barker coded waveform which maintain the same detection capabilities of stealth point target while maintiang acceptable resolution and accuracy of velocity measurement.

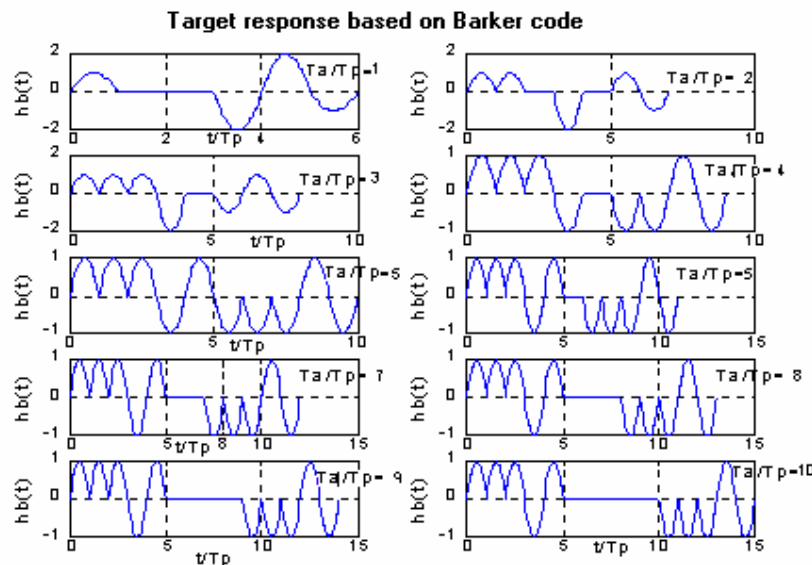


Fig.8 Target response based on Barker code for different absorbing material time delay



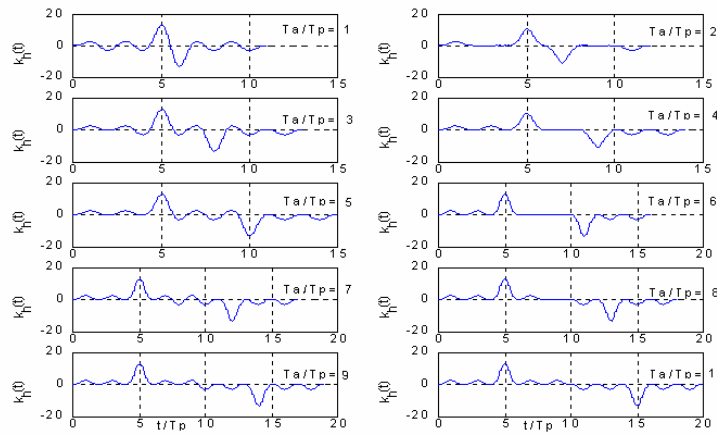


Fig. 9 Autocorrelation function for the target response of Fig. 8

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