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AN ADAPTIVE ANTENNA UTILIZING MUSIC AND LINEARLY CONSTRAINED MINIMUM VARIANCE (LCMV) ALGORITHMS

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ABSTRACT

This paper introduces a new structure based on the MUSIC algorithm and linearly constrained minimum variance (LCMV) to highly suppress the jammers to the GPS receiver. This structure is capable of adjusting the weights of the antenna array in real time to respond to the signals coming from the desired directions while highly suppress the jammers coming from the other directions. The simulations were performed for fixed and moving jammers. It indicates that this structure can give deeper nulls to the jammers directions, up to 120 dB nulls depths for fixed jammers and more than 99 dB depths for the moving one. These nulls are very deep related to that attained by MUSIC algorithm alone especially when the jammer signal power to the GPS signal is low.

KEY WORDS

Adaptive antenna, MUSIC, linearly constrained minimum variance (LCMV), and GPS anti-jamming

I-INTRODUCTION

Global Positioning system (GPS) receiver is the best device in the field of navigation to give a very accurate user position. So it is used in many civilian and military applications. Interference from radar systems and other devices affects the civilian use, otherwise intentionally used jammers affects the military use, so increasing the protection against intentional and unintentional interferences is required. The received GPS signal is about -160 dBW i.e., it is below the receiver thermal noise power by about 20-30 dB. High power jammers like FM, pulse, noise and CW, degrade the behavior of the GPS receiver and cause their code tracking loop and carrier tracking loop to be out of look. Adaptive antenna is suitable to cancel these types of jammers. It utilizes some cancellation techniques based on determining the jammer directions like MUSIC algorithm [1-5] or power inversion [6].

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The main purpose of adaptive antenna is to reduce the jammer signals up to a level where the spread spectrum mechanism can extract the signal. In section II MUSIC (Multiple Signal Classification) is introduced. Short brief about Linearly Constrained Minimum Variance (LCMV) and the proposed structure are discussed in section III. Section IV is assigned for the simulation results. Finally the conclusion is given in section V.

II- MULTIPLE SIGNAL CLASSIFICATION (MUSIC)

One of the high resolution methods for estimating the direction of arrival (DOA) of a narrow band signal in presence of noise is multiple signal classification (MUSIC) algorithm. It is used to describe experimental and theoretical techniques involved in determining the parameters of multiple wavefronts arriving at an antenna array by measuring the signal received at the antenna elements. MUSIC algorithm provides asymptotically unbiased estimate of the number of signals and their directions of arrival [1-3]. This algorithm is very useful when dealing with GPS anti-jamming. Consider the received signal at M-elements uniformly spaced linear array as shown in Fig.1, is linear combination of all the far field incident signals and noise. Thus,

$$\mathbf{X} = \mathbf{V}\mathbf{u} + \mathbf{N} \tag{1}$$

or

$$\begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_M(t) \end{pmatrix} = \begin{pmatrix} \alpha(\theta_1) & \alpha(\theta_2) & \dots & \alpha(\theta_L) \end{pmatrix} \begin{pmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_L(t) \end{pmatrix} + \begin{pmatrix} n_1(t) \\ n_2(t) \\ \vdots \\ n_M(t) \end{pmatrix}$$

$$y = \mathbf{w}^H \mathbf{X} \tag{2}$$

$$\mathbf{w} = (w_1 \quad w_2 \quad \dots \quad w_M)^T$$

Where

\mathbf{X} is $M \times 1$ vector represents the antenna array received signal.

\mathbf{V} a matrix contains the steering vectors associated to the incident signals.

\mathbf{u} a vector represents the incident signals amplitudes.

\mathbf{w} is the $M \times 1$ complex vector represents the array weight vector.

y is the output of the array antenna given by the weighted sum of the array antenna received signal.

\mathbf{N} is $M \times 1$, vector consists of an independent Gaussian noise of variance σ^2 includes channel noise, receiver noise and antenna elements noise.

L is the number of incident signals.

$$\alpha(\theta_i) = \left[1 \quad \exp\left[-j\left(\frac{2\pi l \sin\theta_i}{\lambda}\right)\right] \quad \dots \quad \exp\left[-j\left(\frac{2\pi(M-1) l \sin\theta_i}{\lambda}\right)\right] \right]^T$$

θ_i , is the incident signals directions, where $i=1,2, \dots, L$.

l is the distance between each two antenna elements.

It is assumed that the incident signals are independent of each other and independent on the thermal noise. The covariance matrix of the array received signal \mathbf{R} is given as:

$$\begin{aligned} \mathbf{R} &= E[\mathbf{X} \mathbf{X}^H] \\ &= \mathbf{V} \mathbf{P} \mathbf{V}^H + \sigma_n^2 \mathbf{I} \end{aligned} \quad (3)$$

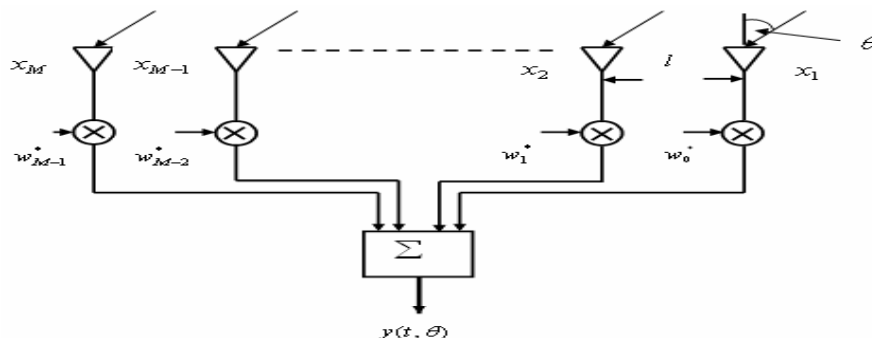


Fig.1. Adaptive array

$$\mathbf{P} = \text{diag} (p_1 \quad p_2 \quad \dots \quad p_L)$$

$p_i = E(u_i(t) u_i^*(t))$ is the power of i^{th} incident signal.

The autocorrelation matrix can be written in terms of its eigenvalues and eigenvectors as follows,

$$\mathbf{R} = \mathbf{Q} \mathbf{\Lambda} \mathbf{Q}^H \quad (4)$$

Where

$$\mathbf{\Lambda} = \text{diag} (\lambda_1 \quad \lambda_2 \quad \dots \quad \lambda_M)$$

$$\mathbf{Q} = (\mathbf{q}_1 \quad \mathbf{q}_2 \quad \dots \quad \mathbf{q}_M)$$

λ_i , represents the i^{th} eigenvalue of \mathbf{R} , where $i=1, 2, \dots M$.

\mathbf{q}_i , is the i^{th} eigenvector associated to the i^{th} eigenvalue of \mathbf{R} where $i=1, 2, \dots M$.

The eigenvalues of \mathbf{R} are in order of decreasing size $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_L > \lambda_{L+1} = \lambda_{L+2} = \dots = \sigma^2$

The signal subspace is represented by the eigenvectors correspond to the L largest eigenvalues.

The remaining $[L+1 : M]$ eigenvectors span the noise subspace. As a result:

$$\mathbf{Q} = (\mathbf{Q}_{\text{Signal}} \quad \mathbf{Q}_{\text{Noise}}) \quad (5)$$

$$\mathbf{Q}_{\text{Signal}} = (\mathbf{q}_1 \quad \mathbf{q}_2 \quad \dots \quad \mathbf{q}_L)$$

$$\mathbf{Q}_{\text{Noise}} = (\mathbf{q}_{L+1} \quad \mathbf{q}_{L+2} \quad \dots \quad \mathbf{q}_M)$$

It is proved in [1] that the columns of $\mathbf{Q}_{\text{Noise}}$ are orthogonal to the space spanned by the columns of \mathbf{V} . i.e.,

$$\mathbf{q}_i \perp \text{span} (\mathbf{V}), \quad i = L+1, L+2, \dots, M \quad (6)$$

The direction of the incident signals can be obtained by getting the minimum of $\xi(\theta)$ in the following formula,

$$\xi(\theta) = (\mathbf{a}^H(\theta) \mathbf{Q}_{\text{Noise}} \mathbf{Q}_{\text{Noise}}^H \mathbf{a}(\theta)) \quad (7)$$

III- MUSIC WITH LCMV STRUCTURE

Consider a linear array of M uniformly spaced elements whose output are individually weighted and then summed to produce the beamformer output, $\mathbf{y} = \mathbf{X}(t)^H \mathbf{w}$.

The main objective of LCMV is to minimize the mean squared output $E(|y|^2)$ subjected to a set of linear constraints on the weight vector \mathbf{w} ,

$$\begin{aligned} \min |y|^2 &= \min \mathbf{w}^H \mathbf{R} \mathbf{w} \\ \text{subjected to} & \\ \mathbf{C}^H \mathbf{w} &= \mathbf{f} \end{aligned} \quad (8)$$

The solution to (8) is

$$\mathbf{w}_o = \mathbf{R}^{-1} \mathbf{C} (\mathbf{C}^H \mathbf{R}^{-1} \mathbf{C})^{-1} \mathbf{f} \quad (9)$$

When there are no useful or jammer signals and only uncorrelated noise (9) can be written as:

$$\mathbf{w}_{qs} = \mathbf{C} (\mathbf{C}^H \mathbf{C})^{-1} \mathbf{f} \quad (10)$$

Where \mathbf{w}_{qs} is the quiescent weight.

\mathbf{R} is given by (3).

The total input signal to the antenna is given by (1).

\mathbf{C} is $M \times K$ matrix, where all of its columns are linearly independent

M is the number of antenna elements.

K the number of constraints.

$\mathbf{f}_{K \times 1}$ the response vector.

Normally $K < M$. If $K = M$ this leads to the weight vector \mathbf{w} can be determined only by the constraints and no degrees of freedom are available to data adaptation. If $K > M$ this means that there is no enough weights satisfying the constraints. If it is required to constrain the known directions of the useful and the intentionally jammer signals to be within certain values, the LCMV minimize the total output power subjected to maintain the directions of the useful and the intentionally jammer signals to be within the required values. So if any jammer signals come from unknown directions the LCMV will assign new nulls to it [7-9]. The constraint part of (8) can be written as:

$$(\mathbf{C}_q^H \quad \mathbf{C}_J^H) \mathbf{w} = (f_{d1} \quad f_{d2} \quad \dots \quad f_{dq} \quad f_{J1} \quad f_{J2} \quad \dots \quad f_{J(K-q)})^T \quad (11)$$

Where

$\mathbf{C}_q = [u(\theta_1), u(\theta_2), \dots, u(\theta_q)]$; is $M \times q$ matrix represents the useful signals vectors.

$$u(\theta_i) = u_i(t) \alpha_{ui} \quad , \quad i = 1, 2, \dots, q \quad (12)$$

$\mathbf{C}_J = [J(\theta_{q+1}), J(\theta_{q+2}), \dots, J(\theta_K)]$; is the $M \times (K - q)$ matrix represents the jammer signals vectors which are coming from known intentionally jammer directions.

The jammer signals vectors can be written as:

$$J(\theta_k) = J_k(t) \alpha_{Jk} \quad , \quad k = q+1, q+2, \dots, K \quad (13)$$

If it is required to widen the region around the angle corresponds to the jammer coming from θ_{q+i}

direction, $i = 1, 2, \dots, K - q$. The constraint matrix and the response vector can be written as follows:

$$\mathbf{C}_J = [J(\theta_{q+1}), \dots, J(\theta_{q+i} - \Delta\theta), J(\theta_{q+i}), J(\theta_{q+i} + \Delta\theta), \dots, J(\theta_K)]$$

$$\mathbf{f} = (f_{d1} \ f_{d2} \ \dots \ f_{dq} \ f_{j1} \ f_{j2} \ \dots \ f_{ji} \ f_{ji} \ f_{ji} \ \dots \ f_{j(K-q)})^T$$

LCMV could not assign deep nulls to the jammer directions if they are not known in advance especially if the jammer power is low. Also the shape of the antenna power pattern will not give exact information about the directions of the jammers. MUSIC algorithm can detect the jammer signals directions blindly but the null depth will not be deep enough especially when the jammer power is low.

Dealing with GPS signals the satellites and the user directions are known so the useful signal directions are exactly known, but the jammer signals directions are unknown. Therefore to deal with the above problem, the proposed structure in Fig.2, based on three steps

- 1) Use MUSIC algorithm as a preprocessor to detect the jammer signals directions.
- 2) Construct the constraint matrix \mathbf{C} which contains both the useful and the jammer signals directions.
- 3) Use LCMV to deal with the useful signals and the jammers from known directions.

For adaptively calculating the weight vector we use Lagrange multiplier to change the constrained equation (8) to unconstrained one, subsequently:

$$\phi = \mathbf{w}^H \mathbf{R} \mathbf{w} + \lambda^H (\mathbf{C}^H \mathbf{w} - \mathbf{f}) + (\mathbf{w}^H \mathbf{C} - \mathbf{f}^H) \lambda \tag{14}$$

Minimizing the output power means taking the gradient of (14) with respect to \mathbf{w}^H and equating the result by zero.

$$\frac{\partial \phi}{\partial \mathbf{w}^H} = 2\mathbf{R} \mathbf{w} + 2\mathbf{C} \lambda = \mathbf{0} \tag{15}$$

λ is $K \times 1$ vector

Utilizing the steepest descent technique to iteratively update the weight vector

$$w(k+1) = w(k) - \mu \frac{\partial \phi}{\partial \mathbf{w}^H} \tag{16}$$

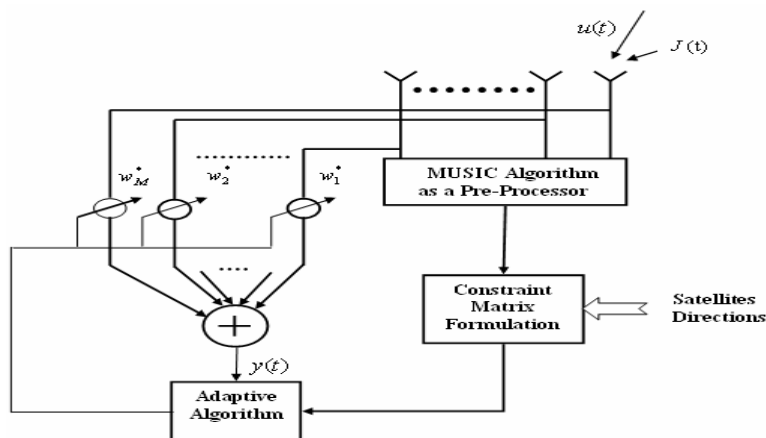


Fig. 2. MUSIC algorithm with Linearly Constrained Minimum Variance (MUSIC_LCMV) structure as jammer suppression for GPS receiver.

Using both (15) and $\mathbf{f} = \mathbf{C}^H \mathbf{w}(k+1)$ which is the constraint part of (8) in (16), λ can be obtained

as:

$$\lambda = \frac{1}{\mu} \left[(\mathbf{C}^H \mathbf{C})^{-1} \mathbf{C}^H (\mathbf{I} - \mu \mathbf{R}) \mathbf{w}(k) - (\mathbf{C}^H \mathbf{C})^{-1} \mathbf{f} \right] \quad (17)$$

Use (17) in (16)

$$\begin{aligned} \mathbf{w}(k+1) &= \mathbf{A} \mathbf{w}(k) - \mu \mathbf{A} \mathbf{R} \mathbf{w}(k) + \mathbf{w}_{qs} \\ &= \mathbf{A}(\mathbf{I} - \mu \mathbf{R}) \mathbf{w}(k) + \mathbf{w}_{qs} \end{aligned} \quad (18)$$

$\mathbf{A} = [\mathbf{I} - \mathbf{C}(\mathbf{C}^H \mathbf{C})^{-1} \mathbf{C}^H]$ is the projecting matrix which projects $(\mathbf{I} - \mu \mathbf{R}) \mathbf{w}(k)$ to the (M-K)-dimensional subspace Ω . This subspace passes through the origin and parallel to the (M-K)-dimensional weight vector hyperplane Π .

The orthogonal matrix A will cancel any component perpendicular to Ω .

$\mathbf{w}_{qs} = \mathbf{C}(\mathbf{C}^H \mathbf{C})^{-1} \mathbf{f}$ is found in the K-dimensional constraint subspace which is the span of the constraint vectors and orthogonal to Π . \mathbf{w}_{qs} is the shortest vector terminates to Π .

Considering the instantaneous value of the autocorrelation matrix, accordingly $\mathbf{R} = \mathbf{X} \mathbf{X}^H$, (18) can take the form,

$$\mathbf{w}(k+1) = \mathbf{A}(\mathbf{w}(k) - \mu \mathbf{X}(k) y(k)) + \mathbf{w}_{qs} \quad (19)$$

$$y(k) = \mathbf{w}^H(k) \mathbf{X}(k).$$

(19) represents the LMS algorithm if A and \mathbf{w}_{qs} are eliminated. Also it can be represented geometrically according to [10], as shown in Fig. 3.

\mathbf{w}_{qs} is represented by OB. It also can be given by GF which is parallel to OB

OP represents the weight vector at iteration k

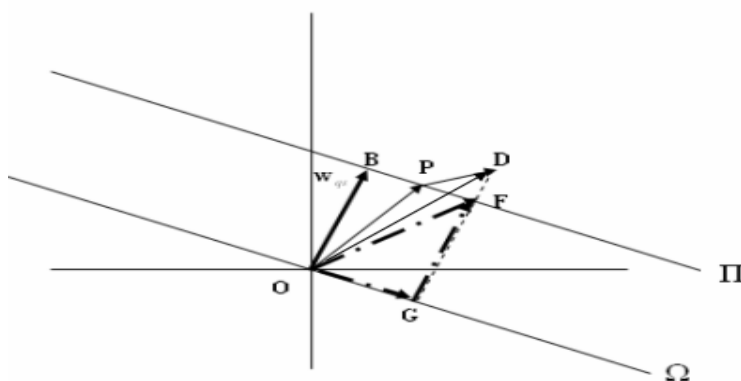


Fig.3. Operation of LCMV

PD represents the value $-\mu \mathbf{X}(k) y(k)$

$\mathbf{w}(k) - \mu \mathbf{X}(k) y(k)$ is represented by OD, which is the weight vector at iteration k+1 in case of

LMS algorithm.

It is clear from Fig. 3 that $\mathbf{w}_{LMS}(k+1)$ is not terminating to $\mathbf{0}$. To make $\mathbf{w}(k+1)$ all the time satisfy the constraint condition, we must project it on Ω and add the quiescent weight to the projecting vector.

OG represented by $\mathbf{A}(\mathbf{w}(k) - \mu \mathbf{X}(k)y(k))$

OF = OG + GF

It is clear that **OF** = $\mathbf{w}(k+1)$ given by (19) terminates to $\mathbf{0}$.

V- SIMULATIONS

Computer simulations were performed using 13 elements uniform linear array arranged in the y-axis with elements spaced half wave length apart. There are six useful GPS signals each with power -165 dBW come from directions $(0^\circ 30^\circ 45^\circ 60^\circ -18^\circ -36^\circ)$. Two jammers come from directions $(-20^\circ 58^\circ)$. The jammer coming from -20° has power -120 dBW and the other coming from 58° has power -100 dBW . Simulation is performed using 200 snap shot. Five cases were simulated.

1) The requirement is to achieve distortion less response in the directions of the useful signals and null the jammer coming form the fixed directions. So the constraint response can be given as:

$\mathbf{f} = (1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0)^T$, and the constraint matrix takes the form:

$(u(0^\circ) \ u(30^\circ) \ u(45^\circ) \ u(60^\circ) \ u(-18^\circ) \ u(-36^\circ) \ J(58^\circ) \ J(-20^\circ))$ The power pattern levels for the desired and the jammer signals of Fig.4, are summarized in TABLE 1.

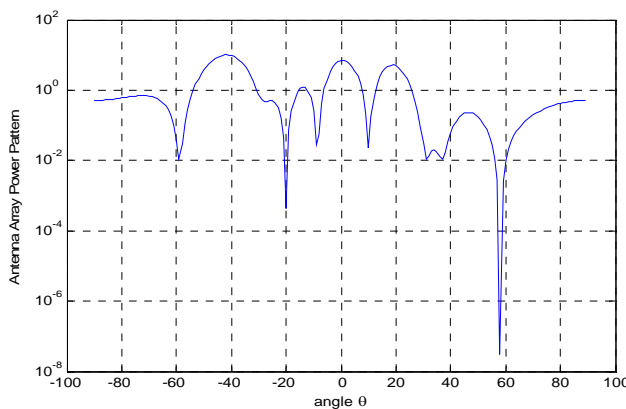


Fig.4. a) Antenna array power pattern for only MUSIC algorithm .

In case of using MUSIC algorithm alone, it is clear that the difference between the useful signals directions level and the

jammer level from direction -20° is ranging between 13.55 dB and 41.9 dB. And at the jammer from direction 58° is in the range between 55.15 dB and 83.5 dB.

In case of MUSIC_LCMV the difference between all the desired signals directions and the jammer form direction -20° is 120 dB and form direction 58° is 117.65 dB.

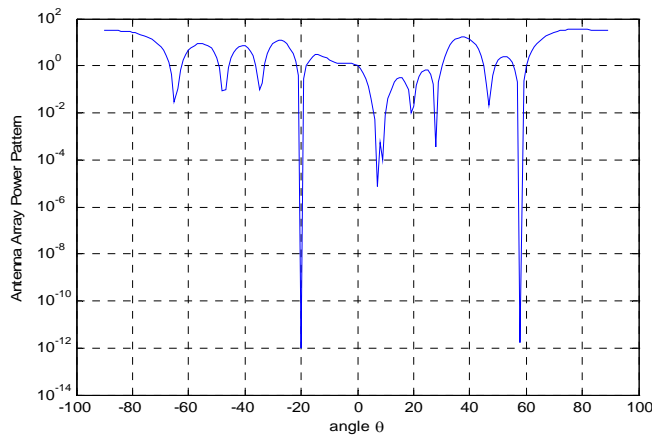


Fig.4. b) Antenna array power pattern for MUSIC algorithm with LCMV structure (MUSIC_LCMV)

TABLE 1. MUSIC against MUSIC_LCMV in case of distortion less response and nulling the jammer direction

Angle θ in degrees	Power Pattern Level in dB	
	MUSIC	MUSIC_LCMV
0°	8.35	0
30°	-15.3	0
45°	-6.54	0
60°	-20	0
-18°	-5.73	0
-36°	7.77	0
-20°	-33.55	-120
58°	-75.15	-117.65

2) The requirement is to achieve controlled response for the useful and the jammer directions signals. It is assumed that the jammers come form fixed directions. The response vector

$\mathbf{f} = (\sqrt{10} \ \sqrt{10} \ \sqrt{10} \ \sqrt{10} \ \sqrt{10} \ \sqrt{10} \ 0.0001 \ 0.0001)^T$ This response vector assure that the antenna must have 10 dB gain in the direction of the all the GPS useful signals. It guarantees that the null depth corresponds to directions 58° and -20° be -80 dB. From Fig.5, and TABLE 2 the antenna power pattern achieves the constraints exactly.

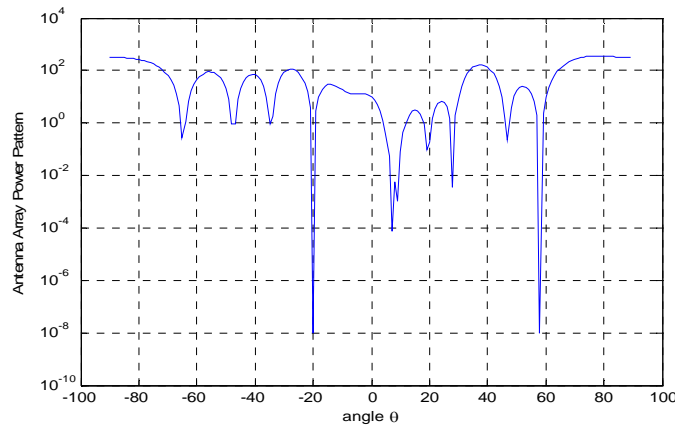


Fig.5. Antenna array power pattern MUSIC_LCMV structure in case of controlling the desired directions gain and the jammer directions nulls

TABLE 2. Summarizing the power pattern level of Fig.5,

Angle θ in degrees	Power Pattern Level in dB	
	MUSIC	LCMV
0°	10	10
30°	10	10
45°	10	10
60°	10	10
-18°	10	10
-36°	10	10
-20°	-80	-80
58°	-80	-80

3) The requirement is to achieve distortion less response in the directions of the useful signals and null the moving jammers directions. So the constraint matrix is give as:

$$(u(0^\circ) u(30^\circ) u(45^\circ) u(60^\circ) u(-18^\circ) u(-36^\circ) J(57^\circ) J(58^\circ) J(59^\circ) J(-21^\circ) J(-20^\circ) J(-19^\circ))$$

The response vector is given as,

$$\mathbf{f} = (1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)^T$$

For simulating the moving jammers, the two jammers were carried by two airplanes 600 km distance from the antenna. Both airplanes move by one mach velocity. One degree movement by the airplane corresponds to 30.7999 second. The simulation is done within 20 second by 1.7 GHz Pentium IV computer. The constraint matrix is constructed to achieve 2° null width for both jammers to assure that the jammers directions lie inside the null. Fig.6 illustrates the power pattern level for each direction given by the constraint matrix. TABLE 3 summarizes the power pattern levels for each direction given by the constraint matrix.

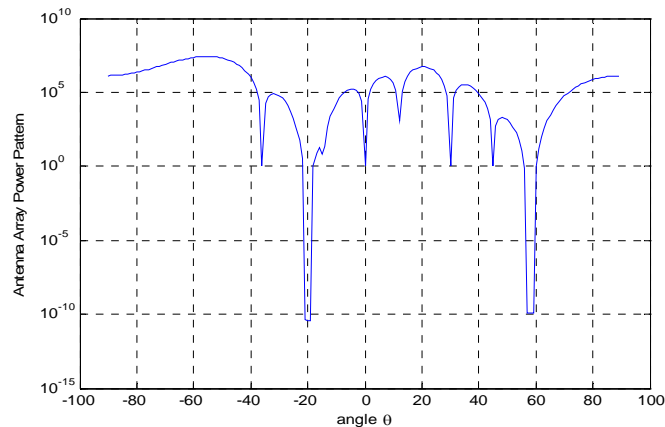


Fig.6. Antenna array power pattern in case of using MUSIC_LCMV with two moving jammers

From Table 3 the difference between all the useful signal directions and the jammer from 58° is ranging between [99.06 : 99.15] dB.

TABLE 3. Summarizing the power pattern level of Fig.6.

Angle θ in degrees	Power Pattern Level in dB MUSIC LCMV
0°	0
30°	0
45°	0
60°	0
-18°	0
-36°	0
-21°	-103.48
-20°	-104.04
-19°	-104.56
57°	-99.06
58°	-99.08
59°	-99.15

The difference between the desired signals and the jammer from -20° is ranging between [103.48 : 104.56] dB. It is obvious that the proposed structure achieves the constraints and highly suppresses the moving jammers. Also the suppression takes in consideration any miss track given by the MUSIC algorithm.

4) The requirement is to achieve 10 dB gain for the useful signals and 2° null of depth - 80 dB in the moving jammers directions. The response vector that achieve the above requirements is:

$$\mathbf{f} = (\sqrt{10} \quad \sqrt{10} \quad \sqrt{10} \quad \sqrt{10} \quad \sqrt{10} \quad \sqrt{10} \quad 0.0001 \quad 0.0001 \quad 0.0001 \quad 0.0001 \quad 0.0001 \quad 0.0001)^T$$

and the constraint matrix is the same like given in case 3).

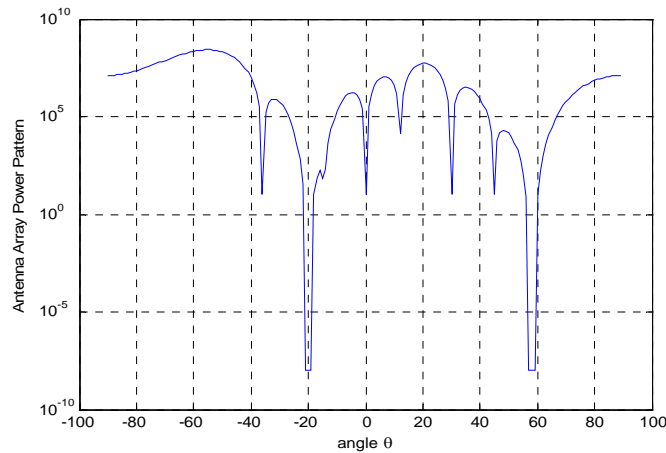


Fig.7. Antenna array power pattern in case of using MUSIC_LCMV with two moving jammers and controlling the desired and jammer directions

TABLE 4. Summarizing the power pattern level of Fig.7.

Angle θ in degrees	Power Pattern Level in dB PI LCMV
0°	10
30°	10
45°	10
60°	10
-18°	10
-36°	10
-21°	-80
-20°	-80
-19°	-80
57°	-80
58°	-80
59°	-80

As shown in from Fig.7.and TABLE 4, the proposed structure achieved the required constraints exactly.

5) This case is exactly like case 1), but after constructing the constraints matrix a new jammer of power -110 dBW form direction 10° is illuminated. Fig 8, illustrates the behavior of the proposed structure. TABLE 5 indicates that a new null of depth 43.4 dB is generated at 10° to cancel the jammer.

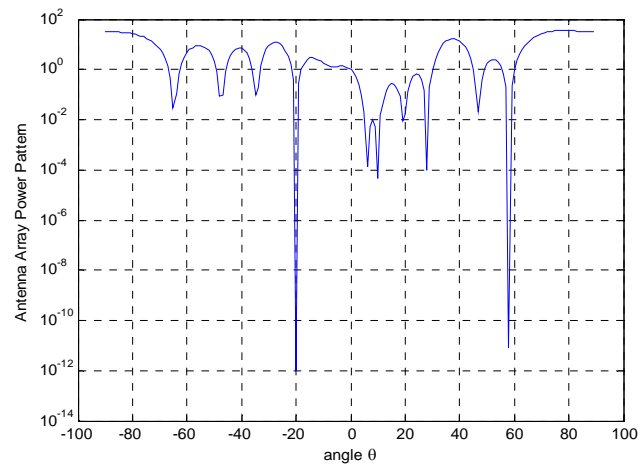


Fig.8. Antenna array power pattern in case of using MUSIC_LCMV when a new jammer from direction 10° is illuminated after updating the constrains matrix by MUSIC algorithm.

TABLE 5. Summarizing the power pattern level of Fig.8.

Angle θ in degrees	Power Pattern Level in dB MUSIC_LCMV
0°	0
30°	0
45°	0
60°	0
-18°	0
-36°	0
-20°	-120
58°	-110.8
10°	-43.4

VI-CONCLUSION

MUSIC algorithm is one of the best methods used for GPS anti jamming but when the input jammer to desired signal ratio is not very high this leads to bad output signal to noise ratio. LCMV with constraining the jammer direction give a very good result for jammer suppression but in case of GPS the jammer direction is unknown. So the proposed method utilizes the MUSIC algorithm as a pre-processor to detect blindly the jammer direction and update the constraint matrix by it. LCMV minimizing the total output power subjected to constraining both the desired and the jammer directions. The simulation takes all the possible situations in consideration like two fixed jammer and the response vector constrained the useful signal to be unity or specific values and the

jammer signals to be zeros or specific values. Two moving jammers with, constraining the useful and jammer signals to take certain values. The effect of a new jammer appears after updating the constraints matrix by the MUSIC algorithm was introduced. All the simulations indicated that the proposed structure is more efficient than the MUSIC alone.

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