
Coupled fixed point theorem for pairs with joint standard limits

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Abstract: *The present study establishes a linked fixed point in the intuitionistic fuzzy metric space for two sets of joint common limits of the range (JCLR property) and common limits of the range (CLR property) that satisfy the contractive requirements. There is weak compatibility between these mappings. Several concepts and theorems from recent research publications were used in this study, including the binary operator, t-norm, t-conorm, intuitionistic fuzzy metric space, and compatible mapping.*

Keywords: *Intuitionistic fuzzy metric space; Common couple fixed point.*

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1 Introduction

One of the most potent and productive nonlinear analysis techniques is fixed point theory [1]. Fixed-point theory can be traced back to the Banach contraction principle [2]. Mathematicians use it to solve existing problems in a variety of situations [3]. There have been numerous generalizations of the Banach contraction [3,4]. Using a specified control function, we can determine that self-maps in a metric space contain fixed points. Therefore, it is possible to establish the presence of fixed points. A number of studies have investigated these regulating functions, including one that examined various fixed-point theorems, after Khan et al. discovered them. Recently, researchers found that partial ordering of metric spaces can alleviate contraction characteristics [7]. Initially, only Ran and Reurings who employed this approach. By incorporating periodic boundary value problems for ordinary differential equations (ODEs), the technique was improved [9,10].

Zadeh introduced the concept of fuzzy sets in [11]. Based on the findings of their article [12], Sedghi et al., for weakly compatible maps, there is a common fixed-point theorem. Utilizing contractive conditions that are integral in nature. There are several contexts in which this is true. Recently, more researchers have been exploring the implications of fuzzy initial value problems for their theoretical frameworks [13]. Fuzzy derivatives, first proposed by Chang and Zadeh, have since gained popularity [14]. It was Dubosi and Prade who initially proposed the extension idea. Recent research has refined differential and integral calculus for functions with fuzzy values [16].

Our research provides several common linked fixed point theorems for mappings under a contractive condition on an intuitionistic fuzzy metric space under compatible and subsequent continuous mappings.

Our study is divided into the following sections: t-norms, t-conorms, intuitionistic fuzzy metric spaces (\mathfrak{D}), and full intuitionistic fuzzy metric spaces ($\mathfrak{f}, \mathfrak{D}$) are used as main icons of our study for achieving our target as shown in Section 2. A summary of the study's conclusions is presented in Section 3.

2 Main results

Here, the t-norms, t-conorms, intuitionistic fuzzy metric spaces (\mathfrak{D}) , and full intuitionistic fuzzy metric spaces $(\mathfrak{f}, \mathfrak{D})$ [17,18,19,20] are used as main icons of our study for achieving our target as shown where, we prescribe a common Couple fixed point in intuitionistic fuzzy metric space for two weakly compatible mappings that fulfill the ψ –contractive criteria and using the concept of the joint common limit of the range (shortly JCLR property) to prove common couple fixed point and proved common common coupled fixed -point theorem by using occasionally weakly compatible (ocw) and (CLR) property in intuitionistic fuzzy metric space.

Theorem 2.1 Let $A, B: X \times X \rightarrow X$ and $S, T: X \rightarrow X$ be mappings on intuitionistic fuzzy metric spaces $(X, M, N, \otimes, \oplus)$ where \otimes is continuous t-norm and \oplus is continuous t-conorm such that

- a) the pairs (A, S) and (B, T) are satisfy JCLR (ST) property,
- b) there exist $\psi \in \varphi, k \in (0,1)$ and $\mathfrak{z}, \eta, \mathfrak{s}, \mathfrak{p} \in X$

$$\begin{cases} M(A(\mathfrak{z}, \eta), B(\mathfrak{s}, \mathfrak{p}), k\mathfrak{S}) \geq \psi \begin{cases} M(A(\mathfrak{z}, \eta), T(\mathfrak{s}), \mathfrak{S}) \otimes M(B(\mathfrak{s}, \mathfrak{p}), S(\mathfrak{z}), \mathfrak{S}) \\ \otimes M(A(\mathfrak{z}, \eta), S(\mathfrak{z}), \mathfrak{S}) \otimes M(B(\mathfrak{s}, \mathfrak{p}), T(\mathfrak{s}), \mathfrak{S}) \\ \otimes M(S(\mathfrak{z}), T(\mathfrak{s}), \mathfrak{S}) \end{cases} \\ N(A(\mathfrak{z}, \eta), B(\mathfrak{s}, \mathfrak{p}), k\mathfrak{S}) \leq \psi \begin{cases} N(A(\mathfrak{z}, \eta), T(\mathfrak{s}), \mathfrak{S}) \oplus N(B(\mathfrak{s}, \mathfrak{p}), S(\mathfrak{z}), \mathfrak{S}) \\ \oplus N(A(\mathfrak{z}, \eta), S(\mathfrak{z}), \mathfrak{S}) \oplus N(B(\mathfrak{s}, \mathfrak{p}), T(\mathfrak{s}), \mathfrak{S}) \\ \oplus N(S(\mathfrak{z}), T(\mathfrak{s}), \mathfrak{S}) \end{cases} \end{cases}$$

- c) The pairs (A, S) and (B, T) are weakly compatible.

Then the pairs (A, S) and (B, T) have common coincident point and unique common fixed point in X .

Proof. The pairs (A, S) and (B, T) are satisfy the JCLR (ST) property, it satisfies the condition

$$\begin{cases} \lim_{\alpha \rightarrow \infty} A(\mathfrak{z}_\alpha, \eta_\alpha) = \lim_{\alpha \rightarrow \infty} S(\mathfrak{z}_\alpha) = S(\mathfrak{C}) = \lim_{\alpha \rightarrow \infty} B(\mathfrak{s}_\alpha, \mathfrak{p}_\alpha) = \lim_{\alpha \rightarrow \infty} T(\mathfrak{s}_\alpha) = T(\mathfrak{C}), \\ \lim_{\alpha \rightarrow \infty} A(\eta_\alpha, \mathfrak{z}_\alpha) = \lim_{\alpha \rightarrow \infty} S(\eta_\alpha) = S(\mathfrak{I}) = \lim_{\alpha \rightarrow \infty} B(\mathfrak{p}_\alpha, \mathfrak{s}_\alpha) = \lim_{\alpha \rightarrow \infty} T(\mathfrak{p}_\alpha) = T(\mathfrak{I}). \end{cases}$$

for all $\mathfrak{C}, \mathfrak{I} \in X$.

We are going to prove $B(\mathfrak{C}, \mathfrak{I}) = T(\mathfrak{C})$, from condition JCLR (ST) property and letting $\alpha \rightarrow \infty$ we have.

$$\begin{cases} M(A(\mathfrak{z}_\alpha, \eta_\alpha), B(\mathfrak{C}, \mathfrak{I}), k\mathfrak{S}) \geq \psi \begin{cases} M(A(\mathfrak{z}_\alpha, \eta_\alpha), T(\mathfrak{C}), \mathfrak{S}) \otimes M(B(\mathfrak{C}, \mathfrak{I}), S(\mathfrak{z}_\alpha), \mathfrak{S}) \\ \otimes M(A(\mathfrak{z}_\alpha, \eta_\alpha), S(\mathfrak{z}_\alpha), \mathfrak{S}) \otimes M(B(\mathfrak{C}, \mathfrak{I}), T(\mathfrak{C}), \mathfrak{S}) \\ \otimes M(S(\mathfrak{z}_\alpha), T(\mathfrak{C}), \mathfrak{S}) \end{cases} \\ N(A(\mathfrak{z}_\alpha, \eta_\alpha), B(\mathfrak{C}, \mathfrak{I}), k\mathfrak{S}) \leq \psi \begin{cases} N(A(\mathfrak{z}_\alpha, \eta_\alpha), T(\mathfrak{C}), \mathfrak{S}) \oplus N(B(\mathfrak{C}, \mathfrak{I}), S(\mathfrak{z}_\alpha), \mathfrak{S}) \\ \oplus N(A(\mathfrak{z}_\alpha, \eta_\alpha), S(\mathfrak{z}_\alpha), \mathfrak{S}) \oplus N(B(\mathfrak{C}, \mathfrak{I}), T(\mathfrak{C}), \mathfrak{S}) \\ \oplus N(S(\mathfrak{z}_\alpha), T(\mathfrak{C}), \mathfrak{S}) \end{cases} \end{cases}$$

we have $B(\mathfrak{C}, \mathfrak{I}) = T(\mathfrak{C})$. Similiar way, we get $B(\mathfrak{I}, \mathfrak{C}) = T(\mathfrak{I})$.

By using the condition b) and letting $\alpha \rightarrow \infty$ and condition JCLR (ST) property and $B(\mathfrak{C}, \mathfrak{I}) = T(\mathfrak{C})$ and $B(\mathfrak{I}, \mathfrak{C}) = T(\mathfrak{I})$ we have

$$\begin{cases} M(A(\mathfrak{C}, \mathfrak{I}), B(\mathfrak{s}_\alpha, \mathfrak{p}_\alpha), k\mathfrak{S}) \geq \psi \begin{cases} M(A(\mathfrak{C}, \mathfrak{I}), T(\mathfrak{s}_\alpha), \mathfrak{S}) \otimes M(B(\mathfrak{s}_\alpha, \mathfrak{p}_\alpha), S(\mathfrak{C}), \mathfrak{S}) \\ \otimes M(A(\mathfrak{C}, \mathfrak{I}), S(\mathfrak{C}), \mathfrak{S}) \otimes M(B(\mathfrak{s}_\alpha, \mathfrak{p}_\alpha), T(\mathfrak{s}_\alpha), \mathfrak{S}) \\ \otimes M(S(\mathfrak{C}), T(\mathfrak{s}_\alpha), \mathfrak{S}) \end{cases} \\ N(A(\mathfrak{C}, \mathfrak{I}), B(\mathfrak{s}_\alpha, \mathfrak{p}_\alpha), k\mathfrak{S}) \leq \psi \begin{cases} N(A(\mathfrak{C}, \mathfrak{I}), T(\mathfrak{s}_\alpha), \mathfrak{S}) \oplus N(B(\mathfrak{s}_\alpha, \mathfrak{p}_\alpha), S(\mathfrak{C}), \mathfrak{S}) \\ \oplus N(A(\mathfrak{C}, \mathfrak{I}), S(\mathfrak{C}), \mathfrak{S}) \oplus N(B(\mathfrak{s}_\alpha, \mathfrak{p}_\alpha), T(\mathfrak{s}_\alpha), \mathfrak{S}) \\ \oplus N(S(\mathfrak{C}), T(\mathfrak{s}_\alpha), \mathfrak{S}) \end{cases} \end{cases}$$

We get $A(\mathfrak{C}, \mathfrak{I}) = S(\mathfrak{C}) = B(\mathfrak{C}, \mathfrak{I}) = T(\mathfrak{C})$,

Repeating this process, we get $A(\mathfrak{I}, \mathfrak{C}) = S(\mathfrak{I}) = B(\mathfrak{I}, \mathfrak{C}) = T(\mathfrak{I})$.

Then the pairs (A, S) and (B, T) have common coupled coincidence points $\mathfrak{C}, \mathfrak{I} \in X$. We assume

$$\begin{cases} A(\mathfrak{C}, \mathfrak{I}) = S(\mathfrak{C}) = B(\mathfrak{C}, \mathfrak{I}) = T(\mathfrak{C}) = a \\ A(\mathfrak{I}, \mathfrak{C}) = S(\mathfrak{I}) = B(\mathfrak{I}, \mathfrak{C}) = T(\mathfrak{I}) = b \end{cases}$$

Where $a, b \in X$, since the pairs (A, S) and (B, T) are weakly compatible.

$$\begin{cases} S(A(\mathfrak{C}, \mathfrak{I})) = A(S\mathfrak{C}, S\mathfrak{I}) \text{ and } S(A(\mathfrak{I}, \mathfrak{C})) = A(S\mathfrak{I}, S\mathfrak{C}), \\ T(B(\mathfrak{C}, \mathfrak{I})) = B(T\mathfrak{C}, T\mathfrak{I}) \text{ and } T(B(\mathfrak{I}, \mathfrak{C})) = B(T\mathfrak{I}, T\mathfrak{C}) \end{cases}$$

We have $S(a) = A(a, b), S(b) = A(b, a), T(a) = B(a, b) \ \& \ T(b) = B(b, a)$.

We can prove $a = A(a, b)$ and $b = A(b, a)$, by using the conditions a),b) and c) we get

$$\begin{cases} M(A(a, b), B(\mathfrak{C}, \mathfrak{I}), k\mathfrak{I}) \geq \psi \left\{ \begin{array}{l} M(A(a, b), T(\mathfrak{C}), \mathfrak{I}) \otimes M(B(\mathfrak{C}, \mathfrak{I}), S(a), \mathfrak{I}) \\ \otimes M(A(a, b), S(a), \mathfrak{I}) \otimes M(B(\mathfrak{C}, \mathfrak{I}), T(\mathfrak{C}), \mathfrak{I}) \\ \otimes M(S(a), T(\mathfrak{C}), \mathfrak{I}) \end{array} \right. \\ N(A(a, b), B(\mathfrak{C}, \mathfrak{I}), k\mathfrak{I}) \leq \psi \left\{ \begin{array}{l} N(A(a, b), T(\mathfrak{C}), \mathfrak{I}) \oplus N(B(\mathfrak{C}, \mathfrak{I}), S(a), \mathfrak{I}) \\ \oplus N(A(a, b), S(a), \mathfrak{I}) \oplus N(B(\mathfrak{C}, \mathfrak{I}), T(\mathfrak{C}), \mathfrak{I}) \\ \oplus N(S(a), T(\mathfrak{C}), \mathfrak{I}) \end{array} \right. \end{cases}$$

This implies

$$a = A(a, b) = S(a) \text{ and } b = A(b, a) = b.$$

Now, we shall prove $a = B(a, b)$ and $b = B(b, a)$. Using the same technique, we have

$$\begin{cases} M(A(\mathfrak{C}, \mathfrak{I}), B(a, b), k\mathfrak{I}) \geq \psi \left\{ \begin{array}{l} M(A(\mathfrak{C}, \mathfrak{I}), T(a), \mathfrak{I}) \otimes M(B(a, b), S(\mathfrak{C}), \mathfrak{I}) \\ \otimes M(A(\mathfrak{C}, \mathfrak{I}), S(\mathfrak{C}), \mathfrak{I}) \otimes M(B(a, b), T(a), \mathfrak{I}) \\ \otimes M(S(\mathfrak{C}), T(a), \mathfrak{I}) \end{array} \right. \\ N(A(\mathfrak{C}, \mathfrak{I}), B(a, b), k\mathfrak{I}) \leq \psi \left\{ \begin{array}{l} N(A(\mathfrak{C}, \mathfrak{I}), T(a), \mathfrak{I}) \oplus N(B(a, b), S(\mathfrak{C}), \mathfrak{I}) \\ \oplus N(A(\mathfrak{C}, \mathfrak{I}), S(\mathfrak{C}), \mathfrak{I}) \oplus N(B(a, b), T(a), \mathfrak{I}) \\ \oplus N(S(\mathfrak{C}), T(a), \mathfrak{I}) \end{array} \right. \end{cases}$$

We get $a = A(a, b) = S(a) = B(a, b) = T(a)$.

Similarly, we shall show that $b = A(b, a) = S(b) = B(b, a) = T(b)$.

Finally, we are going to prove (A, S) and (B, T) have common fixed point in X . For this, we shall prove $a = b$, let supposes $a \neq b$.

$$\begin{cases} M(a, b, \mathfrak{I}) = M(A(a, b), B(b, a), k\mathfrak{I}) \geq \psi \left\{ \begin{array}{l} M(A(a, b), T(b), \mathfrak{I}) \otimes M(B(b, a), S(a), \mathfrak{I}) \\ \otimes M(A(a, b), S(a), \mathfrak{I}) \otimes M(B(b, a), T(b), \mathfrak{I}) \\ \otimes M(S(a), T(b), \mathfrak{I}) \end{array} \right. \\ N(a, b, \mathfrak{I}) = N(A(a, b), B(b, a), k\mathfrak{I}) \leq \psi \left\{ \begin{array}{l} N(A(a, b), T(b), \mathfrak{I}) \oplus N(B(b, a), S(a), \mathfrak{I}) \\ \oplus N(A(a, b), S(a), \mathfrak{I}) \oplus N(B(b, a), T(b), \mathfrak{I}) \\ \oplus N(S(a), T(b), \mathfrak{I}) \end{array} \right. \end{cases}$$

this is contradiction to our supposition. We get $a = b$, then a have common fixed point in X .

Theorem 3.2 Let $A, B: X \times X \rightarrow X$ and $S, T: X \rightarrow X$ be mappings on intuitionistic fuzzy metric spaces $(X, M, N, \otimes, \circledast)$ where \otimes is continuous t-norm and \circledast is continuous t-conorm such that

1. The pair (A, S) and (B, T) are satisfy CLR (ST) property,
2. There exist $\psi \in \Phi, k \in (0,1)$ and $z, \eta, s, p \in X$

$$\begin{cases} M(A(z, \eta), B(s, p), k\mathfrak{S}) \geq \psi(M(S(z), T(s), \mathfrak{S}) \otimes M(A(z, \eta), S(z), \mathfrak{S}) \\ \otimes M(B(s, p), T(s), \mathfrak{S})) \\ N(A(z, \eta), B(s, p), k\mathfrak{S}) \leq \psi(N(S(z), T(s), \mathfrak{S}) \circledast N(A(z, \eta), S(z), \mathfrak{S}) \\ \circledast N(B(s, p), T(s), \mathfrak{S})) \end{cases}$$
3. The pairs (A, S) and (B, T) are occasionally weakly compatible.

Then (A, S) and (B, T) have point of coincidence and unique common fixed point in X .

Proof. The pairs (A, S) and (B, T) are satisfy the (CLR) property, there exist a sequence $z_\alpha, \eta_\alpha, z'_\alpha, \eta'_\alpha \in X$ and satisfies the condition

$$\begin{cases} \lim_{\alpha \rightarrow \infty} A(z_\alpha, \eta_\alpha) = \lim_{\alpha \rightarrow \infty} S(z_\alpha) = S(\mathfrak{C}) \text{ and } \lim_{\alpha \rightarrow \infty} B(z'_\alpha, \eta'_\alpha) = \lim_{\alpha \rightarrow \infty} T(z'_\alpha) = T(\mathfrak{C}'), \\ \lim_{\alpha \rightarrow \infty} A(\eta_\alpha, z_\alpha) = \lim_{\alpha \rightarrow \infty} S(\eta_\alpha) = S(\mathfrak{I}) \text{ and } \lim_{\alpha \rightarrow \infty} B(\eta'_\alpha, z'_\alpha) = \lim_{\alpha \rightarrow \infty} T(\eta'_\alpha) = T(\mathfrak{I}'). \end{cases}$$

for all $\mathfrak{C}, \mathfrak{C}', \mathfrak{I}, \mathfrak{I}' \in X$.

We are showing the pairs (A, S) and (B, T) have common coincidence point. we have

$$\begin{cases} M(A(z_\alpha, \eta_\alpha), B(z'_\alpha, \eta'_\alpha), k\mathfrak{S}) \geq \psi(M(S(z_\alpha), T(z'_\alpha), \mathfrak{S}) \otimes M(A(z_\alpha, \eta_\alpha), S(z_\alpha), \mathfrak{S}) \\ \otimes M(B(z'_\alpha, \eta'_\alpha), T(z'_\alpha), \mathfrak{S})) \\ N(A(z_\alpha, \eta_\alpha), B(z'_\alpha, \eta'_\alpha), k\mathfrak{S}) \leq \psi(N(S(z_\alpha), T(z'_\alpha), \mathfrak{S}) \circledast N(A(z_\alpha, \eta_\alpha), S(z_\alpha), \mathfrak{S}) \\ \circledast N(B(z'_\alpha, \eta'_\alpha), T(z'_\alpha), \mathfrak{S})) \end{cases}$$

Letting $\alpha \rightarrow \infty$, we get

$$\begin{cases} M(S(\mathfrak{C}), T(\mathfrak{C}'), k\mathfrak{S}) \geq \psi(M(S(\mathfrak{C}), T(\mathfrak{C}'), \mathfrak{S}) \otimes 1 \otimes 1) \\ M(S(\mathfrak{C}), T(\mathfrak{C}'), k\mathfrak{S}) \geq M(S(\mathfrak{C}), T(\mathfrak{C}'), \mathfrak{S}) \\ N(S(\mathfrak{C}), T(\mathfrak{C}'), k\mathfrak{S}) \leq \psi(N(S(\mathfrak{C}), T(\mathfrak{C}'), \mathfrak{S}) \circledast 0 \circledast 0) \\ N(S(\mathfrak{C}), T(\mathfrak{C}'), k\mathfrak{S}) \leq N(S(\mathfrak{C}), T(\mathfrak{C}'), \mathfrak{S}) \end{cases}$$

Then, we get $S(\mathfrak{C}) = T(\mathfrak{C}')$, similar way, we get $S(\mathfrak{I}) = T(\mathfrak{I}')$. By using the condition 2), we have

$$\begin{cases} M(A(\eta_\alpha, z_\alpha), B(z'_\alpha, \eta'_\alpha), k\mathfrak{S}) \geq \psi(M(S(\eta_\alpha), T(z'_\alpha), \mathfrak{S}) \otimes M(A(\eta_\alpha, z_\alpha), S(\eta_\alpha), \mathfrak{S}) \\ \otimes M(B(z'_\alpha, \eta'_\alpha), T(z'_\alpha), \mathfrak{S})), \\ N(A(\eta_\alpha, z_\alpha), B(z'_\alpha, \eta'_\alpha), k\mathfrak{S}) \leq \psi(N(S(\eta_\alpha), T(z'_\alpha), \mathfrak{S}) \circledast N(A(\eta_\alpha, z_\alpha), S(\eta_\alpha), \mathfrak{S}) \\ \circledast N(B(z'_\alpha, \eta'_\alpha), T(z'_\alpha), \mathfrak{S})) \end{cases}$$

Letting $\alpha \rightarrow \infty$, we get

$$\begin{cases} M(S(\mathfrak{I}), T(\mathfrak{C}'), k\mathfrak{S}) \geq \psi(M(S(\mathfrak{I}), T(\mathfrak{C}'), \mathfrak{S}) \otimes M(S(\mathfrak{I}), S(\mathfrak{I}), \mathfrak{S}) \otimes M(T(\mathfrak{I}'), T(\mathfrak{I}'), \mathfrak{S})), \\ N(S(\mathfrak{I}), T(\mathfrak{C}'), k\mathfrak{S}) \leq \psi(N(S(\mathfrak{I}), T(\mathfrak{C}'), \mathfrak{S}) \circledast N(S(\mathfrak{I}), S(\mathfrak{I}), \mathfrak{S}) \circledast N(T(\mathfrak{I}'), T(\mathfrak{I}'), \mathfrak{S})) \end{cases}$$

We get $S(\mathfrak{I}) = T(\mathfrak{C}')$. Hence $S(\mathfrak{I}) = T(\mathfrak{C}') = S(\mathfrak{C}) = T(\mathfrak{I}')$ and using the conditions 2) we have

$$\left\{ \begin{array}{l} M(A(\beta_\alpha, \eta_\alpha), B(\mathcal{C}', \mathcal{I}'), k\mathfrak{S}) \geq \psi(M(S(\beta_\alpha), T(\mathcal{C}'), \mathfrak{S}) \otimes M(A(\beta_\alpha, \eta_\alpha), S(\beta_\alpha), \mathfrak{S}) \\ \quad \otimes M(B(\mathcal{C}', \mathcal{I}'), T(\mathcal{C}'), \mathfrak{S})), \\ N(A(\beta_\alpha, \eta_\alpha), B(\mathcal{C}', \mathcal{I}'), k\mathfrak{S}) \leq \psi(N(S(\beta_\alpha), T(\mathcal{C}'), \mathfrak{S}) \otimes N(A(\beta_\alpha, \eta_\alpha), S(\beta_\alpha), \mathfrak{S}) \\ \quad \otimes N(B(\mathcal{C}', \mathcal{I}'), T(\mathcal{C}'), \mathfrak{S})). \\ M(S(\mathcal{C}), B(\mathcal{C}', \mathcal{I}'), k\mathfrak{S}) \geq M(S(\mathcal{C}), B(\mathcal{C}', \mathcal{I}'), \mathfrak{S}), \\ N(S(\mathcal{C}), B(\mathcal{C}', \mathcal{I}'), k\mathfrak{S}) \leq N(S(\mathcal{C}), B(\mathcal{C}', \mathcal{I}'), \mathfrak{S}). \end{array} \right.$$

Then, we get $S(\mathcal{C}) = B(\mathcal{C}', \mathcal{I}')$. Repeating this process, we get $S(\mathcal{I}) = B(\mathcal{I}', \mathcal{C}')$.

Hence, we can have $T(\mathcal{C}') = A(\mathcal{C}, \mathcal{I})$ & $T(\mathcal{I}') = A(\mathcal{I}, \mathcal{C})$. thus, $B(\mathcal{C}', \mathcal{I}') = S(\mathcal{C}) = T(\mathcal{C}') = A(\mathcal{C}, \mathcal{I})$ & $B(\mathcal{I}', \mathcal{C}') = S(\mathcal{I}) = T(\mathcal{I}') = A(\mathcal{I}, \mathcal{C})$. Since the pair (A, S) and (B, T) are occasionally weakly compatible.

$$\left\{ \begin{array}{l} S(\mathcal{C}) = A(\mathcal{C}, \mathcal{I}) = B(\mathcal{C}', \mathcal{I}') = T(\mathcal{C}') = \mathfrak{z}, \\ S(\mathcal{I}) = A(\mathcal{I}, \mathcal{C}) = B(\mathcal{I}', \mathcal{C}') = T(\mathcal{I}') = \eta. \end{array} \right.$$

So,

$$\left\{ \begin{array}{l} S(\mathfrak{z}) = SA(\mathcal{C}, \mathcal{I}) = A(S\mathcal{C}, S\mathcal{I}) = A(\mathfrak{z}, \eta), \\ S(\eta) = SA(\mathcal{I}, \mathcal{C}) = A(S\mathcal{I}, S\mathcal{C}) = A(\eta, \mathfrak{z}). \end{array} \right.$$

And

$$\left\{ \begin{array}{l} T(\mathfrak{z}) = TB(\mathcal{I}', \mathcal{C}') = B(T\mathcal{C}', T\mathcal{I}') = B(\mathfrak{z}, \eta), \\ T(\eta) = TB(\mathcal{C}', \mathcal{I}') = B(T\mathcal{C}', T\mathcal{I}') = B(\eta, \mathfrak{z}). \end{array} \right.$$

we are going to prove $\mathfrak{z} = \eta$, by using the condition 2)

$$\left\{ \begin{array}{l} M(\mathfrak{z}, \eta, k\mathfrak{S}) = M(A(\mathcal{C}, \mathcal{I}), B(\mathcal{C}', \mathcal{I}'), k\mathfrak{S}) \geq \psi(M(S(\mathcal{C}), T(\mathcal{C}'), \mathfrak{S}) \otimes M(A(\mathcal{C}, \mathcal{I}), S(\mathcal{C}), \mathfrak{S}) \\ \quad \otimes M(B(\mathcal{C}', \mathcal{I}'), T(\mathcal{C}'), \mathfrak{S})), \\ N(\mathfrak{z}, \eta, k\mathfrak{S}) = N(A(\mathcal{C}, \mathcal{I}), B(\mathcal{C}', \mathcal{I}'), k\mathfrak{S}) \leq \psi(N(S(\mathcal{C}), T(\mathcal{C}'), \mathfrak{S}) \otimes N(A(\mathcal{C}, \mathcal{I}), S(\mathcal{C}), \mathfrak{S}) \\ \quad \otimes N(B(\mathcal{C}', \mathcal{I}'), T(\mathcal{C}'), \mathfrak{S})). \end{array} \right.$$

then, we get $\mathfrak{z} = \eta$. Now, we show $S\mathfrak{z} = T\mathfrak{z}$, by using the conditions

$$\left\{ \begin{array}{l} M(S\mathfrak{z}, T\mathfrak{z}, k\mathfrak{S}) = M(A(\mathfrak{z}, \eta), B(\eta, \mathfrak{z}), k\mathfrak{S}) \geq \psi(M(S(\mathfrak{z}), T(\eta), \mathfrak{S}) \\ \quad \otimes M(A(\mathfrak{z}, \eta), S(\mathfrak{z}), \mathfrak{S}) \otimes M(B(\eta, \mathfrak{z}), T(\eta), \mathfrak{S})), \\ N(S\mathfrak{z}, T\mathfrak{z}, k\mathfrak{S}) = N(A(\mathfrak{z}, \eta), B(\eta, \mathfrak{z}), k\mathfrak{S}) \leq \psi(N(S(\mathfrak{z}), T(\eta), \mathfrak{S}) \\ \quad \otimes N(A(\mathfrak{z}, \eta), S(\mathfrak{z}), \mathfrak{S}) \otimes N(B(\eta, \mathfrak{z}), T(\eta), \mathfrak{S})). \\ M(S\mathfrak{z}, T\mathfrak{z}, k\mathfrak{S}) \geq M(S\mathfrak{z}, T\eta, \mathfrak{S}) \\ N(S\mathfrak{z}, T\mathfrak{z}, k\mathfrak{S}) \leq N(S\mathfrak{z}, T\eta, \mathfrak{S}) \end{array} \right.$$

Then, we get $S\mathfrak{z} = T\mathfrak{z} = T\eta$. Finally, we prove $S\mathfrak{z} = \mathfrak{z}$ by using the same condition, we have

$$\left\{ \begin{array}{l} (M(S\mathfrak{z}, \mathfrak{z}, k\mathfrak{S})) = M(S\mathfrak{z}, \eta, k\mathfrak{S}) = \\ M(A(\mathfrak{z}, \eta), B(\mathfrak{z}, \eta), k\mathfrak{S}) \geq \psi(M(S\mathfrak{z}, T\mathfrak{z}, \mathfrak{S}) \otimes M(A(\mathfrak{z}, \eta), S\mathfrak{z}, \mathfrak{S}) \otimes M(B(\mathfrak{z}, \eta), T\mathfrak{z}, \mathfrak{S})), \\ N(S\mathfrak{z}, \mathfrak{z}, k\mathfrak{S}) = N(S\mathfrak{z}, \eta, k\mathfrak{S}) = \\ N(A(\mathfrak{z}, \eta), B(\mathfrak{z}, \eta), k\mathfrak{S}) \leq \psi(N(S\mathfrak{z}, T\mathfrak{z}, \mathfrak{S}) \otimes N(A(\mathfrak{z}, \eta), S\mathfrak{z}, \mathfrak{S}) \otimes N(B(\mathfrak{z}, \eta), T\mathfrak{z}, \mathfrak{S})). \end{array} \right.$$

Then UA, B, S, T have a common fixed point in X and uniqueness point in X .

3 Conclusion

As a result of this study, basic icons in intuitionistic fuzzy metric spaces are successfully defined and described. These include the binary operator, compatible mappings, and sequentially continuous mappings. We

also used these symbols to define a shared, linked fixed point in intuitionistic fuzzy metric space for two sets of mutually acceptable and sequentially continuous mappings that satisfy contractive constraints.

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