Bulletin of Faculty of Science, Zagazig University (BFSZU)
e-ISSN: 1110-1555
Volume-2023, Issue-4, pp-221-231
https://bfszu.journals.ekb.eg/journal
Research Paper DOI: 10.21608/bfszu.2023.200417.1265

==

Influence of rotation on micro-stretch the realistic medium with memory-dependent under three phase-lag model

Othman, Mohamed I. A.[1], Atwa, Sarhan[2], Eraki, Ebtesam E.M.[1] and Ismail, Mohamed F.[2]*

[1]Department of Mathematics, Faculty of Science, Zagazig University, P.O. Box 44519, Zagazig, Egypt
[2]Department of Mathematics Engineering of Physics, Higher Institute of Engineering, Shorouk Academy
(*Corresponding author: fekry3281@gmail.com)

***ABSTRACT* : This work is interested with the effect memory-dependent on a thermo-micro-stretch elastic solid under rotation, the medium is studied employing the theory of Green-Naghdi type III (G-N III) and the model of three-phase-lag (3PHL). The governing equations are formulated in the context of G-N III theory and the 3PHL model. Analytical solution to the problem is acquired by utilizing the normal mode method. The boundary conditions created at $z=0$ and applying the boundary aspects discussed to acquire equations that are acceptable with the parameter. Five equations are created as a result. We find the constant values of the solutions by applying the inverse matrix method to the five equations. The magnesium crystal element is utilized as an example to describe the observations of the G-N III theory influenced by rotation on microstretch thermoelastic with those of the 3PHL model. Rotation has been shown to have a significant impact on all physical quantities.**

KEYWORDS: Microstretch thermoelastic, normal mode method, rotation, memory dependent.

Date of Submission: 15-03-2023 Date of acceptance: 08-06-2023

List of Nomenclature

Quantity	Definition
$\boldsymbol{u}$	Displacement vector in micro-stretch medium
$\alpha_{t_1}, \alpha_{t_2}$	linear thermal expansion coefficient where $\beta_1=(3\lambda+2\mu+k)\alpha_{t_1}$, $\nu=(3\lambda+2\mu+k)\alpha_{t_2}$.
λ, μ	Lame's constants of classical elasticity
α, β, γ	Elastic constants
k	Micropolar coupled modulus
δ_{ij}	Kronecker delta
$\alpha_0, \lambda_0, \lambda_1$	Stretch constants
J_0	Equilibrated inertia
J	Micro-inertia
σ_{ij}	Component of stress tensor
k^*	Additional material
k_1^*	Thermal conductivity
Ω	Rotation
c_E	Specific heat at the constant strain
τ_ν	Phase lag of thermal displacement gradient

τ_q	The heat flux phase lag
τ_θ	The temperature gradient phase lag
ρ	Density for microstretch
m_{ij}	Component of couple stress tensor
e	Dilatation
ε_{ijr}	Alternate tensor
$\boldsymbol{\varphi}$	Micro-rotation vector
φ^*	Scalar microstretch function
ψ	the change in void volume fraction

1. INTRODUCTION

Many media, such as solids with micro-cracks, foams, animal bones, inviscid fluids, and voids media with gas-filled pores fall only out the range of micropolar elasticity. As a result, Scientists had to create a mathematical model to examine these media, and they chose microstretch as a mathematical model for these kinds of solids. In micro-stretch elastic solids, there are seven degrees of freedom: one for stretch, three for rotation and three for translation. Material points on micro-stretch bodies have the ability to expand and shrink separately of translation and rotation. The phenomenon of reflection and refraction at a plane connect between a thermo-micro-stretch elastic diffusion medium and a non-viscous fluid, medium is examined by Kumar [1]. Othman et. al. [2] employed the classical coupled theory, Lord-Shulman theory and dual-phase-lag model to explore the impact of gravity and rotation on a thermo-micro-stretch elastic form of media with diffusion. Deswal et al. [3] investigated the reflection of waves at the free surface of a nonlocal thermo micro-stretch elastic solid with the 3PHL model under temperature-dependent properties. Said [4] examined the influence of temperature-dependent properties and rotation on plane waves in an initially stressed, isotropic, homogeneous thermo-microstretch elastic solid. Jahangir et al. [5] illustrated the effects of diffusion and two temperatures on plane waves required to move across thermoelastic media. Abo-Dahab et al. [6] investigated the impacts of stretch and two temperatures on the elastic properties of a generalized thermo-micro-stretch elastic solid. Elhag et al. [7] demonstrated how anti integer derivatives orders examination influences the reflection of partial high thermal waves in the three-phase lag model for micro-stretch elastic solid.

Purkaita et. al. [8] proposed a mathematical formula of thermoelastic interaction in an unlimited space in the framework of Taylor's expansion, which includes a memory-dependent derivative of the function (G-N III) theory heat exchange law, which is described in an integral form of a mutual derivative with a kernel function on a slippery intermission. Sur and Kanoria [9] suggested an innovative magneto-thermoelasticity mathematical model for examining transient phenomena for a fiber-reinforced thick plate with a heat source in the framework of a 3PHL model of generalized thermoelasticity, which is described in an integral form of a mutual derivative on a slippery intermission by utilizing memory-dependent heat exchange. In the papers by Sarkar and Mondal [10], Said [11], Sarkar and Mukhopadhyay [12] and Li and He [13], the authors employed memory-dependent derivative generalized thermoelasticity.

Roy Choudhuri [14] created the 3PHL model for a methodology of heat exchange in which the Fourier rule is changed by a strategic plan to an adjustment of the Fourier rule with varying time translations for the thermal displacement gradient, heat flux, and temperature gradient. Kumar and Chawla [15] utilized 3PHL and DPL thermoelastic models to evaluate the spread of plane waves in an isotropic thermoelastic solid. In the presence of variable periodically various heat sources, one-dimensional thermoelastic unrest in an isotropic, unlimited, functionally graded thermo-visco-elastic solid in the framework of the 3PHL, G-N type III, and G-N type II models are examined by Sur and Kanoria [16]. After five years, the previous theories are employed to examine the effect of gravity field thermoplastic isotropic solid under two-temperature fiber-reinforced by Othman et al. [17]. Sharma et al. [18] illustrated the impact of the 3PHL generalized thermoelastic model on the ideal assessment of 3-dimensional free vibrations of a thermo-viscoelastic medium cylinder that is initially and uniformly undeformed. Many investigations, such as [19-21], have been studied of a homogeneous, isotropic, thermo-micropolar elastic solid under voids under different fields.

In this work, we concentrated our efforts to illustrate thermo-micro-stretch elastic solid under rotation with memory dependent as shown in Fig. 1, the plate is examine employing the 3PHL model and the G-N theory from type III. We began by showing the basic equations and employing non-dimensions. In the second, we employed the normal mode method to transform the partial differential equations to the ordinary differential equations. Afterward, we created the boundary conditions at $z=0$ to find the constant values of the solutions. At last, the calculations are implemented, discussed, and graphed.

2. The description of the problem and basic equations

The system of a mathematical model of a thermo-micro-stretch elastic can be formulated in a 3PHL model as [16, 22]

The equation of motion

$$\sigma_{ji,j} = \rho\,[\,u_{i,tt} + \{\,\boldsymbol{\Omega}\times(\,\boldsymbol{\Omega}\times\boldsymbol{u}\,)\}_i + (\,2\boldsymbol{\Omega}\times\boldsymbol{u}_{,t}\,)_i\,]. \tag{1}$$

The equation of micropolar

$$\varepsilon_{ijr}\sigma_{jr} + m_{ji,j} = \rho J\,[\varphi_{i,tt} + \boldsymbol{\Omega}\times\boldsymbol{\varphi}_{,t}\,] \tag{2}$$

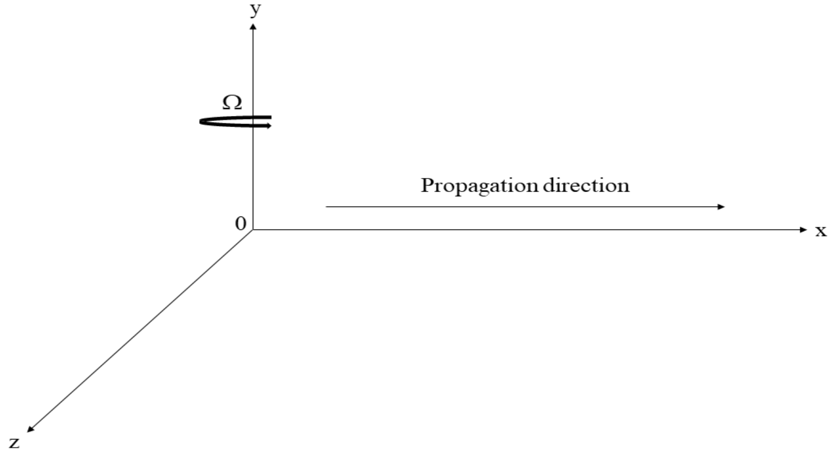

Fig. 1 Geometry of problem.

The equation of micro-stretch

$$\alpha_0\varphi^*_{,ii} + \frac{1}{3}\nu\,T - \frac{1}{3}\lambda_1\varphi^* - \frac{1}{3}\lambda_0 u_{i,i} = \frac{3}{2}\rho J_0\varphi^*_{,tt}. \tag{3}$$

The equation of heat under 3PHL model

Due to the memory-dependent influence, this is additionally a special case of the heat conduction law, as recently announced by Sur and Kanoria [9].

$$k^*(1+\tau_\nu D_{\tau_\nu})T_{,ii} + k_1^*(1+\tau_\theta D_{\tau_\theta})T_{,iit} = [1+\tau_q D_{\tau_q} + \frac{1}{2}\tau_q^2 D_{\tau_q}^2][\rho c_E T_{,tt} + \nu T_0\varphi^*_{,tt} + \beta_1 T_0 e_{,tt}]. \tag{4}$$

Where, 3PHL model, ($\tau_\nu < \tau_\theta < \tau_q \neq 0,\ k^* > 0,\ k_1^* > 0$).

G-N theory of type III, when ($\tau_\nu = \tau_\theta = \tau_q = 0,\ k^* > 0$).

$$D_{\tau_{\omega_i}}\ f\,(t) = \int_{t-\omega_i}^{t} L\,(\,t-\varsigma\,)\ f'(\,\varsigma\,)\,d\varsigma. \tag{5}$$

The parameter ω_i represents the time delay, and $L\,(\,t-\varsigma\,)$ represents the kernel function; for more information, see Caputo and Mainardi [9]. In this work, we utilized $L\,(\,t-\varsigma\,)$ in the following form. $L\,(\,t-\varsigma\,) = q + q_1\,(\,t-\varsigma\,),$ where $q,\ q_1$ are constant.

The constitutive relations

$$\sigma_{ij} = (\lambda\,u_{r,r} + \lambda_0\varphi^* - \beta_1 T)\,\delta_{ij} + \mu\,(u_{i,j} + u_{j,i}) + k\,(u_{j,i}\,\varepsilon_{ijr}\,\varphi_r), \tag{6}$$

$$m_{ij} = \alpha\,\varphi_{r,r}\delta_{ij} + \beta\,\varphi_{i,j} + \gamma\,\varphi_{j,i}\,, \tag{7}$$

$$\lambda_r = \alpha_0\,\varphi^*_{,r}, \qquad e = u_{r,r}. \tag{8}$$

From Eq. (1) and Eq. (7) for $\boldsymbol{u}\,(x,z,t)=u\,(u_1,0,u_3)$ and $\boldsymbol{\Omega}=(0,\Omega,0)$, the equations of motion can be written as

$$(\lambda+\mu)\,e_{,x}+(\mu+k)\,u_{1,ii}+\lambda_0\,\varphi^{*}_{,x}-k\varphi_{2,z}-\beta_1 T_{,x}=\rho[u_{1,tt}-\Omega^2 u_1+2\Omega u_{3,t}], \tag{9}$$

$$(\lambda+\mu)\,e_{,z}+(\mu+k)\,u_{3,ii}+\lambda_0\varphi^{*}_{,z}-k\varphi_{2,x}-\beta_1 T_{,z}=\rho[u_{3,tt}-\Omega^2 u_3+2\Omega u_{1,t}]. \tag{10}$$

From Eqs. (6) and (7) into Eq (2) for $\boldsymbol{\varphi}=(0,\varphi_2,0)$, the equation of micropolar is given by

$$\gamma\varphi_{2,ii}+k(u_{1,z}-u_{3,x})-2k\,\varphi_2=\rho\,J\,\varphi_{2,tt}. \tag{11}$$

We employ the following dimensionless variables

$$(x',z')=\frac{\omega^{*}}{c_1}(x,z),\quad u_i'=\frac{\rho\,\omega^{*}c_1 u_i}{\beta_1 T_0},\quad (t',\tau_\nu',\tau_\theta',\tau_q')=\omega^{*}(t,\tau_\nu,\tau_\theta,\tau_q),\quad \varphi_2'=\frac{\rho c_1^2\varphi_2}{\beta_1 T_0},$$

$$\varphi^{*\prime}=\frac{\rho c_1^2\varphi^{*}}{\beta_1 T_0},\quad T'=\frac{T}{T_0},\quad \sigma_{ij}'=\frac{\sigma_{ij}}{\beta_1 T_0},\quad m_{ij}'=\frac{\omega^{*}m_{ij}}{c_1\beta_1 T_0},\quad \lambda_k'=\frac{\omega^{*}\lambda_k}{c_1\beta_1 T_0},\quad \Omega'=\frac{\Omega}{\omega^{*}},\quad c_1^2=\frac{\lambda+2\mu+k}{\rho}. \tag{12}$$

After introducing the displacement potentials $\Phi\,(x,z,t)$ and $\psi\,(x,z,t)$ which correspond to displacement components, we acquire

$$u_1=\Phi_{,x}+\psi_{,z},\quad u_3=\Phi_{,z}-\psi_{,x}. \tag{13}$$

Substituting Eqs. (12) and (13) in Eqs. (3), (4), (9), (10) and (11), we obtain

$$[(a_1+a_2)\nabla^2+\Omega^2-\frac{\partial^2}{\partial t^2}]\Phi+2\Omega\psi_{,t}+a_3\varphi^{*}-T=0, \tag{14}$$

$$[a_2\nabla^2+\Omega^2-\frac{\partial^2}{\partial t^2}]\psi-2\Omega\,\Phi_{,t}-a_4\,\varphi_2=0, \tag{15}$$

$$[\nabla^2-2a_5-a_6\frac{\partial^2}{\partial t^2}]\varphi_2+\ a_5\nabla^2\psi=0, \tag{16}$$

$$[a_7\nabla^2-\frac{1}{3}a_3-\frac{3}{2}a_{10}\frac{\partial^2}{\partial t^2}]\varphi^{*}+\frac{1}{3}a_8T-\frac{1}{3}a_9\nabla^2\Phi=0, \tag{17}$$

$$(1+\tau_\nu D_{\tau_\nu})\nabla^2 T+a_{11}(1+\tau_\theta D_{\tau_\theta})\nabla^2 T_{,t}=[1+\tau_q D_{\tau_q}+\frac{1}{2}\tau_q^2 D_{\tau_q}^2][a_{12}T_{,tt}+a_{13}\varphi^{*}_{,tt}+a_{14}\nabla^2\Phi_{,tt}]. \tag{18}$$

3. Normal mode analysis

The solution of the considered physical variable can be decomposed in terms of the normal mode method take the following form:

$$[u_i,\Phi,\psi,\varphi^{*},\varphi_2,T,\sigma_{ij},m_{ij},u_i^f,\sigma_{ij}^f](x,z,t)=[\bar{u}_i,\bar{\Phi},\bar{\psi},\bar{\varphi}^{*},\bar{\varphi}_2,\bar{T},\bar{\sigma}_{ij},\bar{m}_{ij},\bar{u}_i^f,\bar{\sigma}_{ij}^f](z)\,e^{ib(x-\xi t)}. \tag{19}$$

Where, $\omega=b\,\xi$ is the frequency, $i=\sqrt{-1}$ is the complex number and b is the wave number in the $x-$ direction.

Using Eq. (19) in Eqs. (14)-(18), we have

$$(\delta_1\mathrm{D}^2+\delta_2)\,\bar{\Phi}-\delta_3\,\bar{\psi}+a_3\bar{\varphi}^{*}-\bar{T}=0, \tag{20}$$

$$\delta_5\,\bar{\Phi}+(a_2\mathrm{D}^2+\delta_4)\,\bar{\psi}-a_4\bar{\varphi}_2=0, \tag{21}$$

$$(a_5\mathrm{D}^2-a_5b^2)\,\bar{\psi}+(\mathrm{D}^2+\delta_6)\,\bar{\varphi}_2=0, \tag{22}$$

$$(\delta_8\mathrm{D}^2+\delta_9)\,\bar{\Phi}+(a_7\mathrm{D}^2+\delta_7)\,\bar{\varphi}^{*}+\delta_{10}T=0, \tag{23}$$

$$(\delta_{12}\mathrm{D}^2-\delta_{13})\,\bar{\Phi}+\delta_{16}\bar{\varphi}^*+(\delta_{14}\mathrm{D}^2+\delta_{15})\,\bar{T}=0, \tag{24}$$

The presence of non-trivial solutions necessitates the fulfilment of the following conditions that are required and sufficient, i.e., the determinant of the preceding equation (20) - (24) needs to be zero, we get

$$(\mathrm{D}^{10}-A\,\mathrm{D}^8+B\,\mathrm{D}^6-C\,\mathrm{D}^4+E\,\mathrm{D}^2-F)\{\bar{\Phi}(z),\bar{\psi}(z),\bar{\varphi}_2(z),\bar{\varphi}^*(z),\bar{T}(z)\}=0. \tag{25}$$

The coefficients of Eq. (25) A,B,C,E,F are defined in Appendix.
Eq. (25) can be factorized as:

$$(\mathrm{D}^2-k_1^2)(\mathrm{D}^2-k_2^2)(\mathrm{D}^2-k_3^2)(\mathrm{D}^2-k_4^2)(\mathrm{D}^2-k_5^2)\{\bar{\Phi}(z),\bar{\psi}(z),\bar{\varphi}_2(z),\bar{\varphi}^*(z),\bar{T}(z)\}=0. \tag{26}$$

Where, $k_n^2,(n=1,2,3,4,5)$ are the roots of the characteristic equation of Eq. (26).
The solution of Eq. (26), which are bounded as $z\rightarrow\infty$ are written:

$$\bar{\Phi}(z)=\sum_{n=1}^{5}M_n\,e^{-k_n z}, \tag{27}$$

$$\bar{\psi}(z)=\sum_{n=1}^{5}H_{1n}\,M_n\,e^{-k_n z}, \tag{28}$$

$$\bar{\varphi}_2(z)=\sum_{n=1}^{5}H_{2n}\,M_n\,e^{-k_n z}, \tag{29}$$

$$\bar{T}(z)=\sum_{n=1}^{5}H_{3n}\,M_n\,e^{-k_n z}, \tag{30}$$

$$\bar{\varphi}^*(z)=\sum_{n=1}^{5}H_{4n}M_n\,e^{-k_n z}. \tag{31}$$

Substituting from Eqs. (27) and (28) in Eq. (13) we acquire

$$\bar{u}_1(z)=\sum_{n=1}^{5}[ib-k_nH_{1n}]M_n e^{-k_n z}, \tag{32}$$

$$\bar{u}_3(z)=\sum_{n=1}^{5}[-k_n-ibH_{1n}]\,M_n\,e^{-k_n z}, \tag{33}$$

By compensation from Eqs. (12) and (19) into Eqs. (6) and by using Eqs. (29) - (33) we infer that the components of the stress tensor, the following components

$$\bar{\sigma}_{xx}(z)=\sum_{n=1}^{5}H_{5n}M_n\,e^{-k_n z}, \tag{34}$$

$$\bar{\sigma}_{yy}(z)=\sum_{n=1}^{5}H_{6n}M_n\,e^{-k_n z}, \tag{35}$$

$$\bar{\sigma}_{zz}(z)=\sum_{n=1}^{5}H_{7n}M_n\,e^{-k_n z}, \tag{36}$$

$$\bar{\sigma}_{xz}(z)=\sum_{n=1}^{5}H_{8n}M_n\,e^{-k_n z}, \tag{37}$$

$$\bar{\sigma}_{zx}(z)=\sum_{n=1}^{5}H_{9n}M_n\,e^{-k_n z}, \tag{38}$$

By compensation from Eqs. (12) and (19) into Eqs. (7) and (8) and by using Eqs. (29) - (31), the couple stress tensor components and the micro-stress tensor have the form

$$\bar{m}_{xy}(z)=\sum_{n=1}^{5} i\, b a_{18} H_{2n} M_n\, e^{-k_n z}, \tag{39}$$

$$\bar{m}_{yx}(z)=\sum_{n=1}^{5} i\, b a_{19} H_{2n} M_n\, e^{-k_n z}, \tag{40}$$

$$\bar{m}_{zy}(z)=\sum_{n=1}^{5} -a_{18} k_n H_{2n} M_n\, e^{-k_n z}, \tag{41}$$

$$\bar{m}_{yz}(z)=\sum_{n=1}^{5} -a_{19}\, k_n\, H_{2n}\, M_n\, e^{-k_n z}, \tag{42}$$

$$\bar{\lambda}_x(z)=\sum_{n=1}^{5} i\, b a_7 H_{4n} M_n\, e^{-k_n z}, \tag{43}$$

$$\bar{\lambda}_z(z)=\sum_{n=1}^{5} -a_7 k_n H_{4n} M_n\, e^{-k_n z}. \tag{44}$$

Where the coefficients a_m , $H_{n'n}$, A, C, F, B, E and $\delta_{m'}$, where $m=(1,2,......,19)$, $m'=(1,2,......,16)$, $n'=(1,2,.....,9)$ are given in **Appendix**

4. Boundary conditions

The boundary conditions for the problem at $z=0$, to determine the constants $M_n, n=(1,2,......,5)$, are

$$\varphi^*=0,\ m_{zy}=0,\ \lambda_z=0,\ T=f_1 e^{i b(x-\xi t)},\ \sigma_{xx}=p\, e^{i b(x-\xi t)},\ \text{at}\ z=0. \tag{45}$$

Where, p is the magnitude of the mechanical force, and f_1 thermal shock. Applying the use of variable expressions implied into the boundary aspects discussed above (45) to acquire equations that are acceptable with the parameters. As a result, five equations are created. When we apply the inverse matrix method to the five equations, we have the value constant $M_n, n=(1,2,......,5)$,

$$\begin{bmatrix} M_1\\ M_2\\ M_3\\ M_4\\ M_5 \end{bmatrix}=\begin{bmatrix} H_{41} & H_{42} & H_{43} & H_{44} & H_{45}\\ -a_{18}k_1H_{21} & -a_{18}k_2H_{22} & -a_{18}k_3H_{23} & -a_{18}k_4H_{24} & -a_{18}k_5H_{25}\\ -a_7k_1H_{41} & -a_7k_2H_{42} & -a_7k_3H_{43} & -a_7k_4H_{44} & -a_7k_5H_{45}\\ H_{31} & H_{32} & H_{33} & H_{34} & H_{35}\\ H_{51} & H_{52} & H_{53} & H_{54} & H_{55} \end{bmatrix}^{-1}\begin{bmatrix} 0\\ 0\\ 0\\ f_1\\ p \end{bmatrix} \tag{46}$$

5. Numerical results and discussions

The analysis has been carried out for magnesium crystal-like material [22]

$\rho=1.47\times10^3\ \mathrm{kg.m^{-3}}$, $\lambda=9.4\times10^{10}\ \mathrm{N.m^{-2}}$, $\mu=4\times10^{10}\ \mathrm{N.m^{-2}}$, $J=0.2\times10^{-19}\ \mathrm{m^2}$, $J_0=1.85\times10^{-19}\ \mathrm{m^2}$, $k=1\times10^{10}\ \mathrm{N.m^{-2}}$, $\alpha_0=0.779\times10^{-9}\ \mathrm{N}$, $\lambda_1=0.5\times10^{10}\ \mathrm{N.m^{-2}}$, $\gamma=0.779\times10^{-9}\ \mathrm{N}$, $T_0=298^\circ\ \mathrm{K}$, $\lambda_0=0.5\times10^{10}\ \mathrm{N.m^{-2}}$, $\beta_1=2.68\times10^6\ \mathrm{N.m^{-2}.k^{-1}}$, $\nu=2\times10^6\ \mathrm{N.m^{-2}.k^{-1}}$, $c_E=1.04\times10^3\ \mathrm{J.kg^{-1}.k^{-1}}$, $k_1^*=1.7\times10^2\ \mathrm{J.m^{-1}.s^{-1}.k^{-1}}$, $\tau_\nu=0.0171\mathrm{s}$, $\tau_\theta=0.031\mathrm{s}$, $\tau_q=0.5\mathrm{s}$, $\omega_1=0.03\,\mathrm{s}$, $\omega_2=0.06\,\mathrm{s}$, $\omega_2=0.09\,\mathrm{s}$, $q=4$, $q_1=5$, $\omega=\omega_0+i\,\breve{\omega}_0$, $\omega_0=3.1099$, $\breve{\omega}_0=3.906$, $b=2$, $p=1.0502$.

In this paper, All physical quantities are calculated for the dimensionless time value $t=1.18$ on the range $0\le z\le1.5$ on the surface $x=1.18$. The numerical strategy introduced here is employed to clarify the variance of physical quantities u_1, u_3, φ^*, σ_{xx}, σ_{zz} and σ_{xz} against the distance z. The graph demonstrates the predicted curves of the theory of G-N III and the model of 3PHL. Figs. 2-7 clarify a comparison among the model of 3PHL and the theory of G-N III in the complete absence and presence of rotation. Fig. 2 shows the variation of u_1 against a distance z. It observed that the two Curves focused on the 3PHL model begin at identical point and decrease until they vanish at $z \geq 1.5$, Furthermore, there are two other curves focused on G-N III theory that begin at an identical point but differ from the previous point and decrease up to vanish at $z \geq 1.5$. Figs. 3 and 5 illustrate the values of u_3 and σ_{xx} Every time begin with positive values and gradually decrease them over the range $0 \leq z \leq 1.5$, after that converge to zero over the range $z \geq 1.5$. It is observed that the values of u_3 and σ_{xx} based on the G-N III theory are small compared with the values of u_3 and σ_{xx} based on the 3PHL model.

Fig. 4 clarifies the change micro-stretch φ^* with distance z. When rotation is present, the values of φ^* every time begin by increasing to a highest, then reduced, and at last, go to zero. In the existence of rotation, the values of φ^* focused on the 3PHL model are also large in comparison to the values of φ^* focused on the G-N III theory. Figs. 6 and 7 clarify the value of σ_{zz} and σ_{xz} every time began with negative values and increase over the range $0 \leq z \leq 1.5$, then go to zero over the range $z \geq 1.5$. It is observed that the values of σ_{zz} and σ_{xz} based on the 3PHL model begins at the exact same point, which is also the relatively similar point in Figs. 6 and 7.

6. Conclusion

In this paper, the problem is interested with the influence rotation on a thermo-micro-stretch elastic solid under memory-dependent, the medium is investigated employing the theory of Green-Naghdi type III and the model of three-phase-lag. Utilizing the normal mode analysis, the problem has been solved, from the results explained above, it can be concluded that:

1- All physical quantities go to zero as distance increases, as well as all functions are continuous

2- The effect of rotation plays a critical role in this investigation of thermoelastic solid deformation.

3- A sort of contrast is established between the model of 3PHL and the theory of G-N III in the complete absence and presence of rotation.

4- The nature of the applied force as well as the type of boundary conditions influence the deformation of a body.

5- The physical quantities are satisfying all the boundary conditions.

6- The investigation can be used in geotechnical engineering, seismology, solid dynamics, seismic analysis, and other fields.

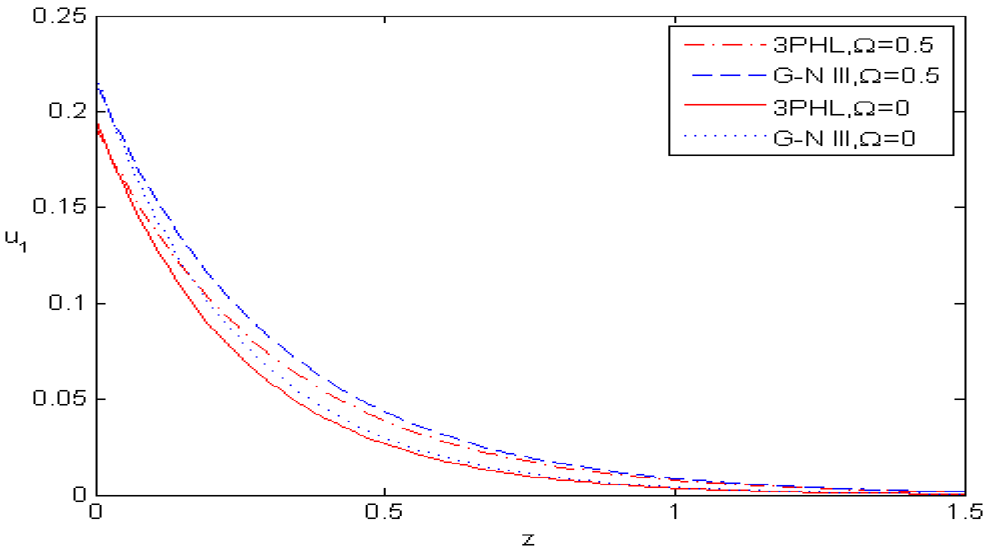

Fig. 2 Variation of the horizontal displacement u_1 against z.

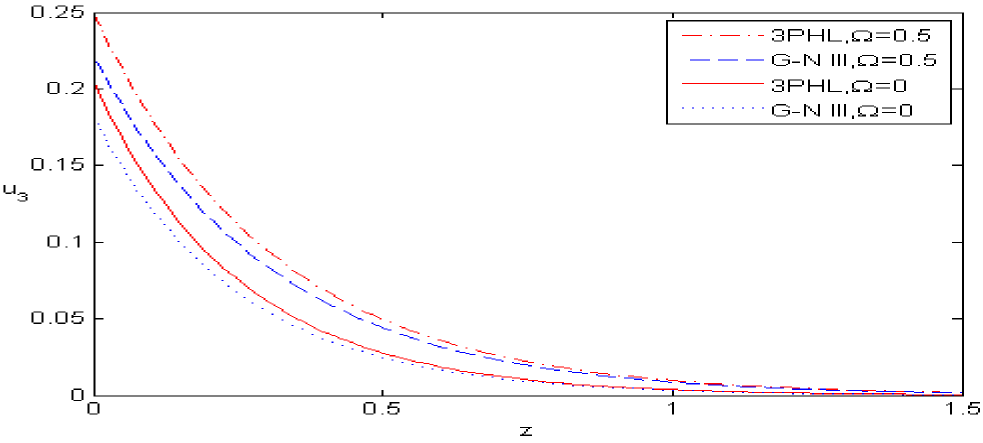

Fig. 3 Variation of the vertical displacement u_3 against z.

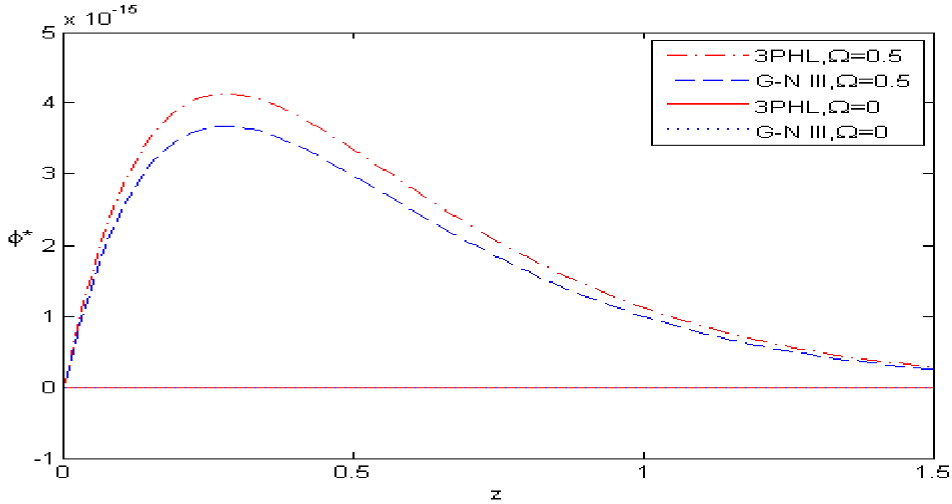

Fig. 4 Variation of the scalar microstretch $\varphi*$ against z.

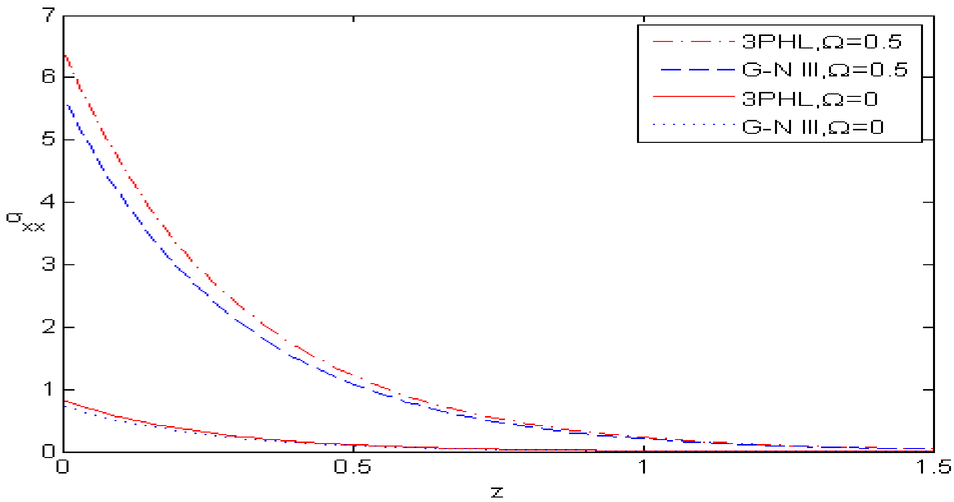

Fig. 5 Variation of the stress component σ_{xx} against z.

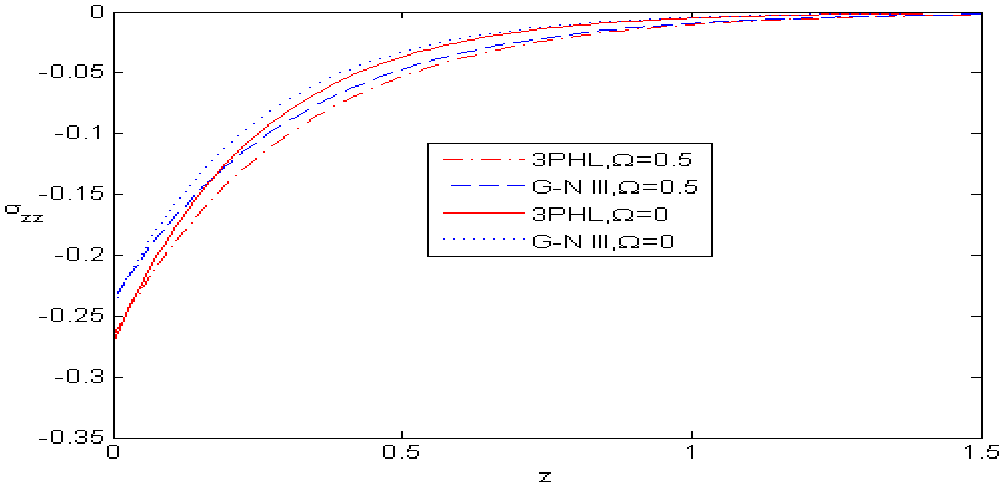

Fig. 6 Variation of the stress component σ_{zz} against z.

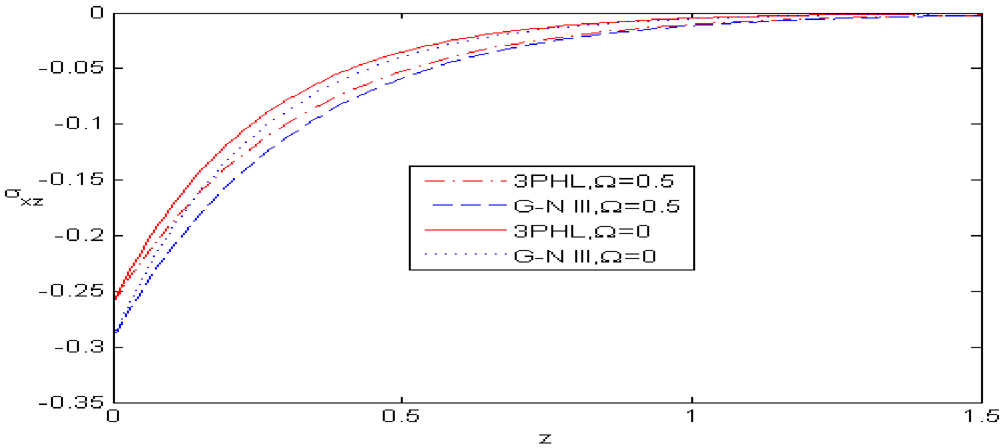

Fig. 7 Variation of the stress component σ_{xz} against z.

Declaration of conflicting interests: The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding: The author received no financial support for the research, authorship, and/or publication of this article.

APPENDIX

$$a_1=\frac{\lambda+\mu}{\rho c_1^2},\quad a_2=\frac{\lambda+k}{\rho c_1^2},\quad a_3=\frac{\lambda_0}{\rho c_1^2},\quad a_4=\frac{k}{\rho c_1^2},\quad a_5=\frac{k c_1^2}{\gamma \omega^{*2}},\quad a_6=\frac{\rho j c_1^2}{\gamma},\quad a_7=\frac{\alpha_0 \omega^{*2}}{\rho c_1^4},\quad a_8=\frac{\nu}{\beta_1},$$

$$a_9=\frac{\lambda_0}{\rho c_1^2},\quad a_{10}=\frac{j_0 \omega^{*2}}{c_1^2},\quad a_{11}=\frac{k_1^* \omega^*}{k^*},\quad a_{12}=\frac{\rho c_E c_1^2}{k^*},\quad a_{13}=\frac{\nu T_0 \beta_1}{\rho k^*},\quad a_{14}=\frac{\beta_1^2 T_0}{\rho k^*},\quad a_{15}=\frac{\lambda}{\rho c_1^2},$$

$$a_{16}=\frac{2\mu+k}{\rho c_1^2},\quad a_{17}=\frac{\mu}{\rho c_1^2},\quad a_{18}=\frac{\gamma \omega^{*2}}{\rho c_1^4},\quad a_{19}=\frac{\beta \omega^{*2}}{\rho c_1^4},\quad \delta_1=(a_1+a_2),\quad \delta_2=-b^2\delta_1+b^2\xi^2+\Omega^2,$$

$$\delta_3=2ib\xi\Omega,\quad \delta_4=b^2\xi^2-a_2 b^2+\Omega^2,\quad \delta_5=\delta_3,\quad \delta_6=a_6 b^2\xi^2-2a_5-b^2,\quad \delta_7=\frac{3}{2}a_{10}b^2\xi^2-a_7 b^2-\frac{1}{3}a_3,$$

$$\delta_8=-\frac{1}{3}a_9,\quad \delta_9=-\delta_8 b^2,\quad \delta_{10}=\frac{1}{3}a_8,\quad \delta_{11}=1+G_1+G_2,\quad \delta_{12}=a_{14}b^2\xi^2,\quad \delta_{13}=a_{14}b^4\xi^2,$$

$$\delta_{14}=(1+G_3)-ib\xi a_{11}(1+G_2),\quad \delta_{15}=-b^2(1+G_3)+ib^3\xi a_{11}(1+G_2)+a_{12}\delta_{11}b^2\xi^2,\quad \delta_{16}=\delta_{11}a_{13}b^2\xi^2,$$

$$G_1=\frac{\tau_q}{\omega_1}[\,(\,q-\frac{q_1}{i\,b\,\xi})\,(1-e^{i\,b\,\xi\,\omega_1})-q_1\,\omega_1\,e^{i\,b\,\xi\,\omega_1}\,],\qquad G_2=\frac{\tau_\theta}{\omega_2}[\,(\,q-\frac{q_1}{i\,b\,\xi})\,(1-e^{i\,b\,\xi\,\omega_2})-q_1\,\omega_2\,e^{i\,b\,\xi\,\omega_2}\,],$$

$$G_3=\frac{\tau_\nu}{\omega_3}[\,(\,q-\frac{q_1}{i\,b\,\xi})\,(1-e^{i\,b\,\xi\,\omega_3})-q_1\,\omega_3\,e^{i\,b\,\xi\,\omega_3}\,],\;\; G_4=\frac{-i\,b\,\xi\,\tau_q^2}{2\,\omega_1}[\,(\,q-\frac{q_1}{i\,b\,\xi})\,(1-e^{i\,b\,\xi\,\omega_1})-q_1\,\omega_1\,e^{i\,b\,\xi\,\omega_1}\,],$$

$$A=(\frac{-1}{a_2a_7\delta_1\delta_{14}})\,[\,a_2a_7\delta_{12}+a_2a_7\delta_1\delta_{15}+a_2a_7\delta_2\delta_{14}+a_2\delta_1\delta_7\delta_{14}+a_7\delta_1\delta_4\delta_{14}+a_4a_5a_7\delta_1\delta_{14}+a_2a_7\delta_1\delta_6\delta_{14}-a_2a_3\delta_8\delta_{14}\,],$$

$$\begin{aligned}B=(\frac{1}{a_2a_7\delta_1\delta_{14}})\,[\,&a_2\delta_7\delta_{12}-a_2a_7\delta_{13}+a_7\delta_4\delta_{12}-a_2\delta_8\delta_{16}+a_4a_5a_7\delta_{12}+a_2a_7\delta_2\delta_{15}+a_2a_3\delta_{10}\delta_{12}+a_2a_7\delta_6\delta_{12}\\&-a_2a_3\delta_8\delta_{15}-a_2a_3\delta_9\delta_{14}+a_2\delta_1\delta_7\delta_{15}+a_2\delta_2\delta_7\delta_{14}+a_7\delta_1\delta_4\delta_{15}+a_7\delta_2\delta_4\delta_{14}-a_2\delta_1\delta_{10}\delta_{16}+a_7\delta_3\delta_5\delta_{14}\\&+\delta_1\delta_4\delta_7\delta_{14}+a_4a_5a_7\delta_1\delta_{15}+a_4a_5a_7\delta_2\delta_{14}+a_4a_5a_7\delta_2\delta_{14}-a_3\delta_4\delta_8\delta_{14}+a_2a_7\delta_2\delta_6\delta_{14}+a_4a_5\delta_1\delta_7\delta_{14}-a_2a_3\delta_6\delta_8\delta_{14}\\&+a_2\delta_1\delta_6\delta_7\delta_{14}+a_7\delta_1\delta_4\delta_6\delta_{14}-a_4a_5a_7b^2\delta_1\delta_{14}-a_3a_4a_5\delta_8\delta_{14}+a_2a_7\delta_1\delta_6\delta_{15}\,],\end{aligned}$$

$$\begin{aligned}C=(\frac{-1}{a_2a_7\delta_1\delta_{14}})\,[&-a_2\delta_7\delta_{13}-a_7\delta_4\delta_{13}-a_2\delta_9\delta_{16}+\delta_4\delta_7\delta_{12}-\delta_4\delta_8\delta_{16}-a_4a_5a_7\delta_{13}-a_2a_3\delta_{10}\delta_{13}-a_2a_7\delta_6\delta_{13}\\&-a_2a_3\delta_9\delta_{15}-a_4a_5\delta_8\delta_{16}+a_2\delta_2\delta_7\delta_{15}+a_2\delta_6\delta_7\delta_{12}+a_7\delta_2\delta_4\delta_{15}+a_7\delta_4\delta_6\delta_{12}-a_2\delta_2\delta_{10}\delta_{16}-a_3\delta_4\delta_8\delta_{15}\\&-a_3\delta_4\delta_9\delta_{14}+a_7\delta_3\delta_5\delta_{15}-a_2\delta_6\delta_8\delta_{16}+\delta_1\delta_4\delta_7\delta_{15}+\delta_2\delta_4\delta_7\delta_{14}+\delta_3\delta_5\delta_7\delta_{14}-\delta_1\delta_4\delta_{10}\delta_{16}+a_4a_5a_7\delta_2\delta_{15}\\&+a_3a_4a_5\delta_{10}\delta_{12}-a_3a_4a_5\delta_8\delta_{15}-a_3a_4a_5\delta_9\delta_{14}+a_2a_7\delta_2\delta_6\delta_{15}+a_4a_5\delta_1\delta_7\delta_{15}+a_4a_5\delta_2\delta_7\delta_{14}+a_2a_3\delta_6\delta_{10}\delta_{12}\\&-a_2a_3\delta_6\delta_8\delta_{15}-a_2a_3\delta_6\delta_9\delta_{14}-a_4a_5\delta_1\delta_{10}\delta_{16}+a_2\delta_1\delta_6\delta_7\delta_{15}+a_2\delta_2\delta_6\delta_7\delta_{14}+a_7\delta_1\delta_4\delta_6\delta_{15}+a_7\delta_2\delta_4\delta_6\delta_{14}\\&-a_2\delta_1\delta_6\delta_{10}\delta_{16}-a_3\delta_4\delta_6\delta_8\delta_{14}+a_7\delta_3\delta_5\delta_6\delta_{14}+\delta_1\delta_4\delta_6\delta_7\delta_{14}-a_4a_5a_7b^2\delta_{12}-a_4a_5a_7b^2\delta_1\delta_{15}-a_4a_5a_7b^2\delta_2\delta_{14}\\&+a_3a_4a_5b^2\delta_8\delta_{14}-a_4a_5b^2\delta_1\delta_7\delta_{14}+a_4a_5\delta_7\delta_{12}\,],\end{aligned}$$

$$\begin{aligned}E=(\frac{1}{a_2a_7\delta_1\delta_{14}})\,[&-\delta_4\delta_7\delta_{13}-\delta_4\delta_9\delta_{16}-a_4a_5\delta_7\delta_{13}-a_4a_5\delta_9\delta_{16}-a_2\delta_6\delta_7\delta_{13}-a_3\delta_4\delta_{10}\delta_{13}-a_7\delta_4\delta_6\delta_{13}-a_3\delta_4\delta_9\delta_{15}\\&-a_2\delta_6\delta_9\delta_{16}+\delta_2\delta_4\delta_7\delta_{15}+\delta_4\delta_6\delta_7\delta_{12}+\delta_3\delta_4\delta_7\delta_{15}-\delta_2\delta_4\delta_{10}\delta_{16}-\delta_3\delta_5\delta_{10}\delta_{16}-\delta_4\delta_6\delta_8\delta_{16}-a_3a_4a_5\delta_{10}\delta_{13}\\&-a_3a_4a_5\delta_9\delta_{15}+a_4a_5\delta_2\delta_7\delta_{15}-a_2a_3\delta_6\delta_{10}\delta_{13}-a_2a_3\delta_6\delta_9\delta_{15}-a_4a_5\delta_2\delta_{10}\delta_{16}+a_2\delta_2\delta_6\delta_7\delta_{15}+a_7\delta_2\delta_4\delta_6\delta_{15}\\&+a_3\delta_4\delta_6\delta_{10}\delta_{12}-a_2\delta_2\delta_6\delta_{10}\delta_{16}-a_3\delta_4\delta_6\delta_8\delta_{15}-a_3\delta_4\delta_6\delta_9\delta_{14}+a_7\delta_3\delta_5\delta_6\delta_{15}+\delta_1\delta_4\delta_6\delta_7\delta_{15}+\delta_2\delta_4\delta_6\delta_7\delta_{14}\\&+\delta_3\delta_5\delta_6\delta_7\delta_{14}-\delta_1\delta_4\delta_6\delta_{10}\delta_{16}+a_4a_5a_7b^2\delta_{13}-a_4a_5b^2\delta_7\delta_{12}+a_4a_5b^2\delta_8\delta_{16}-a_4a_5a_7b^2\delta_2\delta_{15}-a_3a_4a_5b^2\delta_{10}\delta_{12}\\&+a_3a_4a_5b^2\delta_8\delta_{15}+a_3a_4a_5b^2\delta_9\delta_{14}-a_4a_5b^2\delta_1\delta_7\delta_{15}-a_4a_5b^2\delta_2\delta_7\delta_{14}+a_4a_5b^2\delta_1\delta_{10}\delta_{16}\,],\end{aligned}$$

$$\begin{aligned}F=(\frac{1}{a_2a_7\delta_1\delta_{14}})\,[&-\delta_4\delta_6\delta_9\delta_{16}-\delta_4\delta_6\delta_7\delta_{13}-a_3\delta_4\delta_6\delta_{10}\delta_{13}-a_3\delta_4\delta_6\delta_9\delta_{15}+\delta_2\delta_4\delta_6\delta_7\delta_{15}+\delta_3\delta_5\delta_6\delta_7\delta_{15}-\delta_2\delta_4\delta_6\delta_{10}\delta_{16}\\&-\delta_3\delta_5\delta_6\delta_{10}\delta_{16}+a_4a_5b^2\delta_7\delta_{13}+a_4a_5b^2\delta_9\delta_{16}+a_3a_4a_5b^2\delta_{10}\delta_{13}+a_3a_4a_5b^2\delta_9\delta_{15}-a_4a_5b^2\delta_2\delta_7\delta_{15}+a_4a_5b^2\delta_2\delta_{10}\delta_{16}\,],\end{aligned}$$

$$H_{1n}=\frac{-\delta_5\,(k_n^2+\delta_6)}{a_2k_n^2+(a_4a_5+\delta_4+a_2\delta_6\,)k_n^2+\delta_4\delta_6-a_4a_5b^2},\quad H_{2n}=\frac{-(a_5k_n^2-a_5b^2)H_{1n}}{k_n^2+\delta_6},$$

$$H_{3n}=\frac{k_n^2(\delta_1\delta_{16}-a_3\delta_{12})+a_3\delta_{13}+\delta_2\delta_{16}-\delta_3\delta_{16}H_{1n}}{a_3\delta_{14}k_n^2+\delta_{16}+a_3\delta_{15}},\;H_{4n}=\frac{-\delta_8\,k_n^2-\delta_9-\delta_{10}\,H_{3n}}{a_7\,k_n^2+\delta_7},$$

$$H_{5n}=ib\,(a_{15}+a_{16})\,(ib-k_nH_{1n})+a_{15}(k_n^2-ib\,k_nH_{1n}\,)+a_3H_{4n}-H_{3n},$$

$$H_{6n}=ib\,a_{15}(ib-k_nH_{1n})+a_{15}(k_n^2+ib\,k_nH_{1n})+a_3H_{4n}-H_{3n},$$

$$H_{7n}=ib\,a_{15}(ib-k_nH_{1n})+(a_{15}+a_{16})\,(k_n^2+ibk_nH_{1n})+a_3H_{4n}-H_{3n},$$

$$H_{9n}=a_2(ib\,k_n+k_n^2H_{1n})+ib\,a_{17}(k_n-ibH_{1n})-a_4H_{2n},\; H_{9n}=a_2(-ibk_n+k_n^2H_{1n})-ib\,a_{17}(k_n+ibH_{1n})-a_4H_{2n}.$$

References

[1] Kumar, R. (2015). Wave propagation in a microstretch thermoelastic diffusion solid. Analele ştiinţifice ale Universităţii" Ovidius" Constanţa. Seria Matematică, 23(1), 127-170.

[2] Othman, M. I. A., Zidan, M. E., and Mohamed, I. E. (2015). The effect of gravity and rotation on microstretch thermoelastic medium with diffusion by dual phase lag model. Sylwan, 159(2), 284-309.

[3] Deswal, S., Sheoran, D., Thakran, S., and Kalkal, K. K. (2022). Reflection of plane waves in a nonlocal microstretch thermoelastic medium with temperature dependent properties under three-phase-lag model. Mechanics of Advanced Materials and Structures, 29(12), 1692-1707.

[4] Said, S. M. (2021). Effects of phase-lags, rotation, and temperature-dependent properties on plane waves in a magneto-microstretch thermoelastic medium. Mechanics Based Design of Structures and Machines, 49(4), 534-552.

[5] Jahangir, A., Atwa, S., Rehman, N., Usman, M., Ashraf, M. B., and Muhammad, N. (2020). Diffusion effect on plane harmonic waves through thermoelastic microstretch medium in context of Green–Naghdi theory. Indian Journal of Physics, 94, 987-998.

[6] Abo-Dahab, S. M., Kumar, A., and Ailawalia, P. (2020). Mechanical changes due to pulse heating in a microstretch thermoelastic half-space with two-temperatures. Journal of Applied Science and Engineering, 23(1), 153-161.

[7] Elhag, S. H., Bayones, F. S., Kilany, A. A., Abo-Dahab, S. M., Abdel-Salam, E. A. B., Elsagheer, M., and Abd-Alla, A. M. (2022). Noninteger derivative order analysis on plane wave reflection from electro-magneto-thermo-microstretch medium with a gravity field within the three-phase lag model. Advances in Mathematical Physics, 2022, 1-13.

[8] Purkait, P., Sur, A., and Kanoria, M. (2017). Thermoelastic interaction in a two dimensional infinite space due to memory dependent heat transfer. International Journal of Advances in Applied Mathematics and Mechanics, 5(1), 28-39.

[9] Sur, A., and Kanoria, M. (2018). Modeling of memory-dependent derivative in a fibre-reinforced plate. Thin-Walled Structures, 126, 85-93.

[10] Sarkar, N., and Mondal, S. (2019). Transient responses in a two-temperature thermoelastic infinite medium having cylindrical cavity due to moving heat source with memory-dependent derivative. ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik, 99(6), e201800343.

[11] Said, S. M. (2023). A study on the frame of a memory-dependent derivative in a micropolar thermoelastic medium under the effect of the variable thermal conductivity. Mechanics Based Design of Structures and Machines, 51(2), 665-681.

[12] Sarkar, I., and Mukhopadhyay, B. (2021). Thermo-viscoelastic interaction under dual-phase-lag model with memory-dependent derivative. Waves in Random and Complex Media, 31(6), 2214-2237.

[13] Li, Y., and He, T. (2021). The transient response of a functionally graded half-space heated by a laser pulse based on the generalized thermoelasticity with memory- dependent derivative. Mechanics of Advanced Materials and Structures, 28(22), 2299-2309.

[14] Choudhuri, S. R. (2007). On a thermoelastic three-phase-lag model. Journal of Thermal Stresses, 30(3), 231-238.

[15] Kumar, R., and Chawla, V. (2011). A study of plane wave propagation in anisotropic three-phase-lag and two-phase-lag model. International communications in heat and mass transfer, 38(9), 1262-1268.

[16] Sur, A., and Kanoria, M. (2014). Thermoelastic interaction in a viscoelastic functionally graded half-space under three-phase-lag model. European Journal of Computational Mechanics, 23(5-6), 179-198.

[17] Othman, M. I. A., Said, S., and Marin, M. (2019). A novel model of plane waves of two-temperature fiber-reinforced thermoelastic medium under the effect of gravity with three-phase-lag model. International Journal of Numerical Methods for Heat and Fluid Flow, 29(12), 4788-4806.

[18] Sharma, D. K., Sharma, M. K., and Sarkar, N. (2021). Effect of three-phase-lag model on the analysis of three-dimensional free vibrations of viscothermoelastic solid cylinder. Applied Mathematical Modelling, 90, 281-301.

[19] Alharbi, A. M. (2021). Two temperature theory on a micropolar thermoelastic media with voids under the effect of inclined load via three-phase-lag model. ZAMM- Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik, 101(12), e202100078.

[20] Othman, M. I. A., and Abbas, I. A. (2021). 2-D problem of micropolar thermoelastic rotating medium with eigenvalue approach under the three-phase-lag model. Waves in Random and Complex Media, 1-16.

[21] Tiwari, R., and Kumar, R. (2022). Non-local effect on quality factor of micro- mechanical resonator under the purview of three-phase-lag thermoelasticity with memory-dependent derivative. Applied Physics A, 128(3), 190.

[22] Othman, M. I. A., Eraki, E. E., Atwa, S. Y., and Ismail, M. F. (2023). Thermoelastic micro-stretch solid immersed in an infinite inviscid fluid and subject to a rotation under two theories. Engineering Solid Mechanics 11(3), 299-310.