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# A new approach for finding an initial basic feasible solution to a transportation problem

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## Abstract

In this paper, a new approach is proposed to find an initial basic feasible solution (IBFS) for the transportation problem (TP). Finding IBFS simply and effectively at the same time makes it easy to achieve the TP optimal solution. The proposed method aims to find a near-optimal solution in easy steps by "avoiding the bigger cost" (ABC), which has larger unit cost values to reduce the total transportation cost. This technique may be right to be used in big problems of transportation, which might be hard to locate an optimal solution manually, or by expensive software. The solution algorithm of the new method is included.

## Keywords

Transportation Problem; Linear Programming; Optimization; initial basic feasible solution; ABC method

## Introduction

The transportation model is a special case of linear programming [1]. The transportation problem is concerned with moving different amounts of products from a set of sources to a set of destinations. The transportation problem was introduced first by F.L. Hitchcock in 1941. Hitchcock presented a study entitled “The Distribution of a Product from Several Sources to Numerous Localities”[2]. This presentation is considered the first important contribution to the solution of the transportation problems. In 1947, T.C. Koopmans presented an independent study, not related to Hitchcock’s, called “Optimum Utilization of the Transportation System”[3]. Both Hitchcock and Koopmans’s presentations aided in the transportation strategies that incorporate many shipping sources and multiple destinations. The transportation problem got its name because many of its applications involve determining how to optimally transport goods[4].

Researchers frequently attempted to develop approaches that provide results close to the optimal solution or optimal solution[5].

Mishra [6] compared the three most well-known strategies for determining the transportation problem's initial feasible solution. The three methods are the North-west Corner Method, Least Cost Method, and Vogel’s Approximation Method. The author outlined the differences between the

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methods and supported that with step-by-step illustrations. He explained that the decision-maker should compare the different results and choose the best solution among them to determine the size of movements from each source to each destination.

Kousalya [7] proposed a new method for determining the initial basic feasible solution to the transportation problem. In his work, he compared the new method with the least cost method (LCM)، It is one of the most popular ways to find an IBFS, and discovered that the new method provided a lower or equivalent cost solution when compared to the least cost method. This study aims to reduce transportation costs by using a new strategy to eliminate steps to find the optimal solution to the transportation problem. To demonstrate his strategy, Kousalya applied his new method to two numerical examples and show the results compared to the solution of the least cost method and the optimal solution.

Jude et al [8] devised a new method to find an initial basic feasible solution to the transportation problem. Jude's new method is called the "Inverse Coefficient of Variation Method" (ICVM). This new method relies on statistics to find the least cost of transportation. The author used arithmetical and standard deviation to create the inverse coefficient of variation [(CV)-1], calculated the inverse coefficient for all rows and columns, and began solving the problem by the lowest cost cell corresponding to the lowest inverse coefficient. The author has proved using solved examples the ability of this method to find results very close to the optimal solution in most cases.

This paper presents a new approach to finding an initial basic feasible solution (IBFS) called the "Avoiding the Bigger cost" ABC method. This method eliminates cells that have a high transportation cost unit, which leads to a reduction in the total transportation cost. A solution algorithm has been introduced to demonstrate how to solve this approach. Many examples were also solved to clarify the results and compare them with other ways to find IBFS.

## The General Structure of the Transportation Problem

Transportation economics is one of the crucial subjects for each car professionals and economists[9]. It is the technology that researches the motion of humans and items economically over area and time. Traditionally, transportation economics has been the concept of the intersection of microeconomics and transportation engineering. There are a variety of organizations everywhere in the international that use the technology of transportation economics to optimize their associated work. There are a variety topics that are in transportation economics. Out of those programs are, the transportation problem, replacement, routing bus schedule, and lots of different subjects[10]. The transportation problem can be taken into consideration as one of the maximum well-known subjects of transportation economics. It offers to locate the distribution of products among numerous sources and numerous destinations. Optimality a define as minimizing the whole transportation cost, time, or distance. It will be maximizing general profit, safety, or some other goal that desires to be maximized.

Classical Transportation Problem (CTP) is formulated as [11] :

$$\mathrm{Min.}\,Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \qquad (1)$$

Subject to:

$$\sum_{j=1}^{n} x_{ij} = a_i, i = 1, 2, \dots, m; \quad (2)$$

$$\sum_{i=1}^{m} x_{ij} = b_j, j = 1, 2, \dots, n; \quad (3)$$

$$x_{ij} \geq o \ \forall \text{ i and j} \quad (4)$$

Where:

- Cij is the cost of transporting one unit of the commodity from origin I to destination j.
- Xij is the amount to be shipped from origin I to destination j.
- ai is the supply availabilities at origin I.
- bj is the demand requirements at destination j.

If we have two sources and two destinations, the problem can be represented as follows:

Table (1) 2-sources and 2-destinations T.P.

| | | Destinations | | |
|---|---|---|---|---|
| | | D1 | D2 | ai |
| Sources | S1 | x11 (c11) | x12 (c12) | a1 |
| | S2 | x21 (c21) | x22 (c22) | a2 |
| | bj | b1 | b2 | |

Minimize Z = c11 x11 + c12 x12 + c21 x21 + c22 x22

Subject to:

x11 + x12 = a1 x11 + x21 = b1

x21 + x22 = a2 x12 + x22 = b2

and xij ≥ 0, for all i and j

The solution to the Transportation problems generally includes the following three steps [12]:

- Finding an initial basic feasible solution.
- Optimality test for the initial feasible basic solution (examine if the transportation cost is minimum or not).
- Moving towards optimality.

A transportation model wherein the entire supply and total demand are unequal are known as an unbalanced transportation problem. It's continually feasible to balance an unbalanced transportation problem[13].

A feasible solution is said to be optimal if it minimizes the total transportation cost.

There are many methods to obtain an Initial Basic Feasible Solution. Some simple strategies to find the preliminary basic viable solution are[14-16]:

- North-West Corner Method (NWCM)
- Row Minima Method (RMM)
- Column Minima Method (CMM)
- Least Cost Method (LCM)
- Vogel Approximation Method (VAM)

**ABC Method Solution Algorithm**

**Step 1:**

1. Check whether the transportation problem is balanced or unbalanced.

i.e. $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ (5)

2. If not, add a dummy source (or destination) with an availability (or requirement) that equals the difference between the total availabilities and the total requirements.

**Step 2:**

1. The transportation problem tableau includes (m*n) cells, where "m" represents the total sources and "n" represents the total destinations.
2. Based on that find the square root of the (m*n). (i.e$\sqrt{m * n}$).
3. Sort the unit costs in ascending order.
4. Prepare a sample of "the lowest" unit costs which equals $\sqrt{m * n}$.
5. If there are more unit costs have the same lowest values, the sample should include all of them.
6. Consider the minimum of the chosen unit costs of this sample to start with. In case of the existence of more than one unit cost with the same lowest values, choose any one of them to start with.

**Step 3:**

1. Check whether the source availability or the destination requirement for the chosen cell has a lower value.
2. Calculate the sum of the other unit costs for the row or the column, which has the lower value. This should not include the chosen unit cost.
3. Repeat "Step 3" for the next unit cost of the chosen sample of unit costs. If all unit costs in the chosen sample are considered, go to "Step 4".

**Step 4:**

1. Sort the calculated values in "Step 3 - point 2" descending.
2. Choose the cell of the sample which is corresponding to the largest calculated value.
3. Fill this cell with the minimum of its source availability and its destination requirement.
4. Now consider the cell with the next largest calculated value and go to "Step 4 – point 3"
5. If all the calculated values are considered, go to "Step 5".

**Step 5:**

1. Now either the transportation table cells are completely filled or there may be values that are not filled yet. If they are all filled stop.
2. If the remaining cells are 4 or more "not in one row or one column" go to "Step 2 – point 2" to prepare a new sample based on the remaining number of cells.

## Numerical Example

I have solved seven numerical examples with methods: North-West Corner Method (NW), Least Cost Method (LC), Vogel Approximation Method (VAM), and Avoiding the Bigger cost Method (ABC) to ensure the effectiveness of the proposed method and compare it with the known methods for solving the transfer problem. Which gives results close to the optimal solution and perhaps the optimal solution in some cases. An example of them has also been resolved in detail to show how to solve the transportation problem with the new approach.

Table (2) illustrates a transportation problem example including all availabilities, requirements by tons, and the transportation cost by dollars per ton.

| | D1 | D2 | D3 | Avail. |
|---|---|---|---|---|
| S1 | 2 | 7 | 4 | 5 |
| S2 | 3 | 3 | 1 | 8 |
| S3 | 5 | 4 | 7 | 7 |
| S4 | 1 | 6 | 2 | 14 |
| Requirements | 7 | 9 | 18 | 34 |

First, take a sample ($\sqrt{m*n}$) as shown in step 2 and complete step 3 to select the cell to start with.

Table (3) testing the sample values

| Cell index | Cell value | Less | Sum. | priority |
|---|---|---|---|---|
| C23 | 1 | row | 6 | 4th |
| C41 | 1 | column | 10 | 2nd |
| C11 | 2 | row | 11 | 1st |
| C43 | 2 | row | 7 | 3rd |

Then start filling in the cells that have been prioritized for filling and solve the problem as in steps 4 and 5.

Table (4) making the first moving

| | D1 | D2 | D3 | Availabilities | |
|---|---|---|---|---|---|
| S1 | $\boxed{2}$ 5 | $\boxed{7}$ | $\boxed{4}$ | 5 | ~~0~~ |
| S2 | $\boxed{3}$ | $\boxed{3}$ | $\boxed{1}$ | 8 | |
| S3 | $\boxed{5}$ | $\boxed{4}$ | $\boxed{7}$ | 7 | |
| S4 | $\boxed{1}$ | $\boxed{6}$ | $\boxed{2}$ | 14 | |
| Req. | 7 | 9 | 18 | | |
| | ~~2~~ | | | | |

Table (5) making the second moving

| | D1 | D2 | D3 | Avail. | |
|---|---|---|---|---|---|
| S1 | $\boxed{2}$ 5 | $\boxed{7}$ | $\boxed{4}$ | 5 | ~~0~~ |
| S2 | $\boxed{3}$ | $\boxed{3}$ | $\boxed{1}$ | 8 | |
| S3 | $\boxed{5}$ | $\boxed{4}$ | $\boxed{7}$ | 7 | |
| S4 | $\boxed{1}$ 2 | $\boxed{6}$ | $\boxed{2}$ | 14 | ~~0~~ |
| Req. | 7 | 9 | 18 | | |
| | ~~2~~ | | | | |
| | ~~0~~ | | | | |

Table (6) making the third moving

| | D1 | D2 | D3 | Avail. | |
|---|---|---|---|---|---|
| S1 | $\boxed{2}$ 5 | $\boxed{7}$ | $\boxed{4}$ | 5 | ~~0~~ |
| S2 | $\boxed{3}$ | $\boxed{3}$ | $\boxed{1}$ | 8 | |
| S3 | $\boxed{5}$ | $\boxed{4}$ | $\boxed{7}$ | 7 | |
| S4 | $\boxed{1}$ 2 | $\boxed{6}$ | $\boxed{2}$ 12 | 14 | ~~12~~ \| ~~0~~ |
| Req. | 7 | 9 | 18 | | |
| | ~~2~~ | | ~~6~~ | | |
| | ~~0~~ | | | | |

Table (7) making the fourth moving

| | D1 | D2 | D3 | Avail. | |
|---|---|---|---|---|---|
| S1 | $\boxed{2}$ 5 | $\boxed{7}$ | $\boxed{4}$ | 5 | ~~0~~ |
| S2 | $\boxed{3}$ | $\boxed{3}$ | $\boxed{1}$ 6 | 8 | 2 |
| S3 | $\boxed{5}$ | $\boxed{4}$ | $\boxed{7}$ | 7 | |
| S4 | $\boxed{1}$ | $\boxed{6}$ | $\boxed{2}$ | 14 | ~~12~~ \| ~~0~~ |

| | | | | |
|---|---|---|---|---|
| | 2 | | 12 | |
| Req. | 7 | 9 | 18 | |
| | ~~2~~ | | 6 | |
| | ~~0~~ | | ~~0~~ | |

Table (8) making the final moving

| | D1 | D2 | D3 | Avail. | |
|---|---|---|---|---|---|
| S1 | 2<br>5 | 7 | 4 | 5 | ~~0~~ |
| S2 | 3 | 3<br>2 | 1<br>6 | 8 | 2 |
| S3 | 5 | 4<br>7 | 7 | 7 | |
| S4 | 1<br>2 | 6 | 2<br>12 | 14 | ~~12~~ ~~0~~ |
| Req. | 7 | 9 | 18 | | |
| | ~~2~~ | | 6 | | |
| | ~~0~~ | | ~~0~~ | | |

Total Cost (Z) = **76**

Here are the results of solving many transportation problems, showing the results for the ABC method and other methods

Table (9) illustrates the comparison in the results between the proposed approach and the most well-known methods for solving transportatio problems with an optimal solution

| Allocations | NW | LC | Vogel | Approach | optimal |
|---|---|---|---|---|---|
| X11=2;X12=7;X13=4<br>X21=3;x22=3;x23=1<br>X31=5;x32=4;x33=7<br>X41=1;x42=6;x43=2<br>a1=5;a2=8;a3=7;a4=14<br>a1=7;a2=9;a3=18 | 102 | 104 | 80 | 76 | 76 |
| X11=12;X12=11;X13=8; X14=13<br>X21=10;x22=7;x23=12;x24=9<br>X31=9;x32=8;x33=10;x34=6<br>a1=14;a2=16;a3=20<br>b1=11;b2=15;b3=13;b4=11 | 453 | 378 | 378 | 378 | 378 |
| X11=4;X12=3;X13=4<br>X21=10;x22=7;x23=5<br>X31=8;x32=8;x33=3<br>X41=5;x42=6;x43=6<br>a1=11;a2=12;a3=10;a4=7<br>b1=16;b2=10;b3=14 | 230 | 199 | 199 | 199 | 183 |
| X11=6;X12=8;X13=10<br>X21=7;x22=11;x23=11<br>X31=4;x32=5;x33=12<br>a1=150;a2=175;a3=275<br>b1=200;b2=100;b3=300 | 5925 | 4550 | 5125 | 5025 | 4525 |
| X11=7;X12=5;X13=9;X14=11<br>X21=4;x22=3;x23=8;x24=6<br>X31=3;x32=8;x33=10;x34=5<br>X41=2;x42=6;x43=7;x44=3<br>a1=30;a2=25;a3=20;a4=15<br>b1=30;b2=30;b3=20;b4=10 | 540 | 435 | 470 | 415 | 415 |
| X11=10;X12=0;X13=20; X14=11<br>X21=12;x22=7;x23=9;x24=20<br>X31=0;x32=14;x33=16;x34=18<br>a1=20;b2=25;a3=15<br>b1=10;b2=15;b3=15;b4=20 | 640 | 480 | 480 | 480 | 460 |
| X11=2;X12=5;X13=6;X14=3<br>X21=9;x22=6;x23=2;x24=1<br>X31=5;x32=2;x33=3;x34=6<br>X41=7;x42=7;x43=2;x44=4<br>a1=6;a2=9;a3=7;a4=12<br>b1=10;b2=4;b3=6;b4=14 | 149 | 83 | 92 | 83 | 83 |

## Results and Discussion

After solving the previous transportation problems with the most famous methods used to find transportation and comparing them with the ABC method, it was found that this method is very effective for solving the transportation problem, as it gives results close to the optimal solution and perhaps the optimal solution in some cases. Taking a sample whose value is equal to the square root of the sum of the possible destinations contributed to finding the solution faster, which enhances the importance and value of this approach. This gives it an advantage for use, especially in transportation problems with huge sources and destinations.

## Conclusion

This paper presented a new approach "Avoiding the Bigger cost" (ABC) to solving the initial basic feasible solution (IBFS) for thetransportation problem. The best methods are not always the most complicated and this is what distinguishes this method, which is simple in solution and effective in results. If you can get better results in easier ways, it is a better one to use.

A solution algorithm was created for this method. The steps for solving the new approach were illustrated through examples, which showed the effectiveness of this solution compared to other methods. This method will be good for use in large transportation problems that are difficult to find an optimal solution by computational methods and in light of the difficulty of using software because of its high prices.

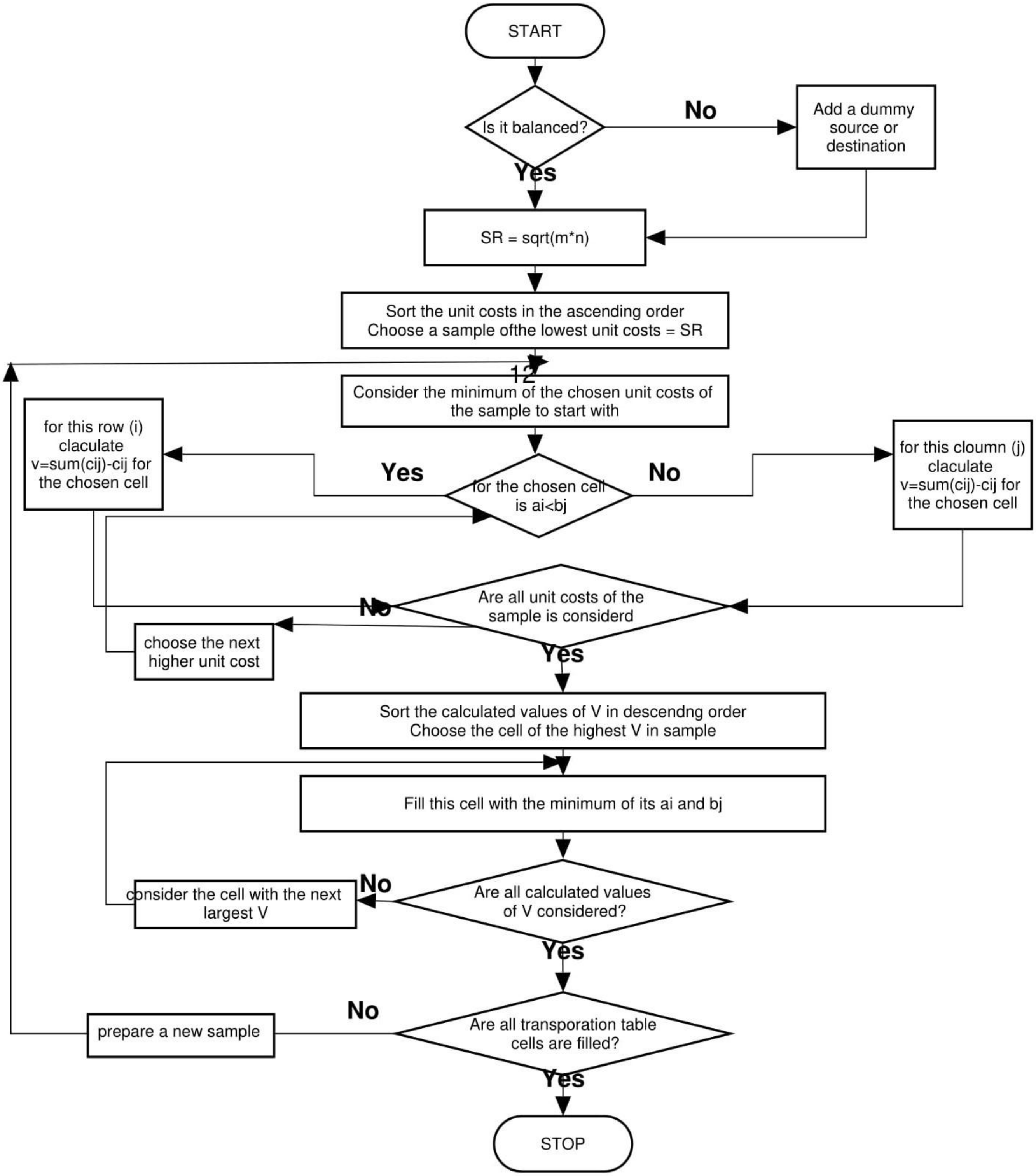


Fig (1). ABC method flow chart

## References


[1] H. A. Taha, *Operations research: an introduction*: Macmillan, 1992.

[2] F. L. Hitchcock, "The distribution of a product from several sources to numerous localities," *Journal of mathematics and physics,* vol. 20, pp. 224-230, 1941.

[3] T. C. Koopmans, "Optimum utilization of the transportation system," *Econometrica: Journal of the Econometric Society,* pp. 136-146, 1949.

[4] A. A. Hlayel and M. A. Alia, "Solving transportation problems using the best candidates method," *Computer Science & Engineering,* vol. 2, p. 23, 2012.

[5] M. H. Abdelati, M. I. Khalil, K. A. Abdelgawwad, and M. Rabie, "ALTERNATIVE ALGORITHMS FOR SOLVING CLASSICAL TRANSPORTATION PROBLEMS," *Journal of Advanced Engineering Trends,* vol. 39, pp. 13-24, 2020.

[6] S. Mishra, "Solving Transportation Problem by Various Methods and Their Comaprison," *International Journal of MathematicsTrends and Technology(IJMTT),* vol. 44, p. 6, 4 April2017 2017.

[7] P. Kousalya and P. Malarvizhi, "A New Technique for Finding Initial Basic Feasible Solution to Transportation Problem," *International Journal of Engineering and Management Research (IJEMR),* vol. 6, pp. 26-32, 2016.

[8] O. Jude, O. B. Ifeanyichukwu, I. A. Ihuoma, and E. P. Akpos, "A New and Efficient Proposed Approach to Find Initial Basic Feasible Solution of a Transportation Problem," *American Journal of Applied Mathematics and Statistics,* vol. 5, pp. 54-61, 2017.

[9] G. Shenoy, U. K. Srivastava, and S. C. Sharma, *Operations Research for management*: New Age International, 1986.

[10] S. Roy and G. Maity, "Minimizing cost and time through single objective function in multi-choice interval valued transportation problem," *Journal of Intelligent and Fuzzy Systems,* vol. 32, pp. 1697-1709, 2017.

[11] J. AlRajhi, K. Alkhulaifi, H. A. Abdelwali, M. AlArdhi, and E. E. Ellaimony, "An Algorithm for Solving Bi-criteria Large Scale Transshipment Problems," *Global Journal of Research In Engineering,* 2014.

[12] R. Patel, D. B. Patel, and D. P . H. Bhathawala, "On Optimal Solution of a Transportation Problem," *Global Journal of Pure and Applied Mathematics,* vol. 13, pp. 6201-6208, 2017.

[13] N. Girmay and T. Sharma, "Balance An Unbalanced Transportation Problem By A Heuristic approach," *International Journal of Mathematics and its applications,* vol. 1, pp. 13-19, 2013.

[14] E. Hosseini, "Three new methods to find initial basic feasible solution of transportation problems," *Applied Mathematical Sciences,* vol. 11, pp. 1803-1814, 2017.

[15] M. Uddin, A. Khan, C. Kibria, and I. Raeva, *Improved Least Cost Method to Obtain a Better IBFS to the Transportation Problem* vol. 6, 2016.

[16] M. Ary and D. Syarifuddin, "Comparison the Transportation Problem Solution Betwen Northwest-Corner Method and Stepping Stone Method with Basis Tree Approach," in *Internasional Seminar on Scientific Issue and Trends (ISSIT)(pp. A 35-44). Yogyakarta*, 2011.