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Transient response of loop antenna using SEM -FDTD Method

By

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Abstract:

The transient response of a square strip loop antenna is investigated using the singularity expansion method (SEM). The required natural modes of the loop are obtained to good approximation using the finite difference time domain (FDTD) method together with Prony's method. In order to generate a certain mode, a plane wave is used to excite the loop antenna with both a frequency near to the natural loop mode (resonance) frequency and with a spatial distribution approximating the mode loop current. The transient response of a loop is studied for an incident short damped sinusoidal plane wave, representing a high power microwave pulse, with different frequencies of the incident wave.

Keywords:

Loop antenna transient response, Hybrid singularity expansion method-Finite difference time domain method

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1. Introduction:

Investigation of the transient response of antennas is important in studying EMP effects, high power microwave weapons, localized waves, ultra-wideband communication and radar systems, EMP sensors ,.etc [1-6]. Recently, attention has been given to the different aspects of high power microwave weapons, their effects on different electronic systems, methods of protection, methods of measurement and standards [7-15]. Analysis of the transient response of different types of antennas has been implemented using time domain integral equations (IE), frequency domain IE together with Fourier transform, SEM, FDTD, in addition to high frequency techniques such as time domain physical optics, time domain geometric theory of diffraction and time domain pulsed plane waves [16-22].

The SEM has the advantage that once the antenna natural modes are obtained one can solve different excitation problems in a simple and fast way using a few numbers of natural modes [22]. Such transient problems include exciting the antenna with different waveforms (either as a transmitting antenna or a receiving one) and with different angles of incidence of a plane wave. The solution is particularly simple after the early time period when the incident wave has traversed the antenna.

The natural modes complex frequencies correspond to the resonant modes of the antenna with damping factors accounting for antenna radiation during current oscillation. However, obtaining the natural modes is usually cumbersome, based on solving homogenous integral equations numerically and searching for the complex natural frequencies [23]. The natural modes of the circular, elliptic and polygonal loops were investigated [24-26].

Recently, the basic natural modes of the dipole antenna having minimum damping, which are required for solving transient problems, were obtained in a simple way by using the FDTD method [27]. The modes were obtained by exciting the antenna with a transient plane wave containing a frequency spectrum concentrated near the frequency of the required mode and with a spatial field distribution similar to that of the required mode current. After the transient excitation has died out, the excited antenna oscillates in the intended mode with an exponentially damped sinusoidal waveform, which is used to find the mode complex frequency. Once the antenna modes have been obtained, it is straight forward and simple to obtain the transient response of the antenna to different excitation waveforms and different incidence angles.

This method is applied in the present work to the square strip loop antenna. When exciting the loop to find the modes, modes other than the required mode may also be excited with a small amplitude, thus Prony's method is used to separate the damped sinusoidal waveforms constituting the transient waveform, which correspond to different modes. The transient response of the loop is then investigated for short pulse waves resembling high power microwaves with different frequencies. The results are compared with a purely FDTD solution and the results are found to agree reasonably.

2. The SEM method for loop antenna

The electric field integral equation for the current distribution I(p) on the loop antenna with exciting electric field E_i (tangential to the loop perimeter) is [28]

$$L^t I = -s\varepsilon_0 E^t{}_i \qquad (1)$$

L^t is a tangential operator given by [23, 28] where

$$L^t(I)_{(r)} = \frac{s^2}{c^2}\int G I(r')dp' - \nabla\int G\nabla' . I(r')dp' \qquad (2)$$

where G is given by

G= exp(-sR/c)/(4πR) (3)

The operators ∇ and ∇' operate on the observation point r and the source point r', respectively, and the integration is performed over the loop perimeter p'. s is the Laplace transform variable, c is the speed of light, ε_0 is the free space permittivity and R is the distance between a current point at p' and an observation point at p on the loop surface. The natural modes current distribution on the loop antenna, is nearly sinusoidal with N full cycles for mode N [26] (similar to the current distribution at the resonant frequencies of the loop antenna). For a loop antenna lying in the x-z plane with perimeter 2L we thus assume the model current distribution for mode N along the perimeter p (constituting x- and z- directions) to be:

$$I(p) = Sin(p\frac{N\Pi}{L}) \qquad (4)$$

where (p = zero) is at the center of an x- directed arm.

The complex natural frequency s_n of mode n is given by

$$s_n = j\, w_n - \sigma_n \qquad (5)$$

where the damping factor is σ_n and j is the imaginary unit. The complex natural frequencies should be obtained numerically.

For an incident transient wave on a loop, the transient excited current on the loop can be obtained in SEM method mainly in terms of the natural modes of the loop, with additional polynomial and entire functions of s, whose contributions are usually neglected. The Laplace transform of the excited current is given by [23, 28]

$$I(p,s) = -\varepsilon_0 s \sum_n (s-s_n)^{-1} < L' I_n, I_n >^{-1} < E_i, I_n > I_n \qquad (6)$$

where the summation is over the complex frequencies s_n and their complex conjugates, and the operator:

$L' = dL/ds$

and the scalar product is defined by

$$< f.g > \equiv \int_p f(p).g(p)dp \qquad (7)$$

The Laplace transform of the excited current thus takes the form

$$I(p,s) = \sum_n \frac{-\varepsilon_0 s}{s - s_n} \frac{c_n}{B_n} I_n(p) \qquad (8)$$

where c_n is the mode coupling coefficient, given for the thin wire natural modes by

$$c_n = \int E^i(s).I_n(p)dp \qquad (9)$$

And B_n is the mode normalization constant

$$B_n = < L' I_n, I_n > \qquad (10)$$

The normalization constant B_n is evaluated at $s=s_n$, thus when considering the conjugate complex frequency s_n^* the normalization constant become B_n^* .

For a normally incident uniform plane wave on the plane of the loop with the electric field oriented in the z- direction, the mode coupling coefficient becomes

$$C_n = 2E_z^i(s)\int_0^{\frac{L}{2}} \sin(Z + \frac{L}{4}) \frac{n\Pi}{L} dz$$

$$= 2E_{z(s)}^i \left(\frac{-\cos(z + L/4)\frac{n\pi}{L}}{n\Pi / L} \right)_0^{\frac{L}{2}}$$

$$= 2E^i_z(s)\,[\cos(n\pi/4) - \cos(3n\pi/4)]/(n\pi/4) = f_n E^i_z(s) \qquad (11)$$

where the length of the loop arm is L/2 and the z- directed arm starts at p=L/4. The factor 2 appears due to integration on both arms oriented in the z- direction.

For the dipole antenna the value of B_n can be obtained analytically for thin dipoles [23, 28]. For the loop antenna this value may be evaluated numerically. The investigated loop is a square strip loop to simplify the required calculations and to make simple comparison with FDTD solution. For this purpose we need the detailed from the operator L at the different loop arms. For the current distribution given by Eqn.(4), and for an observation point on arms 2 or 4 in x- direction, the tangential operator becomes as follows due to the four source arms (arms 1 and 3 are in the z- direction):

$$L^t\, I \Big|_{2,4}^{r\to x} = \left(-\frac{\partial}{\partial x}\int G \frac{\partial}{\partial z'} I(z')dz' \right)_{1',3',p'\to z'} + \left(\frac{s^2}{c^2}\int G\; I(x')dx' - \frac{\partial}{\partial x}\int G \frac{\partial}{\partial x'} I(x')dx' \right)_{2',4',p'\to x'} \qquad (12)$$

For an observation point on arms 1 or 3 in the z- direction,

$$L^t\ I\ \Big|_{1,3}^{r\to z} = \left(\frac{s^2}{c^2}\int GI(z')dz' - \frac{\partial}{\partial z}\int G\frac{\partial}{\partial z'}I(z')dz'\right)_{1',3',p'\to z'} - \left(\frac{\partial}{\partial z}G\frac{\partial}{\partial x'}I(x')dx'\right)_{2',4',p'\to x'} \quad (13)$$

In order to obtain the operator $L' = \frac{\partial L}{\partial S}$ we use the form of G in Eqn.(3), thus the derivative of the term containing G becomes

$$\frac{dG}{ds} = -\frac{e^{-sR/c}}{4\Pi c} \quad (14)$$

The derivative of the term containing s^2 G becomes

$d(s^2G)/ds = -s^2 \exp(-sR/c)/(4\pi c)+2s \exp(-sR/c)/(4\pi R)$ (15)

Due to the current symmetry on parallel arms for odd modes, the integral of the scalar product in B_n , Eqns.(10, 7), needs to be performed on only one of each two parallel arms and this result is doubled.

When making the numerical integration for B_n an equivalent cylindrical wire of radius a is used instead of the strip wire of the loop. For the self element (observation point at the same source segment Δl), the integral of G becomes [29]

$$\frac{1}{4\Pi}\int\frac{e^{-sR/C}}{R}dl = \frac{1}{2\Pi}\ln\left(\frac{\Delta l}{a}\right) + \frac{s}{4\Pi c}$$

3. The loop current for an incident plane wave with damped sinusoidal time dependence

Consider an incident plane wave with damped sinusoidal time dependence and whose electric field is oriented in the z- direction, given by

$$E(t) = 2E_0\cos(w_K t)e^{-\sigma_k t} \quad (17)$$

where w_k is the radian frequency and σ_k is the damping constant of the incident wave. The Laplace transform of E(t) is (with E_0=1)

$$E(s) = \frac{1}{s-a} + \frac{1}{s-a^*} \quad (18)$$

where * denotes the complex conjugate and

$a = j\, w_k - \sigma_k$ (19)

The damped sinusoidal wave can be used to represent an high power microwave signal. The induced current due to E_z , thus equals

$$I(p,s) = -\varepsilon_0\sum_{Sn} sI_n(p)\,f_n\ \ /\{B_n\,(s-s_n)\}\ \sum_{a_j}\frac{1}{s\ -a_j} \quad (20)$$

To obtain the time dependence of the current, the inverse Laplace transform is performed. By using partial fractions,

$$\frac{s}{(s-s_n)(s-a_j)} = \frac{s_n}{(s-s_n)(s_n-a_j)} + \frac{a_j}{(s-a_j)(a_j-s_n)} \qquad (21)$$

The required inverse Laplace transform for a general term in eqn.(21) is thus

$$L^{-1}\frac{1}{s-b} = e^{bt} \qquad (22)$$

here b = s_i or a_i .Thus the transient response becomes

$$I(P,t) = -\varepsilon_0 \sum_{Sn} I_n(p) f_n \Big/ B_n \sum_{a_i} \left(\frac{s_n e^{s_n t}}{s_n - a_j} + \frac{a_j e^{a_j t}}{a_j - s_n}\right) \qquad (23)$$

The exponential terms including s_i represent natural oscillations of the loop, whereas those including a_i represent forced oscillations.

4. Calculating the natural modes of the loop using FDTD Method

The FDTD method is used to find both the natural loop modes and also to check the loop current for an incident transient plane wave. The conventional Yee algorithm is used to obtain the scattered field in free space [18, 19]. Liao absorbing boundary conditions are used to enclose the computation domain.

Excitation of the natural modes of the loop

The natural damped sinusoidal modes of the loop take the form

$I_n(t) = i_o \cos(\omega_n t) \exp(-\sigma_n t)$ (24)

where ω_n and σ_n are represented in Eqn.(5). The main modes required for the SEM solution are the least damped modes [24]. Other natural modes exist with higher damping factors, but such modes are not relevant in the SEM.

To excite a certain mode 'n' on the loop, a plane wave is used to excite the loop with approximate frequency and spatial distribution of field along the loop arm parallel to the electric field as the mode current required to be excited, having the form

$$E_i = E_0 \sin(\omega_n t)\sin(\frac{\omega_n t}{8})\sin[\frac{n\Pi}{L}(z - L/4)] \qquad (25)$$

In order to limit the period of the incident plane wave, few cycles of the natural frequency ω_n are used, modulated by a half sine wave of a lower frequency (ω_n /8 in Eqn.(25)), Fig.(1a), in order to make the field continuous with time, thus the spectrum of this wave becomes concentrated around ω_n, so the other natural modes are only negligibly excited. After the termination of this excitation, the current oscillates on the loop mainly with the intended natural mode.

When the loop is excited in a high order mode, more than one mode may be excited.

The damped sinusoidal waveforms of the modes were separated using Prony's method which is used to separate terms of the form $e^{(j\omega_n - \alpha_n)}$ with different complex frequencies in a certain transient waveform.

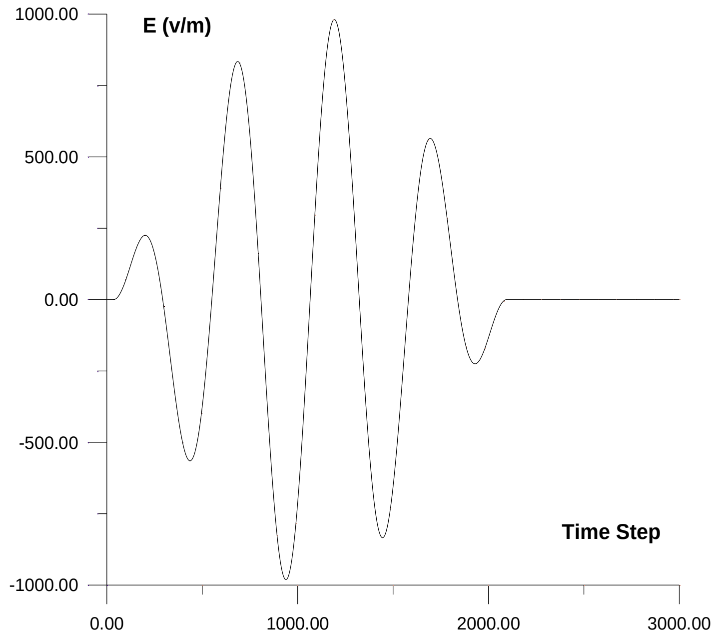

Fig. (1a) incident waveform to excite the first mode

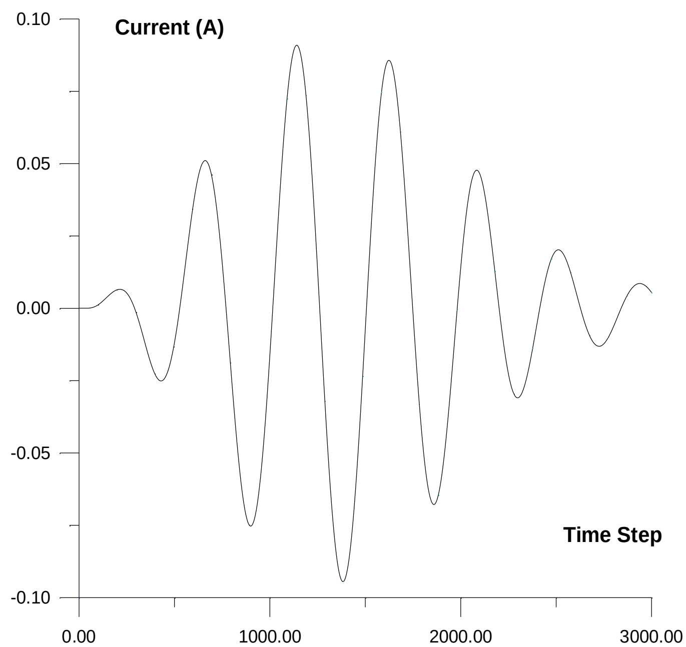

Fig. (1-b) The current at the center of the arm parallel to the incident electric field versus time (first mode) when excited by a plane wave with the waveform of Fig.(1a)

5. Results and discussion

5.1 The natural modes of the loop

Fig.(1b) shows the variation of the excited current, at the center of an arm parallel to the incident electric field, versus time using the exciting waveform of Fig. (1a), where the frequency of the incident plane wave is nearly equal to the frequency of the first natural mode, and the direction of the incident plane wave is normal to the plane of the loop. The forced excitation due to the incident wave terminates after 4 cycles then the damped oscillation of the first natural mode begins. The loop used is a strip loop whose mean arm length is 73 cells and strip width of 3 cells. The charts which are used to obtain the equivalent radius of a strip dipole are used approximately to obtain the equivalent radius of the strip in the loop antenna [30]. With strip thickness to width ratio of 1/3, the ratio of the equivalent cylindrical dipole radius to strip width is 0.38. With the loop of 3 cells wide, its equivalent radius is 1.14 cells with cell length

$\Delta x = \Delta z = 2.016x10^{-4}\, m$

For the first natural mode the loop perimeter is nearly equal to one wavelength, which corresponds to a frequency of 5 GHz. The time step used equals the cell length divided by c*sqrt(3) to satisfy the stability condition [18, 19].

An incident plane wave parallel to one arm produces a first mode with two half cycles on the loop perimeter with maxima at the centers of the arms parallel to the incident electric field and their currens are in the same direction. Fig (2) shows the current distribution of the first mode on the four loop arms. It is to be noted that the three cells at the ends of each arm overlap with the adjacent orthogonal arm, thus the current changes its direction at these cells. If the current distributions on the four arms are connected together, the resulting current on the loop perimeter becomes continuous and nearly sinusoidal. Due to the excitation by a plane wave incident in the direction normal to the loop plane with electric field parallel to one arm, only the odd modes are excited since the N^{th} mode current has 2N half cycles, and for even modes the currents in parallel arms have reversed directions , thus their excitation coefficients are zero, Eqn.(11).

Table (1) shows the normalized complex resonance frequencies of the first three odd modes where the normalized values are defined as

$$\omega_n^* \frac{P}{2\Pi C} = \frac{\omega_n^*}{\omega_{1p}} \qquad (26)$$

where

ω_{1p} corresponds to the radian frequency when the loop perimeter P is considered to be one wavelength. In Ref. 24 the ratio of the loop perimeter to radius is identified by the parameter $\Omega = 2\ln\frac{p}{r}$. The results for the present case are shown in table 1 with Ω=11.1 . The results of [24] are also shown at the same value of Ω and the results of [26] are for a square loop with Ω=12.8 . The results show reasonable agreement.

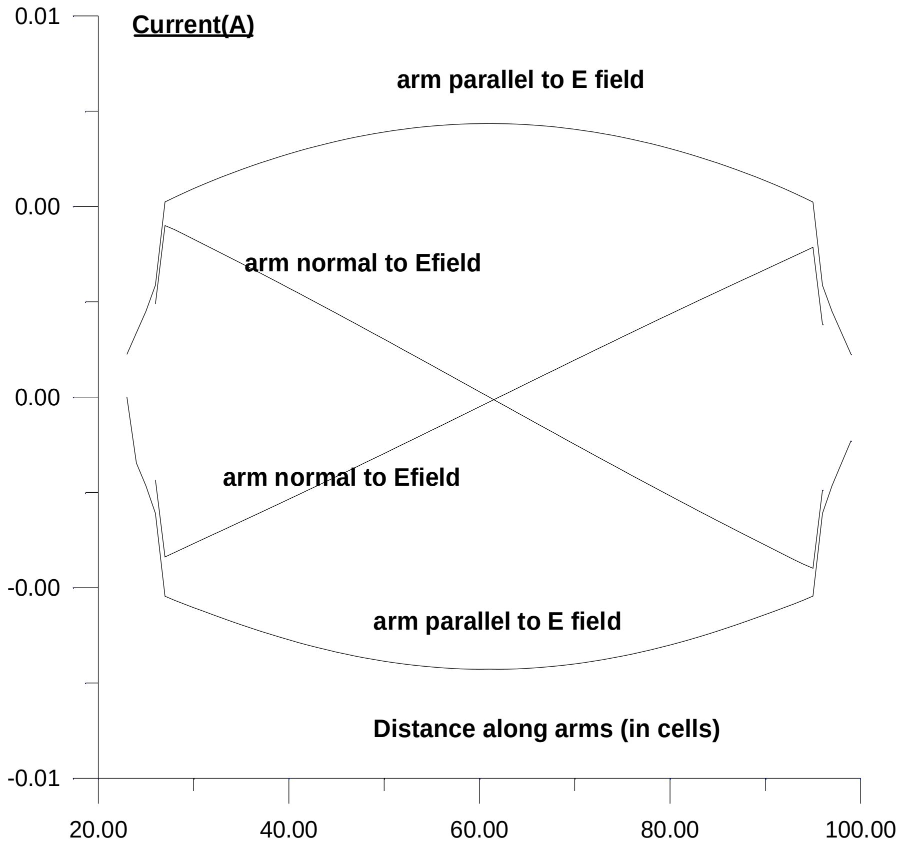

Fig.(2) The current distribution of the first mode on the four arms of the loop

Table 1. Normalized complex resonance frequencies for odd modes of the loop

Mode n	1	3	5
f_n/f_{1p}	1.198	3.12	5.22
σ_n / ω_{1p}	-0.172	-0.215	-0.902
f_n/f_{1p} [24]	1.1	3.07	
σ_n / ω_{1p} [24]	-0.13	-0.255	
f_n/f_{1p} [26]	1.094	3.15	
σ_n / ω_{1p} [26]	-0.098	-0.23	

Table 2 shows the normalization constants of modes compared with those of a dipole of half the loop length (half wavelength dipole). The dipole modes normalization constants for thin dipoles are given by [23, 28]:

$$B_n = \left(\frac{Jn\Omega_d}{4c}\right) \tag{27}$$

where

$$\Omega_d = 2\ln(\frac{l}{r}) \tag{28}$$

where l and r are the dipole length and radius, respectively.

Table 2 Modes normalization constants for the loop compared with those of the dipole

Mode n	1	3
loop	2.6e-9 + j 1e-8	5.8e-9 + j 2.08e-8
dipole	j 0.8e-8	j 2.42e-8

The results show that the normalization constants of the loop modes are nearly equal to the corresponding ones for the dipole, although the loop perimeter is double the dipole length. This behavior is found to be due to the effect of the orthogonal arms.

5.2 Loop response to an incident damped sinusoidal plane wave

High power microwave weapons use few cycles of microwave frequency in order to penetrate devices operating at microwave frequencies. Here we use a damped sinusoidal incident plane wave to investigate such conditions. In order to investigate loop response to an incident plane wave of short period (wide spectrum), we use a large damping constant $\sigma_k=1.9*10^{10}$, which equals the inverse of the damping time constant of the sinusoidal wave envelope. If the frequency of the sinusoid is low, the high damping constant makes the waveform to be nearly an impulse.

The loop with the above mentioned dimensions (with first mode frequency near 6GHz) is considered with incident plane waves of different frequencies, normally incident on the loop plane. Figures (3, 4), show the loop responses with f_k=5GHz and 2 GHz. Fig. (3) shows a comparison between the solution obtained using the present SEM method and the solution using a FDTD method, and reasonable agreement is found. The first natural mode of the loop is clearly dominant in Figs.(3, 4) because the excitation frequencies are far from the frequencies of the higher order natural modes. Once the loop modes are characterized, the responses are obtained in nearly no time and for different incident waveforms or exciting voltages.

Comparison of the responses of the loop with the responses of half wavelength dipole resonating at nearly the resonance frequency of the loop [27] for the studied excitation waveforms shows that the dipole response is higher by about 50%. This near equality can be explained by the nearly equal values of the modes normalization constants, table 2, and that the coupling coefficients are nearly the same because the coupling of the incident field to a loop arm parallel to it is less than for the dipole because of its shorter length, thus the two arms of the loop parallel to the incident electric lead to a coupling coefficient nearly equal to that of the dipole.

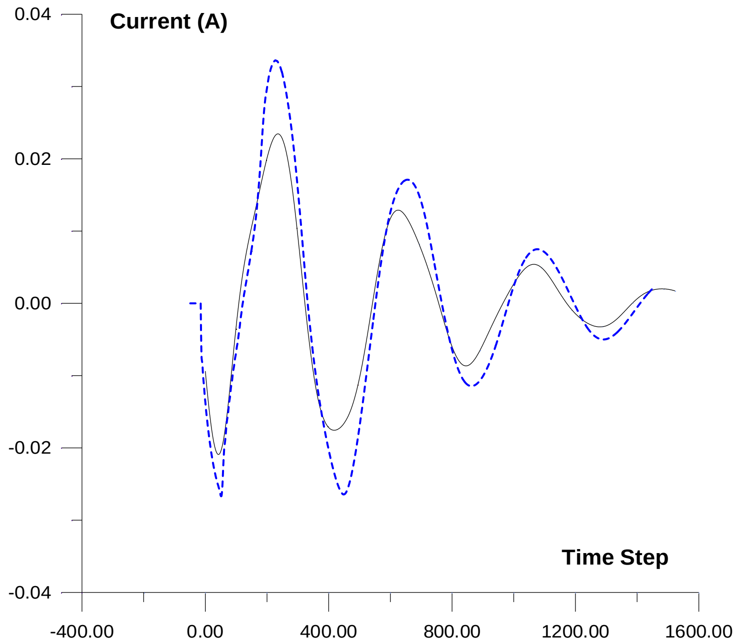

Fig. (3) The transient current at the center of the loop arm parallel to the incident electric field when the loop is excited by a damped sinusoidal plane wave with frequency 5GHz (… FDTD method , __ SEM method)

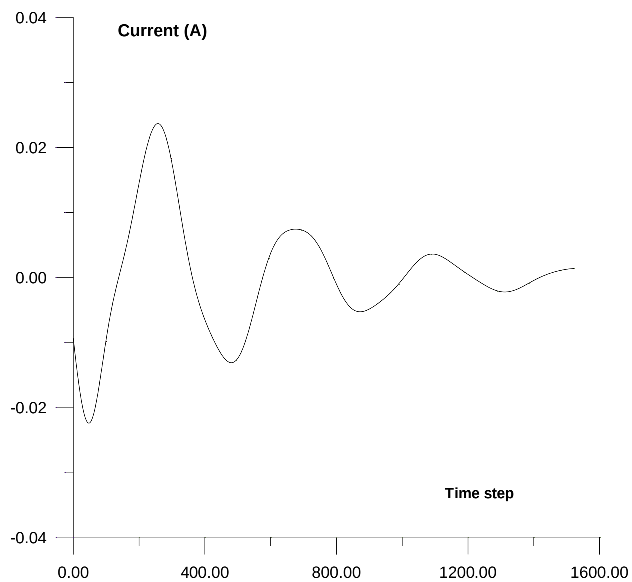

Fig. (4) The transient current induced at the center of the arm parallel to the incident electric field versus time using SEM method, fk=2GHz

6. Conclusion

The computationally efficient SEM method is used to investigate the transient response of loop antenna using the natural complex frequencies of the loop. A FDTD approach together with Prony's method is used to obtain the approximate complex natural modes of the loop. The required modes normalization constants are obtained numerically. The obtained natural modes are used to find the response of the loop to a highly damped sinusoidal plane wave (resembling high power microwave pulse). The response of the loop is found to be nearly equal to the response of a dipole with length equal to half the loop perimeter.

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