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# Actuator fault detection and isolation of nonlinear systems by robust fuzzy observers

*By*

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***Abstract:***

This paper presents a model-based technique for fault detection and isolation (FDI) of actuators of a benchmark which schematizes a hydraulic process made up of three tanks. Takagi Sugeno's model approach is used for describing the dynamic of the system. In the same way, the fuzzy membership functions used for constructing Takagi and Sugeno's model are combined with local unknown input observers to form robust fuzzy observer. Sufficient conditions for the existence of this fuzzy observer are derived. The stability as well as eigen-value constraints conditions are presented and solved in the LMI framework. For the observer gives a good estimation without amplifying noise and with a convergence faster than the dynamic of the system a eigen-value assignment is necessary. Robust residual signals, generated by these fuzzy observers robust to unknown inputs are dedicated to supervise actuators. These residuals are sensitive to faults acting on one actuator and are also insensitive to faults on the others by considering faults such unknown disturbances. This permits to carry out directly the isolation of the faulty actuator.

***Keywords:***

Nonlinear system, FDI, T-S fuzzy model, unknown input fuzzy observers, quadratic stability, LMI approach

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## 1. Introduction:

The multiple model approach to nonlinear systems modelling has become one of the most active research areas in the recent years. Unlike the classical approaches which attempt to find a global and consequently complex-model, the multiple model approach is based on Takagi-Sugeno decomposition [1] of the input space of the system in small areas. Simple local models, usually linear, are used to describe the system in each area. The global model (multi-model) is an interpolation of the local models using weighting functions which assess the local validity of the corresponding models and provide smooth transition between local models. In the same way, for the system observation, a number of local linear observers [2] are designed and the state estimation is given by a fuzzy fusion of local observers.

For LTI systems, the robust residual generation by unknown input observer [3] currently reached a maturity degree in FDI. A FDI-Toolbox is developed in Matlab© programming environment. It includes a number of functions to the design of observer-based and parity space FDI systems including both residual generation and evaluation [4]. The application of these scientific results to multi-model cases seems very promising. A FDI scheme proposed in [5] is based on Takagi-Sugeno fuzzy models for normal operation and for each fault. A fuzzy decision making approach is used to isolate incipient and abrupt faults of a pneumatic servomotor actuated industrial valve. The continuous unknown input fuzzy observer (UIFO) [6] [7] allows to account for the good estimation of the states of nonlinear systems represented by Takagi-Sugeno model in spite of the presence of unknown disturbances. In this paper, by considering faults in the equations of the model, other conditions are added to the inherent constraints of the UIFO to generate residuals for detecting actuator faults especially for non linear systems case. The linear matrix inequality approach (LMI) [8] is used to analyse the global stability of the UIFO.

The paper is organised as follows: In Section 2, the Takagi Sugeno model is discussed and quadratic stability condition, used to ensure the stability of the UIFO, is underlined. The design of UIFO in presence of actuator/system faults is presented in Section 3, where also existence and convergence conditions are demonstrated and fault detectability conditions are analysed. The proposed UIFO is used to generate residuals that are necessary for detecting and isolating actuators faults of a three tanks process where simulation results, in section 4, demonstrate the effectiveness of the proposed study. Finally, the conclusions are drawn in section 5.

## 2. Takagi-Sugeno (T-S) model:

### 2.1 Modelling:

Real physical systems are often non linear. As it is delicate to synthesize an observer for a nonlinear system, the idea of multiple models is preferred. The approach is to apprehend the total behavior of a system by a set of models (linear or affine), each model characterizing the local behavior of the system. The local models are then aggregated by an interpolation mechanism to form the global model.
The continuous dynamic model proposed by [1] is described by fuzzy IF-THEN rules, which represent local linear input-output relations of nonlinear systems. The $i^{th}$ rule of this fuzzy model is of the following form:
IF $w_1(t)$ is $N_{i1}$ and ...and $w_g(t)$ is $N_{ig}$ THEN

$$\begin{cases} \dfrac{dx(t)}{dt} = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t) + D_i u(t) \end{cases} \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^r$, $y(t) \in R^m$ and $A_i$ , $B_i$ , $C_i$ and $D_i$ are time invariant matrices of appropriate dimensions. The vector w(t) is termed the premise variable or decision variable which may depend on the known inputs and/or the measured state variables, whereas $N_{ij}$ is a fuzzy set, M is the number of If-Then rules and g the number of premises variables.
Given the input vector u(t), the global state and output of the system are inferred as follows:

$$\begin{cases} \dfrac{dx(t)}{dt} = \sum_{i=1}^{M} \mu_i(w(t))(A_i x(t) + B_i u(t)) \\ y(t) = \sum_{i=1}^{M} \mu_i(w(t))(C_i x(t) + D_i u(t)) \end{cases} \tag{2}$$

$N_{ij}(w_j(t))$ is the grade of membership of the premise variable $w_j(t)$ to $N_{ij}$ . The function $\mu_i(w(t))$ is normalised and satisfies, for all t, the following constraints:

$$\begin{cases} \sum_{i=1}^{M} \mu_i(w(t)) = 1 \\ 0 \le \mu_i(w(t)) \le 1 \end{cases} \qquad \forall i = 1...M \tag{3}$$

At each time, $\mu_i(w(t))$ quantifies the relative contribution of each local model to construct the global model or multi-model. Choosing the number M of local models of that multi-model may be intuitively achieved taking account the number of operating point. However, determining matrices $A_i$, $B_i$, $C_i$ and $D_i$ needs the use of specific identification technique [9]. From a practical point of view, these matrices are those used to describe the local functioning around the $i^{th}$ operating point. That is exactly the case at the $i^{th}$ operating point, when $\mu_i(w(t))=1$ and $\mu_j(w(t))=0$ with $j \neq i$.

## ***2.2 Stability analysis***

The stability of the T-S autonomous system described by the following equation:

$$\frac{dx(t)}{dt} = \sum_{i=1}^{M} \mu_i(w(t))(A_i x(t)) \qquad (4)$$

is verified using the following theorem:

***Theorem*** [10]: The continuous fuzzy system described by (4) is globally-asymptotically stable if it there is a common positive symmetric matrix X such that the following inequality holds:

$$\begin{aligned} &A_i^T X + X A_i < 0 \\ &\forall i = 1 \ldots M \end{aligned} \qquad (5)$$

This theorem, which gives a sufficient condition for ensuring stability, is an extension of the second Lyapunov theorem. The matrix inequality in (5) can be solved using the Linear Matrix Inequality (LMI) method [8]. However, if the number of local models is large, it might be difficult to find common matrix *X* and it's well known that, in a lot of cases, a common positive definite matrix doesn't exist, whereas the system is stable. To overcome this limitation, authors in [11] propose an analytic way to find a Lyapunov but non quadratic function that guarantees the stability of the global T-S continuous fuzzy model. This is an alternative way used to design fuzzy regulators and/or fuzzy observers. The Lyapunov approach will be used in the next section to study the stability of the estimation error.

## 3. Unknown Input Fuzzy Observer design (UIFO)

Let us consider a nonlinear system represented by the following multi-model subject to unknown inputs and faults:

$$\begin{cases} \frac{dx(t)}{dt} = \sum_{i=1}^{M} \mu_i(w(t))(A_i x(t) + B_i u(t) + E_i d(t) + R_i f(t) + \Delta x_i) \\ y(t) = \sum_{i=1}^{M} \mu_i(w(t))(C_i x(t) + D_i u(t) + F_i d(t)) \end{cases} \tag{6}$$

where $E_i \in R^{n\times q}$ is the unknown input matrix over state variables, $R_i \in R^{nxp}$ the actuator/system faults matrix, $dx_i \in R^{n\times 1}$ is introduced to take into account the operating point of the system and $F_i \in R^{m\times q}$ the unknown input matrix over outputs variables.
In this paper, we consider that there is no non-linearity in the system output equation, (i.e $C_i = C$ and $F_i = F$ $\forall i = 1...M$) and there is no input term on y(t) ($D_i = 0, \forall i = 1...M$). In this situation the output equation is:

$$y(t) = Cx(t) + Fd(t) \tag{7}$$

This is a very common situation in practice because the output equation represents the relation between measurements and the system state variables.

### 3.1 Existence conditions of the UIFO to fault detection

The structure of UIFO results of the aggregation of local observers [6][7]and the obtained analytical form is particularly adapted for studying the stability and the convergence property of the state reconstruction error and residuals. The numerical aspect related to the determination of matrices of this observer in view of fault detection will be also analysed. For a non-linear dynamic system described by the T-S fuzzy model (6), an unknown input fuzzy observer can be designed to estimate the system state vector. For the unknown input fuzzy observer design, it is assumed that the fuzzy system model is locally observable, i.e., all pairs ($A_i$, $C_i$) (i = 1...M) are observable. Using the same idea in T-S fuzzy model, an UIFO utilises M number of loca1 linear time-invariant observers as below:

$$\begin{cases} \frac{dz(t)}{dt} = \sum_{i=1}^{M} \mu_i(w(t))(N_i z(t) + G_i u(t) + L_i y(t) + \Delta z_i) \\ \hat{x}(t) = z(t) - Hy(t) \end{cases} \tag{8}$$

where $N_i \in R^{n\times n}$, $G_i \in R^{n\times r}$, $L_i \in R^{n\times m}$ are the gains of the $i^{th}$ local observer, $dz_i \in R^{n\times 1}$ is a constant vector and $H \in R^{n\times m}$ is a matrix transformation. Indeed, the observer only uses known variables u and y, variable d being not measured.
These matrices have to fulfil some properties in order for (8) to be a good observer. The objective is to ensure the convergence of the estimated state towards the true state.
The estimation error is defined as:

$$e(t) = x(t) - \hat{x}(t) \tag{9}$$

and the residual is:

$$r(t) = y(t) - C\hat{x}(t) \tag{10}$$

The estimation error dynamic is given by the following differential equation:

$$\begin{cases} \dfrac{de(t)}{dt} = \sum_{i=1}^{M} \mu_i(w(t))(N_i e(t) + (PA_i - L_i C - N_i P)x(t) + (PB_i - G_i)u(t) + \\ (PE_i - L_i F - N_i HF)d(t) + PR_i f(t) + (P\Delta x_i - \Delta z_i) - HF\dfrac{dd(t)}{dt}) \end{cases} \tag{11}$$

## *3.1.1 Existence conditions of the UIFO*

If the following sufficient conditions for the existence of UIFO are satisfied:

$$\begin{aligned} &P = I + HC \\ &PA_i - L_i C = N_i P \\ &PB_i = G_i \\ &PE_i - L_i F - N_i HF = 0 \\ &P\Delta x_i = \Delta z_i \\ &HF = 0 \\ &\forall i = 1...M \\ &\text{and } \sum_{i=1}^{M} \mu_i(w(t))N_i \text{ is stable} \end{aligned} \tag{12}$$

then equation (11) is reduced to:

$$\frac{de(t)}{dt} = \sum_{i=1}^{M} \mu_i(w(t)) N_i e(t) + PR_i f(t) \tag{13}$$

also when no fault occurs (f(t) = 0):

$$\frac{de(t)}{dt} = \sum_{i=1}^{M} \mu_i(w(t)) N_i e(t) \tag{14}$$

## ***3.1.2 Global convergence of the UIFO***

From theorem mentioned in section 2.2, if a common positive symmetric matrix X exists such that the following inequality holds:

$$\begin{aligned} & N_i^T X + X N_i < 0 \\ & \forall i = 1 \ldots M \end{aligned} \tag{15}$$

Then the continuous fuzzy observer described by (8) is globally- asymptotically convergent and the error dynamic is globally- asymptotically stable and the rate decay of state error estimation and residual (if no faults f(t)=0) is strongly depending on the matrix N such :

$$N = \sum_{i=1}^{M} \mu_i(w(t)) N_i \tag{16}$$

## ***3.2 Eigen-value assignment***

To ensure good convergence dynamic of the UIFO, stability in the presence of uncertainty such as noises and for avoiding any delayed detection the eigen-values of all local observers are assigned within a region *S(a,b)* witch is the intersection between a circle of center (0,0) and radius *b*, and a strip in the left hand side of the complex plane with real part smaller than *a*, then the UIFO is stable in the S region if there exist a matrix X definite positive such that for i = 1,..M and in case of $F = 0$ and $E_i = E$, we have:

The solution of the system (12) depends on the generalized inverse $(CE)^-$ of (CE).
*T*o propose an observer design strategy, inequalities (17) are solved by LMI formulation.

$$\begin{cases}\begin{bmatrix}-bX & PA_i^T X - C^T W_i^T \\ XPA_i - W_i C & -bX\end{bmatrix} < 0 \\ (PA_i)^T X + XPA_i - C^T W_i^T - W_i C + 2a < 0\end{cases} \qquad \forall i = 1...M \tag{17}$$

### *3.3 Fault detectability conditions*

a) A well known fault detectability condition for LTI systems [3] is $Rf(t) \neq 0$, and then for multi-model representation:

$$\begin{aligned}&R_i f(t) \neq 0\\ &\forall i = 1...M\end{aligned} \tag{18}$$

That is the structures of the distribution matrix of faults $R_i$ must be chosen so that faults don't mask themselves. In more over for fault diagnosis [3] and [12], impose that the number of disturbances and/or faults q+p must be strictly lower than the number m of outputs.

b) From (13) generally, it is difficult to have a solution for the equation $PR_i = 0$ when $i = 1…M$, but this is possible when $R_1 = R_2… = R_M = R$.
Indeed, when we consider the same distribution matrix of faults acting on all local models and when $PR = 0$, the residual still equal to zero even in presence of faults, this situation is far from the desired objective. Then, in presence of faults, the distribution matrix R must in addition to conditions mentioned in (18) satisfies the relation in (19), inherent to the structure of the observer:

$$H \neq -R(CR)^{-} \tag{19}$$

Contrainst (19) is deduced from $P = I + HC$ and $PR \neq 0$ where $(CR)^-$ is the generalized inverse matrix of (CR).
-The UIFO is useful for detecting actuator and system faults only if in addition to conditions in (17), the two conditions in (18) and (19) are satisfied.
-When all matrices $E_i$ are zero, we obtain the fuzzy proportional observer proposed by Ma and al [2] for fuzzy control of nonlinear systems.

## *4. Example:*

Dynamic models may be linear, in the simplest cases, or nonlinear. They may include ordinary differential or partial derivatives equations. For the need of diagnosis, we can

use the same mathematical model that used for control laws. [13].
To illustrate the performance of the unknown input fuzzy observer for state estimation and residual generation, we consider the system studied in [14] and illustrated in Figure (1). The plant consists of three cylinders $T_1$, $T_2$ and $T_3$ with cross section A. They are connected serially with one another by cylindrical pipes. A single so-called "nominal" outflow valve is located at $T_2$. The out-flowing liquid is collected in a tank which supplies the pumps $Q_1$ and $Q_2$, where $q_1$ and $q_2$ are the mass flow rates. The pumps are controlled such that a well-defined incoming mass flow corresponds to the reference input introduced to the pump controller. The three water levels $x_1$, $x_2$ and $x_3$, governed by the constraint $x_1 > x_3 > x_2$, are measured via pressure sensors. The process model is given by (20) and the numerical values of the physical parameters of the system are listed bellow:

$$\begin{cases} A\dfrac{dx_1(t)}{dt} = q_1(t) - \alpha_1 S_n(2g(x_1(t)-x_3(t)))^{1/2} + d(t) \\ A\dfrac{dx_2(t)}{dt} = q_2(t) + \alpha_3 S_n(2g(x_3(t)-x_2(t)))^{1/2} - \alpha_2 S_n(2g(x_2(t)))^{1/2} - d(t) \\ A\dfrac{dx_3(t)}{dt} = \alpha_1 S_n(2g(x_1(t)-x_3(t)))^{1/2} - \alpha_3 S_n(2g(x_3(t)-x_2(t)))^{1/2} - 2d(t) \end{cases} \tag{20}$$

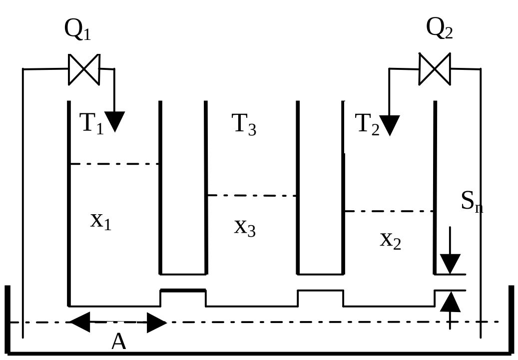


***Figure (1):** Three Tank System*

In (21), $\alpha_i$ (i = 1,2,3) are scaling constants for the relation between the cross-section of the connecting outlet pipes and the mass flows going through them and the unknown input d(t) denotes the additional mass flows into the tanks caused by leaks or plugging in the various tanks or pipes. Simulations were carried out initially in absence of faults by reaching three operating points (M = 3) according to the variations of the inputs like shown in Figure (2). The unknown input d(t) is chosen a random signal.
$\alpha_1 = 0.78$, $\alpha_2 = 0.78$, $\alpha_3 = 0.75$, $A = 154\text{x}10^{-4}$ et $S_n = 5\text{x}10^{-5}$ and $g = 9.8 N.Kg^{-1}$

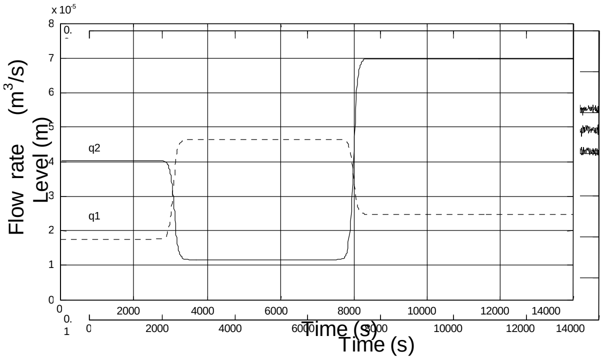


**Figure (2):** *Variations of the inputs* **Figure (3):** *Variations of the outputs*

Each output signal is disturbed by a Gaussian signal of zero mean and standard deviation equal to $5x10^{-3}$. Figure (3) illustrates the estimated outputs with the changes of operating points at the times: 3100s and 8000s. The selected initial conditions for the state estimation are different from the state values and the area of S defined by a = 0.02 and b = 0.08 makes it possible the observer to quickly converge with a good stability.
The residuals generated with the unknown inputs fuzzy observers are presented in Figure (4). In this study, two residuals are generated to supervise the two actuators. It can be noted that these residuals are zero mean and are different from zero at the beginning of the estimation due to the difference between initial conditions of real and estimated states. The residuals are also slightly different from zero at times of transitions from an operating point to another. These imperfections are directly related to modelling uncertainties. However, considering that they are of low amplitudes and, if necessary, by using adaptive thresholds, these residuals are considered of zero mean.

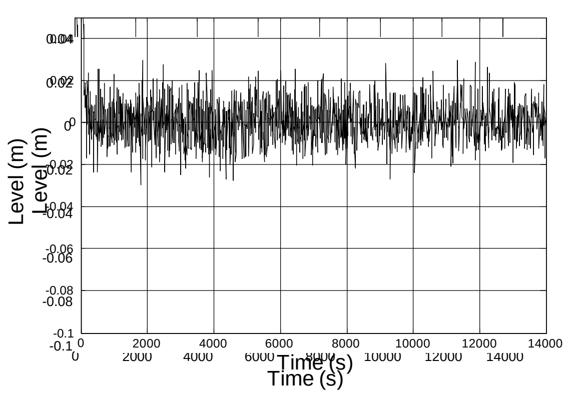


**Figure (4):** *Residuals without faults: First actuator(left:a), Second actuator(right:b)*

A fault is now considered on pump 2, with a 20% loss of its nominal flow value, which appears at the time t = 7000s, just before the change to the third operating point (Figure 5).

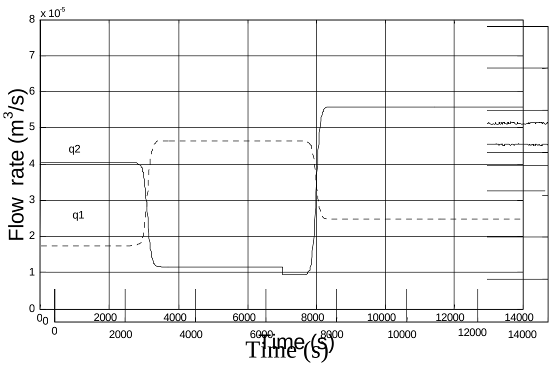


***Figure (5):*** *Inputs with a fault on Q2* ***Figure (6):*** *Estimated outputs with a fault on* $Q_2$

The dynamic evolution of all levels is consequently affected by this fault. Figure (6) illustrates that the outputs are different from those without faults. The presence of a fault generates changes of operating points, and thus decreases the initial performances. In spite of the presence of unknown inputs and noises, the continuous fuzzy observer well estimates these changes. As shown in Figure (8), the residual dedicated to supervising the second actuator moves slightly away from zero at t = 7000s because of fault's low amplitude. At t = 8000s, the control law $q_2$ increases from the value of $0.9259\text{x}10^{-5}m^3/s$ to $5.5830\text{x}10^{-5}m^3/s$, a transition from the second operating point to the third one takes place in spite of the fault. The fault amplitude (20% of $q_2$) increases, also the residual increases significantly at that time like clearly noticed in Figure 8. One can also note in Figure (7), that the residual dedicated to the first actuator remains insensitive to the fault which is considered by the observer as an unknown input and thus directly eliminated from this residual.

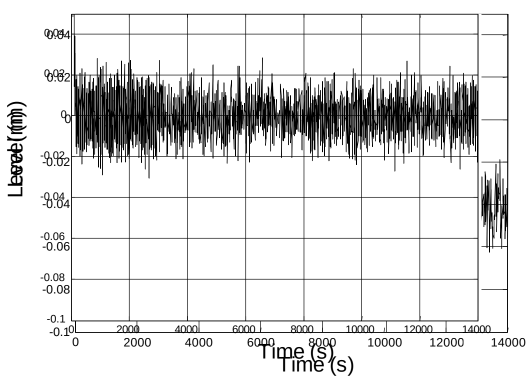


***Figure (7):*** *Residual dedicated to the first actuator*

***Figure (8):*** *Residual dedicated to the second actuator*

A same procedure is used to detect and isolate faults on the first actuator. One then obtains a DOS scheme (Dedicated Observers Scheme) that allows the detection of simultaneous actuators faults.

## *6. Conclusions*

In view of fault detection and isolation, the design of unknown input fuzzy observers for a T-S model has been proposed in this paper. Such observer relies on the existence of a quadratic Lyapunov function ensuring asymptotic convergence. The stability and convergence of the fuzzy observer requires however the consideration of coupling constraints between these local observers, these constraints lead to the resolution of a LMI problem by finding a common Lyapunov matrix X such as the fuzzy observer is stable and convergent. The reconstruction of the state vector is then possible. The simulation results show that the estimations are constraint to satisfying several conditions of existence and convergence. For FDI purpose, the output estimation is required. The direct application of this observer when taking into account unknown inputs and noises on outputs for residual generation, shows the well detection of faulty actuators with a simple procedure of isolation.

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