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Testing Exponentiality Against NBUC Class with Some Applications in Health and Engineering Fields

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ABSTRACT

The concept of convex order, which pertains to the arrangement of observations in a dataset, holds significant academic value and finds extensive application across diverse disciplines such as health and engineering. It serves as a robust mathematical tool for the purpose of ranking and comparing distinct alternatives. Nevertheless, the extent to which its performance aligns with the "new better than used" criterion remains mostly ambiguous. The objective of this study is to examine and analyze the exponential property of the idea of new better than used in convex ordering (NBUC), based on the concepts of aging and classes of life distributions. To examine its ramifications in the disciplines of health and engineering. For this purpose, a new test statistic for testing exponentiality versus NBUC property. Some efficiency criteria for the proposed test are estimated and compared with previous tests. Finally, real data examples are used as practical applications for the proposed test..

1. Introduction

Reliability theory is grounded in the principles of statistics and probability, and it pertains to the examination of system reliability. System reliability, in this context, denotes the capacity of a system or its constituent parts to execute their intended functions under prescribed conditions for a designated duration. The field of study encompasses the examination and anticipation of system and component failure patterns, to establish mathematical models and techniques for assessing reliability metrics, including failure probabilities, mean time between failures (MTBF), and failure rate. For more details, see for examples: [7], [30] and [31].

The examination of exponentiality in reliability theory holds significant importance, particularly within the field of engineering. This analysis provides engineers with vital insights into the overall performance and failure patterns exhibited by systems and components. There are other elements that contribute to the significance of this matter, one of which is the estimation of failure rates. The exponential distribution is frequently employed in the field of reliability engineering because of its capacity to accurately represent constant failure rates. Engineers can obtain accurate estimates of the failure rate by conducting tests on observed failure data that conforms to an exponential distribution.

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This is very helpful when predicting future failures and planning maintenance schedules. This helps engineers decide whether the system achieves the needed reliability levels and whether improvements or modifications are required. Additionally, it helps them to analyze warranty claims and evaluate if the observed failure patterns correspond to what is expected from an exponential distribution which leads to recognizing any potential manufacturing or design issues. Many authors focus on this importance, see for example, : [14], [17], [18], [34], [5] and [33].

Additionally, the examination of exponentiality plays a significant function within the realm of healthcare, as it facilitates comprehension and anticipation of illness progression. This is of utmost importance in the context of patient diagnosis and treatment. Moreover, the evaluation of therapies or interventions in the field of health holds significant importance. The tasks encompassed in this domain include the estimation of outbreak patterns, the evaluation of treatment effectiveness, the formulation of strategies for public health initiatives, and the assessment of vaccination efficacy. See for example, [12], [20] and [15].

Failure occurs when a unit or compoenent fails to perform the necessary function. Checking out whether the survival dataset has an increasing failure rate (IFR), a constant failure rate, or a decreasing failure rate (DFR) is part of the analysis of failure behavior. The constant failure rate property and the memoryless property are the two main characteristics of the exponential distribution. Due to these two characteristics, the exponential distribution leads to classes of life distributions. The dataset has been assembled two claims are made concerning it: first, that the data are exponential, and second, that they possess IFR property. A statistical test is performed to determine which of the two hypotheses or claims is true and to support one of the two claims. Recently, the classification of life probability distributions has helped to develop new highly efficient statistical tests. for more details: [8], [16], [25], [26], [9] and [2].

Statisticians and reliability analysts have studied testing exponentiality concerns based on various classes of life distributions from a number of classes: see for example: [4], [6] and [22].

Testing exponentiality against various classes of life distributions is a topic of great interest. These kinds of life distributions have applications in engineering, social science, biology, maintenance, and health. See for example: [27], [29], [10], [23], [24], [25], [28] and [11].

In this paper, the investigation for testing the exponential property versus the NBUC property is conducted for some observations. On the other hand, the arrangement of observations in a random sample, known as the convex ordering, plays a vital role in revealing significant insights on the distribution and structure of the data. It facilitates the identification of outliers, computation of statistical measures, establishment of comparisons, execution of hypothesis tests, and generation of visual representations. Hence, it assumes a crucial function in both the process of exploratory data analysis and the realm of statistical analysis.

The remainder of the paper is arranged as follows: The remainder of this section is dedicated to defining the NBUC class of life distribution. Testing the exponentiality versus NBUC is derived in Section 2. In Section 3, the Pitman asymptotic for some asymmetric distributions is obtained. The critical values and power estimates for Monte Carlo null distributions are simulated in Section 4. Additionally, based on the statistical test that is provided in Section 1, several real data applications are studied in Section 5. Finally, in Section 6, we list concluding remarks.

Here, the basic definition is provided to explain how our test is constructed as follows:

Definition 1.1 *For any distribution* $\boldsymbol{F}$, $\boldsymbol{F}$ *is NBUC, if and only if*

$$\int_x^\infty \overline{F}(y+u)du \leq \overline{F}(y)\int_x^\infty \overline{F}(u)du, x, y > 0. \tag{1.1}$$

For more details, see [3].

2 Testing Against the NBUC Class

The test statistic depends on the moment inequalities for testing H_o:F is exponential against an alternative that H_1:F belongs to NBUC belongs to the NBUC class and is not exponential is proposed.

We use laplace transform to construct the test as follows:

$$\delta = \int_0^\infty \overline{\nu}(x)e^{-sx}dx\int_0^\infty \overline{F}(y)e^{-sy}dy - \int_0^\infty\int_0^\infty \overline{\nu}(x+y)e^{-s(x+y)}dxdy$$

$$= I_1I_2 - I_3. \tag{2.1}$$

Since, $\overline{\nu}(x) = E(X-x)I(X>x)$, we easily get that

$$I_1 = E\int_0^X (X-x)e^{-sx}dx = E\left[\int_0^X Xe^{-sx}dx - \int_0^X xe^{-sx}dx\right].$$

After some simplifications, we obtain

$$I_1 = \frac{1}{s}E\left[X + \frac{1}{s^2}e^{-sX} - \frac{1}{s}\right]. \tag{2.2}$$

Also,

$$I_2 = \int_0^\infty \overline{F}(y)e^{-sy}dy = E\int_0^X I(X>y)e^{-sy}dy = E\left[\int_0^X e^{-sy}dy\right].$$

Then,

$$I_2 = \frac{1}{s}[1 - Ee^{-sX}]. \tag{2.3}$$

Finally,

$$I_3 = \int_0^\infty \int_0^\infty \overline{\nu}(x+y)e^{-s(x+y)}dxdy, v = x+y, u = y, |J| = 1$$

$$= \frac{1}{s^2}EX - \frac{1}{s^2}EXe^{-sX} + \frac{2}{s^2}EXe^{-sX} - \frac{2}{s^3} + \frac{2}{s^3}Ee^{-sX}. \tag{2.4}$$

Then,

$$\delta = I_1 I_2 - I_3$$
$$= \frac{1}{s^2}\left[EX + \frac{1}{s}Ee^{-sx} - \frac{1}{s}\right][1 - Ee^{-sX}]$$
$$- \frac{1}{s^2}EX + \frac{1}{s^2}EXe^{-sX} - \frac{2}{s^2}EXe^{-sX} + \frac{2}{s^3} - \frac{2}{s^3}Ee^{-sX}.$$

After some simplifications, we obtain

$$\delta = \frac{1}{s^2}\left[-EXEe^{-sX} - \frac{1}{s}(Ee^{-sX})^2 - EXe^{-sX} + \frac{1}{s}\right]. \tag{2.5}$$

Depends on a random sample $X_1, X_2, \dots, X_n$ and using Equ. (2.5), we estimate δ by:

$$\hat{\delta} = \frac{1}{n(n-1)}\sum_{l_1 \neq l_2} \frac{1}{s^2}\left[\frac{1}{s} - X_{l_1}e^{-sX_{l_1}} - \frac{1}{s}e^{-s(X_{l_1}+X_{l_2})} - X_{l_1}e^{-sX_{l_2}}\right]. \tag{2.6}$$

Setting

$$\phi(X_1, X_2) = \frac{1}{s^2}\left[\frac{1}{s} - X_1 e^{-sX_1} - \frac{1}{s}e^{-s(X_1+X_2)} - X_1 e^{-sX_2}\right].$$

$$E[\phi(X_1, X_2)|X_1] = \frac{1}{s^2}\left[\frac{1}{s} - X_1 e^{-sX_1} - \frac{1}{s}e^{-sX_1}\int_0^\infty e^{-sx}dF(x) - X_1\int_0^\infty e^{-sx}dF(x)\right]$$
$$= \frac{1}{s^2}\left[\frac{1}{s} - X_1 e^{-sX_1} - \frac{e^{-sX_1}}{s(s+1)} - \frac{X_1}{s+1}\right]. \tag{2.7}$$

$$E[\phi(X_2, X_1)|X_1] = \frac{1}{s^2}\left[\frac{1}{s} - \int_0^\infty xe^{-sx}dF(x) - \frac{1}{s}e^{-sX_1}\int_0^\infty e^{-sx}dF(x) - e^{-sX_1}\int_0^\infty xdF(x)\right]$$
$$= \frac{1}{s^2}\left[\frac{1}{s} - \frac{1}{(s+1)^2} - \frac{e^{-sX_1}}{s(s+1)} - e^{-sX_1}\right]. \tag{2.8}$$

Under H_o,

$$\sigma_o^2 = V\big[E[\phi(X_1, X_2)|X_1] + E[\phi(X_2, X_1)|X_1]\big].$$
$$= V\left[\frac{1}{s^2}\left[\frac{2}{s} - \frac{2e^{-sX_1}}{s(s+1)} - X_1 e^{-sX_1} - \frac{X_1}{s+1} - e^{-sX_1} - \frac{1}{(s+1)^2}\right]\right]. \tag{2.9}$$

Then,

$$\sigma_o^2 = \frac{1}{s^4}V\left[\frac{2}{s} - \frac{2e^{-sX_1}}{s(s+1)} - X_1 e^{-sX_1} - \frac{X_1}{s+1} - e^{-sX_1} - \frac{1}{(s+1)^2}\right].$$

After some simplifactions, we get

$$\sigma_o^2 = \frac{5+4s(2+s)}{(1+s)^4(1+2s)^3} \tag{2.10}$$

when $s = 1, \sigma_o^2 = \frac{17}{432}$.

3 Pitman Asymptotic Efficiency (PAE)

To assess the effectiveness of this test, we use the Pitman asymptotic efficienies (PAE) are computed for Generalized linear failure, Weibull and Gamma families in this section. The PAE is defined by:

$$PAE(\delta_1(\theta)) = \frac{1}{\sigma_o}\left|\frac{d}{d\theta}\delta_1(\theta)\right|_{\theta \to \theta_o}. \tag{3.1}$$

then,

$$\delta_1(\theta) = \int_0^\infty \bar{\nu}_\theta(x)e^{-sx}dx\int_0^\infty \bar{F}_\theta(y)e^{-sy}dy - \int_0^\infty xe^{-sx}\bar{\nu}_\theta(x)dx. \tag{3.2}$$

and

$$\frac{\partial\delta_1(\theta)}{\partial\theta} = \int_0^\infty \bar{\nu}_\theta(x)e^{-sx}dx\int_0^\infty \bar{F}_\theta'(y)e^{-sy}dy + \int_0^\infty \bar{\nu}_\theta'(x)e^{-sx}dx\int_0^\infty \bar{F}_\theta'(y)e^{-sy}dy$$
$$- \int_0^\infty xe^{-sx}\bar{\nu}_\theta'(x)dx. \tag{3.3}$$

After some simplifications, we obtain

$$PAE(\hat{\delta}_n) = \frac{1}{\sigma_o}\left|\frac{1}{(s+1)}\int_0^{\infty} \bar{\nu}'_{\theta_o}(x)e^{-sx}dx + \frac{1}{(s+1)} \times \int_0^{\infty} \bar{F}'_{\theta_o}(x)e^{-sx}dx - \int_0^{\infty} xe^{-sx}\bar{\nu}'_{\theta_o}(x)dx\right|. \quad (3.4)$$

For efficiency calculations, the following three families of alternatives are frequently used.

1. The PAE of $\hat{\delta}_n$ for Weibull distribution:

After using $\theta_o = 1$,we get

$$\bar{F}_\theta(x) = e^{-x} \text{ and } \bar{F}'_\theta(x) = -xe^{-x}ln(x). \quad (3.5)$$

Upon using Equs.(3.4) and (3.5), we obtain $PAE(\hat{\delta}_n) = 1.20813$

2. The PAE of $\hat{\delta}_n$ for Generalized Linear Failure Rate distribution (GLFR):

After using $\theta_o = 1$, we get

$$\bar{F}_\theta(x) = e^{-\left(\alpha x+\frac{b}{2}x^2\right)}$$

and

$$\bar{F}'_\theta(x) =.-(1-e^{-\left(\alpha x+\frac{b}{2}x^2\right)})ln(1-e^{-\left(\alpha x+\frac{b}{2}x^2\right)}). \quad (3.6)$$

Upon using Equs.(3.4) and (3.6), we obtain $PAE(\hat{\delta}_n) = 0.641109$

3. The PAE of $\hat{\delta}_n$ for Makeham family:

After $\theta_o = 0$, we get

$$\bar{F}_\theta(x) = e^{-[x+\theta(x+e^{x}-1)]}\text{and}\bar{F}'_\theta(x) = e^{-x}(1-x-e^{-x}). \quad (3.7)$$

Upon using Equs.(3.4) and (3.7), we obtain $PAE(\hat{\delta}_n) = 0.287193$

4 Monte Carlo Null Distribution with critical values

The Monte Carlo null distribution critical points for our test $\hat{\delta}_n$ are presented in this section. These critical points were calculated using 1000 samples of size 5(1)50 from a standard exponential distribution with the Mathematica (11) program. Table **1** displays thehe upper percentile points of the statistic $\hat{\delta}_n$

From Figure (1), It is obvious that the critical values are decreasing as the sample size increasing and are increasing as the confidence levels increasing.

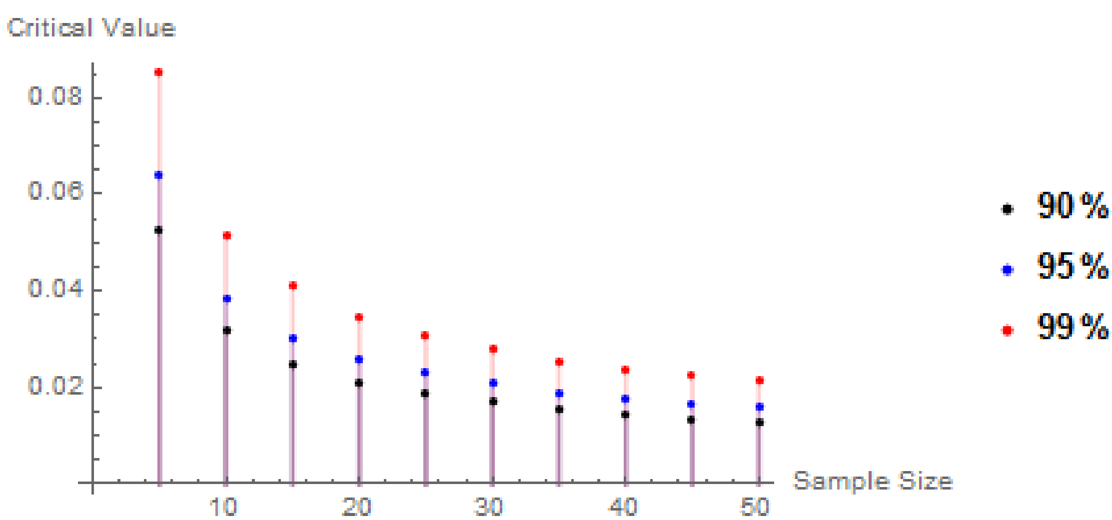

Figure 1: The relation between Sample Size and Critical Values

Table 1: Critical values of proposed test $\hat{\delta}_n$

n	90%	95%	99%	n	90%	95%	99%
5	0.0524075	0.0640333	0.085201	28	0.0170867	0211999	0.0287409
6	0.0449624	0.0550161	0.0753383	29	0.0173391	0214659	0.0285591
7	0.0402761	0.048953	0.0664083	30	0.0168879	0209572	0.0282702
8	0.0364348	0.044685	0.0605111	31	0.0162845	0202296	0.0273348
9	0.0333924	0.0409134	0.0562085	32	0.0162789	0199895	0.0274572
10	0.0316644	0.0386183	0.0515845	33	0.0160403	0200167	0.026327
11	0.0296022	0.0362597	0.0498059	34	0.0158383	0194211	0.0259668
12	0.0288255	0.035161	0.0475857	35	0.0153787	0187579	0.0253027
13	0.0267624	0.0326431	0.0439159	36	0.0151192	0185195	0.0253416
14	0.0260868	0.0325314	0.0431767	37	0.0150087	0185857	0.024751
15	0.0249832	0.0302663	0.0411107	38	0.0147174	0183595	0.0246153
16	0.024018	0.02876	0.0396025	39	0.0145089	0179697	0.0239174
17	0.0230344	0.0277395	0.0371467	40	0.014325	0178975	0.0237524
18	0.0224051	0.0274049	0.037569	41	0.0141531	0177392	0.0235973
19	0.0216765	0.026909	0.036132	42	0.0139851	0172686	0.0232914
20	0.0211305	0.0258291	0.0344109	43	0.0135143	0166491	0.0223321
21	0.0204952	0.0245983	0.0328007	44	0.0136362	0165994	0.022118
22	0.0202178	0.0247643	0.0335052	45	0.013582	0168474	0.0225509
23	0.0195158	0.0238419	0.032135	46	0.0132053	0162853	0.0221813
24	0.01896	0.0231172	0.0307754	47	0.0132365	0163362	0.021679
25	0.0187574	0.0230276	0.0307234	48	0.0133489	0162602	0.0217963
26	0.0180775	0.0224826	0.0304948	49	0.0129067	0162342	0.0219317
27	0.0181037	0.022154	0.0298297	50	0.0129771	.015912	0.0212876

Table 2: The Power estimates of the statistic $\hat{\delta}_n$ at $s = 2$

n	θ	Weibull	Gamma	GLFR
	2	0.8264	0.7715	0.4926
10	3	0.9949	0.9801	0.8968
	4	1.0000	0.9983	0.9883
	2	0.9911	0.9097	0.9157
20	3	1.0000	0.9982	0.9995
	4	1.0000	1.0000	1.0000
	2	0.9998	0.9645	0.9933
30	3	1.0000	0.9999	1.0000
	4	1.0000	1.0000	1.0000

5 Power of The Test

The power of the proposed test at a significant level of 95% with appropriate parameters is computed in this section based on simulated data for $n = 10{,}20$, and 30 and with respect to the Weibull, Gamma, and Generalized Linear Failure Rate Distribution. Table 2 displays the test's power.

Table 2 shows that $\hat{\delta}_n$ at $s = 2$ has a good performance for Weibull, Gamma and Generalized linear failure rate distribution and the power estimates increase when the sample size increases.

6 Applications

In this section, we extend our test to some actual data from the fields of medicine and engineering, using a 95% confinement level as follows:

Application 1: [1] presents a group of 40 patients suffering from blood cancer (leukemia) from a Saudi Arabian ministry of health hospital and the ordered values in years are

0.315	0.496	0.616	1.145	1.208	1.263	1.414
2.025	2.036	2.162	2.211	2.370	2.532	2.693
2.805	2.910	2.912	3.192	3.263	3.348	3.348
3.427	3.499	3.534	3.767	3.751	3.858	3.986
4.049	4.244	4.323	4.381	4.392	4.397	4.647
4.753	4.929	4.973	5.074	4.381		

We see that $\hat{\delta}_n = 0.0670345$ which is greater than the critical value of the Table 1. Then we reject the null hypothesis of exponentially and accept H_1 which states that the data set have NBUC property.

Application 2: Consider the real data given in [13] and have been utilized in [32]. The intervals between 25 customers arrivals at a facility are presented in this data set.

1.80	2.89	2.93	3.03	3.15	4.65
3.43	3.48	3.57	3.85	3.92	4.84
3.98	4.06	4.11	4.13	4.16	4.91
4.23	4.34	4.37	4.53	4.62	4.99
5.17					

We see that $\hat{\delta}_n = 0.0994745$ which is greater than the critical value of the Table **1**.Then we reject the null hypothesis of exponentially and accept H_1 which states that the data set have NBUC property.

Application 3: Goldfish were subjected to various dosages of methy 1 mercury in an experiment at Florida State University to explore how the poisoning affected the life lengths of the fish. At one dosage level, [19] reported that the ordered times to death in week were:

6.000	6.143	7.286	8.714	9.429
9.859	10.143	11.571	11.714	11.714

We see that $\hat{\delta}_n = 0.108022$ which is greater than the critical value of the Table 1.Then we reject the null hypothesis of exponentially and accept H_1 which states that the data set have NBUC property.

Application 4: [21] represents failure times in hours, for a particular type of electrical insulation in an experiment in which the insulation was subjected to a continuously increasing voltage stress as follows

0.205	0.363	0.407	0.770	0.720	0.782
1.178	1.255	1.592	1.635	2.310	

We see that $\hat{\delta}_n = 0.0355937$ which is less than the critical value of the Table 1.Then we accept H_o and conclude that this data set have exponentially property and hence the used method is not significant.

Concluding Remraks

A non-parametric statistical test is proposed for testing the exponential property against the NBUC property for some observations relevant to the Engineering and Health sectors. The efficiency criteria of the proposed test show good performance compared to tests presented previously in the literature. Therefore, it is recommended to credit the decisions made based on applying the proposed test. The proposed test can be applied to assess the efficacy of all treatment methods in all different fields in medical research regardless of knowing the nature of the used treatment method. But it is not recommended to apply this test in case of comparing two treatment methods. It is also recommended to develop new non-parametric statistical tests with high efficiency and using these tests as tools for assessing the different proposed treatments. The proposed test is not limited to be applied in the medical research but it can be extenend to the industrial engineering field to assess some manufacturing schemes aiming to producing reliable products.

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