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Integrated MEMS-Based IMU/GPS navigation system for motion dynamical vehicles

By

Hossam M. Hendy[*] Y. Z. Elhalwagy[**] G. A. Al-Sheikh[***]

Abstract:

Aided navigation system, in particular GPS/INS integrated navigation system, is a growing approach for both civilian and military applications. The availability of low cost MEMS-based inertial sensors aided with GPS dominated enormous research for both land and small airborne vehicle navigation. This paper addresses improving the performance of MEMS-based inertial sensors, for integration with GPS for land vehicle navigation applications, so that it becomes comparable to that of tactical grade sensors. Experimentally, the general performance characteristics of MEMS-based inertial sensors and their variations are investigated. This investigation helps finding a proper algorithm to efficiently model the variation of those performance characteristics. In addition, the problem of the integrated system is discussed and its KALMAN filter formulation is outlined. Such a filter calculates the best estimates for a set of parameters from all data collected up to the processing epoch; and it also can predict the subsequent progress of the process; The basic schemes commonly used for integrating inertial and GPS are given. The paper addresses issues of reducing sources of errors and consequently increases the accuracy of the required measured quantities. An experimental work has been conducted in this study for error modeling characterization for physical low cost IMU. The sensor data required for estimating the error coefficients is obtained using the turntable associated with testing facilities and acquisition system. A strap down inertial navigation (SDINS) algorithms has been carried out and verified using accurately real measurement data. Finally, an integrated GPS/INS system is developed, verified and tested in a field environment. A vehicle test was performed under varying operational conditions. Experimental results show an adequate performance for an in progress project. The performance analysis results are very relevant to system design and platform trajectory.

[*] Egyptian Armed Forces
[**] Egyptian Armed Forces
[***] MSA

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1. Introduction:

Basically, navigation is the art of knowing where you are, how fast you are moving and in which direction. Inertial navigation is accomplished by an Inertial Measurement Unit (IMU) that integrates the output of a set of sensors to compute position, velocity, and attitude. IMU is a sensor assembly constructed by inertial sensors such as accelerometers and gyros. An IMU can measure the linear acceleration and angular velocity of a vehicle to calculate its navigational information, and is the core equipment in an inertial navigation system [1], [2]. An ordinary IMU consists of three accelerometers and three gyros. In general, the sensors in an ordinary IMU have an orthogonal configuration. Therefore, this IMU cannot detect the faults of the sensors. In this case, the navigation system may have an enormous error due to these faults.

Inertial Navigation is a dead reckoning technique, so it suffers from one serious limitation: drift rate errors constantly accumulate with time. Because its drift errors always accumulate, an inertial navigation system that operates for a long time must be updated periodically with accurate positioning information. This can be accomplished by using an external navigation reference such as GPS. The objective of any INS is to determine the location, velocity and acceleration of an object with respect to some reference frame. Convenient INS achieve these objectives by performing first and second integrations to acceleration – vector components in the three main directions x, y, and z. Thus, the accuracy of INS constituted of gyroscopes and accelerometers relies on the internal errors of these sensors.

The position accuracy requirements can be satisfied using GPS and/or DGPS. However, the raw data output rate of a typical GPS receiver is limited to 1-2Hz. For land and/or small airborne vehicle navigation applications the GPS data rate is too low. In this case high rate IMU data can serve to interpolate the GPS position data. Another aspect of INS usage is the ability of extrapolation in GPS data gaps. Additionally, satellite based navigation systems cannot supply the orientation parameters with the required temporal resolution and the necessary accuracy. On the other hand also the INS navigation accuracy can be improved by GPS information; through data integration the sensor errors of the INS can be estimated.

Integrated GPS/INS systems have been developed in order to overcome the inherent drawbacks of each system. Moreover, such a system is well suited for trajectory determination, as it can be easily described using position and attitude information. In such an integrated system, low data rate, high accuracy GPS measurements can be used to estimate and to correct the error states of the INS within a dedicated Kalman filter.

The integration of GPS with INS can be implemented using a Kalman filter in such modes as loosely, tightly and ultra-tightly coupled. In all these integration modes the

INS error states, together with any navigation state (position, velocity, and attitude) and other unknown parameters of interest, are estimated using GPS parameters.

In principle the GPS/INS integration can be done in two different ways. Firstly a near hardware connection between both systems is conceivable. Thereby the INS sensors support the tracking loop of the GPS receiver. A fast reacquisition of the satellite signal after a gap could be one potential advantage of such a tight linkage. Secondly there is the software sided integration in which both sensors are physically separated and only their measurements will be integrated. This strategy can be subdivided: in a central filtering method (closely coupled filtering) and a decentralized approach (loosely coupled filtering). In the loosely coupled integration mode GPS raw measurements are pre-analyzed in a local KALMAN filter to determine the GPS positions and velocities in a geographic coordinate system [1-4]. In a second KALMAN filter these positions are combined with the INS raw measurements to calculate more reliable positions and velocities, and also orientation parameters and INS sensor errors.

In closely coupled Kalman filters only one central filter combines the raw data of both systems. This approach has the advantage also to take into account GPS data also if less than 4 satellites are visible [1]. A disadvantage of this concept consists in a rather complicated integration of additional sensors. These techniques based on Kalman filter integration have a long history in navigation; they have the advantage of being able to provide position and orientation parameter estimates in real time. Given a correct stochastic model they calculate optimal solutions based on all past data.

This paper addresses issues of reducing sources of errors and consequently increases the accuracy of the required measured quantities. To accomplish this, various approaches has been considered, simulated, experimentally measured and tested. First, an experimental work has been done in this study for error modeling and characterization for physical low cost IMU. The sensor data required for estimating the error coefficients is obtained using the turntable associated with testing facilities and acquisition system. In order to obtain the proper sensor data, a test procedure for the turntable is designed. Using the test procedure, a rate test and multi-position test are carried out. Second, a strap down inertial navigation (SDINS) algorithms has been carried out and verified using accurately real measurement data. Finally, an integrated GPS/INS system is developed, verified and tested in a field environment. A vehicle test was performed under varying operational conditions. Experimental results show an adequate performance for an in progress project. The performance analysis results are very relevant to system design and platform trajectory.

2. Mathematical Modeling and Equations Mechanization

2.1. Optimal Estimator (Kalman Filter) :

Kalman Filter is a recursive algorithm designed to compute corrections to a system based on external measurements. The corrections are weighted according to the filter current estimate of the system error statistics. The derivations of the filter equations require some knowledge of linear algebra and stochastic processes. The filter equations can be cumbersome from an algebraic point of view. The filter is very powerful in several aspects, it supports estimations of past, present, and even future states, and it can do so even when the precise nature of the modeled system is unknown [4-9]. The Kalman Filter has a general problem of trying to estimate the state $x \in \Re^n$ of a discrete-time controlled process that is governed by the linear stochastic difference equation:

$$x_k = \phi_{k-1}\, x_{k-1} + G_k\, u_k + w_{k-1} \tag{1}$$

with a measurement $z \in \Re^m$ that is

$$Z_k = H_k\, x_k + v_k \tag{2}$$

The random variables w_k and v_k represent the process and measurement noise respectively. They are assumed to be independent white with normal probability distributions

$$p(w) = N(0, Q) \tag{3}$$

$$p(v) = N(0, R) \tag{4}$$

Where the process noise covariance Q, and measurement noise covariance R matrices might change with each time step. However, here, they are assumed to be constant. Let's define $\hat{x}_k^- \in \Re^n$ as a priori state estimate at step k given the knowledge of the process prior to step k, and $\hat{x}_k \in \Re^n$ as a posteriori state estimate at step k given the measurement Z_k. A priori and a posteriori estimate errors and error covariance are:

$$e_k^- = x_k - \hat{x}_k^- \,, \qquad P_k^- = E\left[e_k^-\ e_k^{-T}\right] \tag{5}$$

$$e_k^- = x_k - \hat{x}_k \,, \qquad P_k = E\left[e_k\ e_k^T\right] \tag{6}$$

The linear combination between a posteriori state estimate $\hat{x}_k$, a priori state estimate $\hat{x}_k^-$, and a weighted difference between an actual measurement Z_k and measurement prediction $H\ \hat{x}_k^-$ is: -

$$x_k = x_k^- + K_k * \left[Z_k - H_k * x_k^-\right] \tag{7}$$

In the equation $\left[Z_k - H_k * x_k^-\right]$ is called measurement innovation or the residual. K_k matrix is also called as Kalman gain matrix and can be shown as:

$$K_k = P_k^- * H_k^T \left[H_k * P_k^- * H_k^T + R_k\right]^{-1} \tag{8}$$

The Kalman Filter estimates a process by using a form of feedback control: the filter estimates the process state at some time and then obtains feedback in the form of noisy measurements.

As such, the equations for the Kalman Filter fall into two groups: time update equations and measurement update equations. The time update equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain the a priori estimates for the next time step. The measurement update equations are responsible for the feedback i.e. for incorporating a new measurement into the a priori estimate to obtain an improved a posteriori estimate [4]. After each time and measurement update pair, the process is repeated with the previous a posteriori estimates used to project or predict the new a priori estimates in recursive nature as shown in Fig. 1.

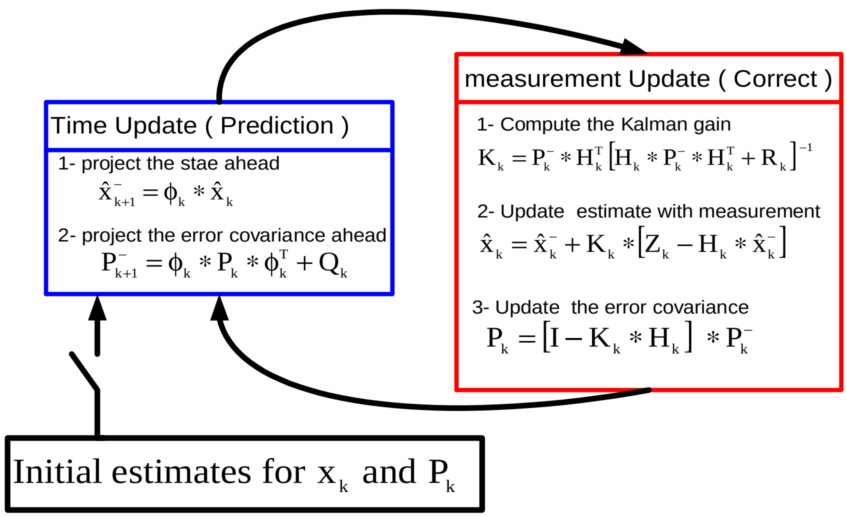


Figure (1): KALMAN filter architecture

2.2. The Attitude Dynamics Using Quaternion:

Using quaternion method of transformation [9] as the orientation states is an improvement over angular representations since quaternion are singularity-free, have fewer issues with normalization, and trade bulky trigonometric functions with more convenient polynomial operations. Quaternion models have been used in orientation Kalman filters; but they have yet to be used widely in Kalman filters for general navigation purposes [6]. In this work, quaternion method rotates a vector from the n-frame to the b-frame. This is because the IMU measures are in the b-frame and need to be rotated into the n-frame.

$$q_n^b = (q_0 \quad q_1 \quad q_2 \quad q_3)^T \tag{9}$$

The quaternion should satisfy the following normality condition to represent orientations to frame.

$$q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1 \tag{10}$$

The b-frame is attached to the rate gyros, so the gyro signals measure ω_{ib}^b the angular velocity of the b-frame with respect to the i-frame projection to the b-frame.

This can be decomposed into ω_{ie}^{b} the angular velocities of the e-frame with respect to the i-frame projection to the b-frame, ω_{en}^{b} the angular velocities of the n-frame with respect to the e-frame projection to the b-frame, and ω_{nb}^{b} the angular velocities of the b-frame with respect to the n-frame frame projection to the b-frame,.

$$\omega_{ib}^{b} = \omega_{ie}^{b} + \omega_{en}^{b} + \omega_{nb}^{b} \tag{11}$$

$$\begin{pmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 0 \end{pmatrix}\begin{pmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{pmatrix} \tag{12}$$

the transformation matrix C_b^n are accomplished by using quaternion [5].

$$C_b^n(q_n^b) = \left(C_n^b(q_n^b)\right)^{-1} = \left(C_n^b(q_n^b)\right)^T$$
$$= \begin{pmatrix} q_1^2 + q_0^2 - q_2^2 - q_3^2 & 2(q_1 q_2 + q_3 q_0) & 2(q_1 q_3 - q_2 q_0) \\ 2(q_1 q_2 - q_3 q_0) & q_2^2 + q_0^2 - q_1^2 - q_3^2 & 2(q_2 q_3 + q_1 q_0) \\ 2(q_1 q_3 + q_2 q_0) & 2(q_2 q_3 - q_1 q_0) & q_3^2 + q_0^2 - q_1^2 - q_2^2 \end{pmatrix}^T \tag{13}$$

The Euler angles can also be determined from quaternion by the following.

The pitch angle: $\theta = \sin^{-1}\left[-2\,(q_1 q_3 - q_0 q_2)\right]$ (14)

The roll angle: $\phi = \sin^{-1}\left[\dfrac{2\,(q_2 q_3 + q_1 q_0)}{\cos(\theta)}\right]$ (15)

The yaw angle: $\psi = \sin^{-1}\left[\dfrac{2\,(q_1 q_2 + q_3 q_0)}{\cos(\theta)}\right]$ (16)

3. ***Integrated Navigation System***

The most commonly used method of integrating GPS and INS is to use GPS position and velocity measurements to correct errors in the INS. This is generally referred to as loosely coupled integration. This is because the GPS and INS are treated as individual systems. This algorithm is also sometimes referred to as decentralized filtering since two KALMAN filters are used: one to process the GPS measurements, and a second to perform the integration. This algorithm is commonly used due to its simplicity and ease of hardware implementation. However, it is clear that for applications where there is a restricted view of the sky, there is an improvement that can be made by combining the two systems at the measurement level.

Figure 2 shows that the raw measurements are collected from the IMU and are converted to position, velocity, and attitude measurements in the desired coordinate system using the INS algorithms (quaternion algorithm).

The position is then used to form predicted range measurements (corrected for the lever arm offset from the GPS antenna to the INS) to each of the satellites.

These ranges are then differenced with the raw GPS measurements to estimate the INS errors feeding the integrating filter. The decentralized filtering algorithm is commonly used to integrate GPS and INS simply.

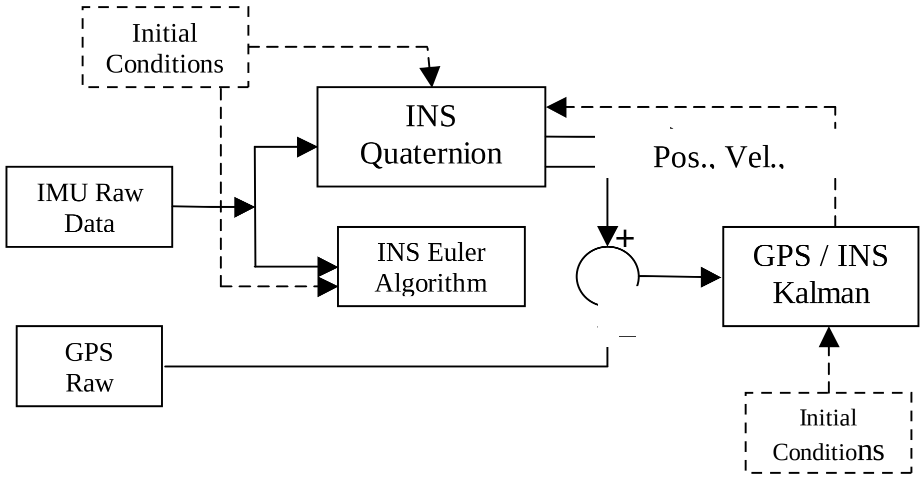


Figure (2): *flowchart of loosely integration algorithm*

4. Simulation and Experimental results:

In this section INS only results from the recorded IMU data [10] with quaternion algorithm that has been carried out in this paper, GPS only data [10], and the GPS/INS integrated work from KALMAN filter algorithm using loosely integration also carried out in this work, all are off line tests, and finally the integrated data recorded from an integrated GPS/INS software (Un available code), all are figured out to compare there performance (quaternion INS algorithm and KALMAN filter algorithm with the saved data from a complete integrated navigation system to insure there performance).

The integrated navigation algorithm has been carried out, verified, and tested based on a typical recorded data for specified trajectory utilizing high resolution measurement kit (LN200 tactical IMU and Novatel OEM4 GPS integrated using ready software) [10]. The condition for recording and analyzing these data is illustrated in the following:

A land vehicle (car) was used to set a reference trajectory during which an output data for GPS and IMU units has been saved. The GPS receiver and the IMU unit were fixed on the car with a lever arm from IMU to GPS in body frame: X=1.72m, Y=0.30m, Z=-1.49m and with initial conditions set as the following; Roll = +0.5 deg; Pitch = +14.0 deg; Yaw = +31.8 deg; with initial Coordinate: Lat. = 51 5’ 47.10419"; Lon. = -114 21’ 47.08920”; Height = 1182.908; the height includes the antenna height.

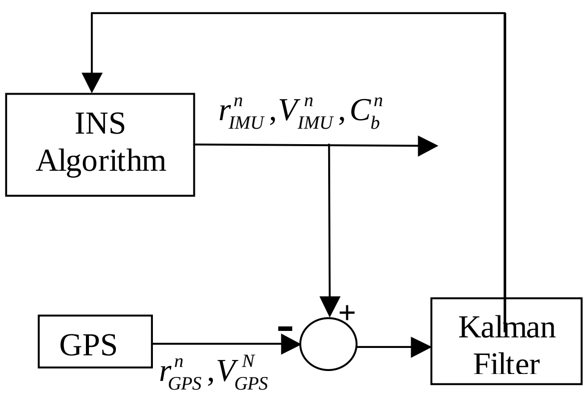
Since both sensors cannot be installed at the same place in the host vehicle, the position and velocity of the IMU are different from those of the GPS.

This is called the lever-arm effect. The lever-arm correction for the position and velocity can be written as:

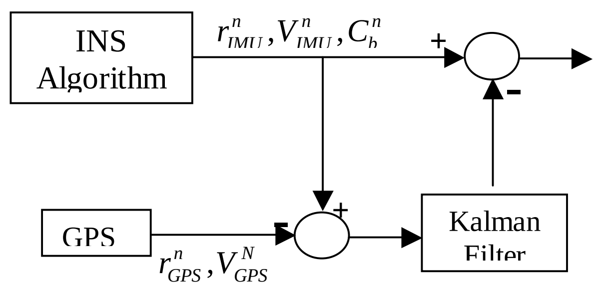
$$\underline{r}^{n}_{IMU} = \underline{r}^{n}_{GPS} - \begin{pmatrix} \frac{1}{M+h} & 0 & 0 \\ 0 & \frac{1}{(N+h)\cos\varphi} & 0 \\ 0 & 0 & -1 \end{pmatrix} C^n_b \Delta \underline{r}^{b} \tag{18}$$

$$\underline{v}^{n}_{IMU} = \underline{v}^{n}_{GPS} - C^n_b \Omega^b_{nb} \Delta \underline{r}^{b} = \underline{v}^{n}_{GPS} + (\Omega^n_{ie} + \Omega^n_{en}) C^n_b \Delta \underline{r}^{b} - C^n_b \Omega^b_{ib} \Delta \underline{r}^{b} \tag{19}$$

$\Delta \underline{r}^{b}$ is the offset vector of the GPS antenna from the centre of the IMU in the body frame. The estimated errors in the navigation components are fed back to the algorithm or fed forward to the output. In the feed forward method, the inertial system operates as if there was no aiding: it is unaware of the existence of the filter or the external data. The disadvantage of the feed forward method is that the algorithm can experience unbounded error growth, which makes unbounded error observations delivered to the Kalman filter. This causes a problem to the linear filter since only small errors are allowed due to the linearization process [7].



(a)



(b)

Figure (3): *(a)The feedback algorithm (b)The feed forward algorithm*

After the correction of level arm the results are recorded and shown in Figs. 4 - 13. The IMU output response and the navigation states resulted from the navigation algorithm using quaternion transformation method and the output data of integration are presented.

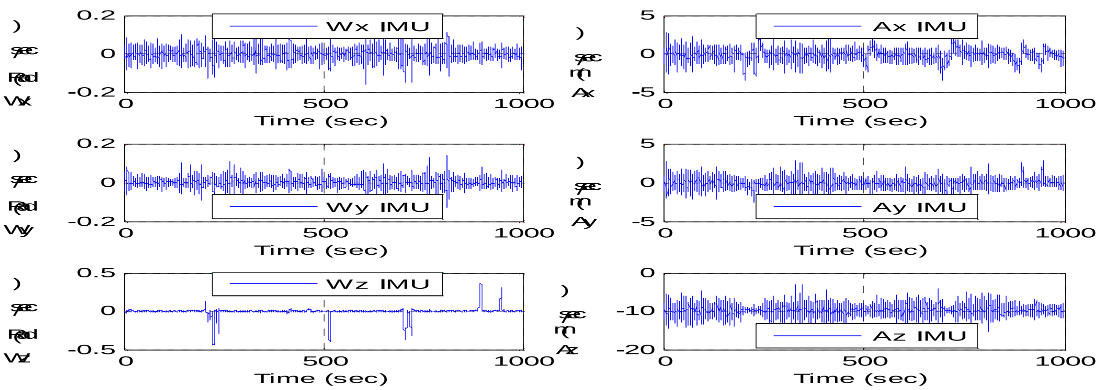


Figure (4): *IMU output response* (LN200)

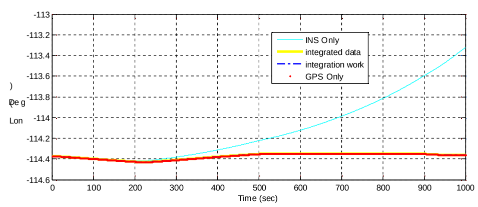


Figure (5): *Longitude*

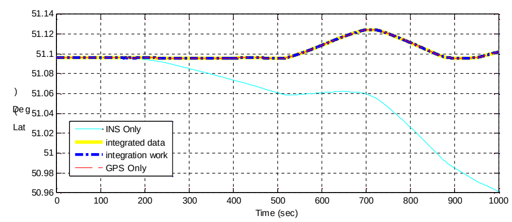


Figure (6): *Latitude*

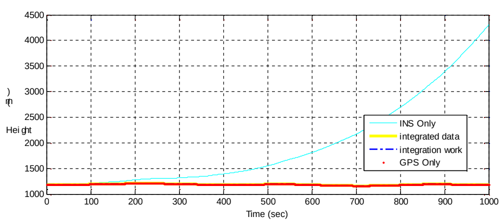


Figure (7): *Height*

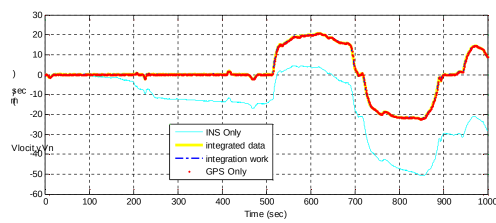


Figure (8): *North velocity*

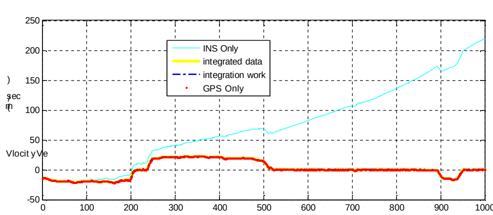


Figure (9): *East velocity*

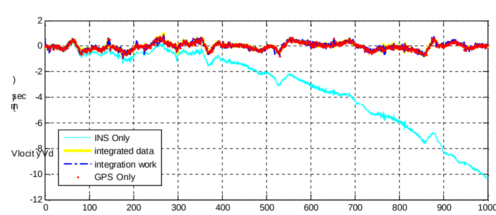


Figure (10): *Down velocity*

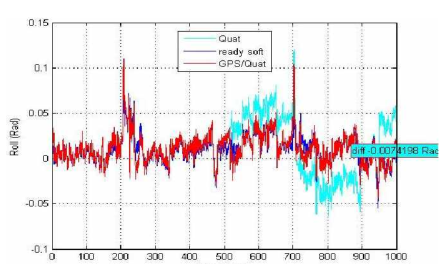


Figure (11): *Roll*

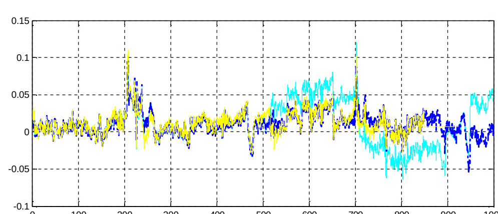


Figure (12): *Pitch*

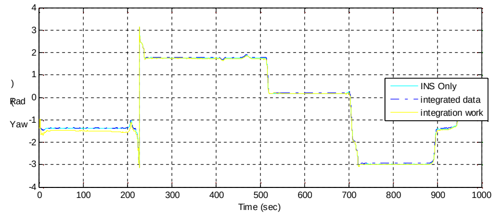


Figure (13): *Yaw*

From the previous figures, the navigation algorithm utilizing quaternion transformation technique is tested. As a result, integration between GPS and INS is carried out while utilizing Quaternion algorithm and using kalman filter in a loosely coupled integration scheme. Results of the final path and the integration for 1000 sec. run time are recorded and presented. Figure (14) shows that trajectories of the vehicle from the ready software and the proposed integrated navigation system including kalman filter algorithm are identical.

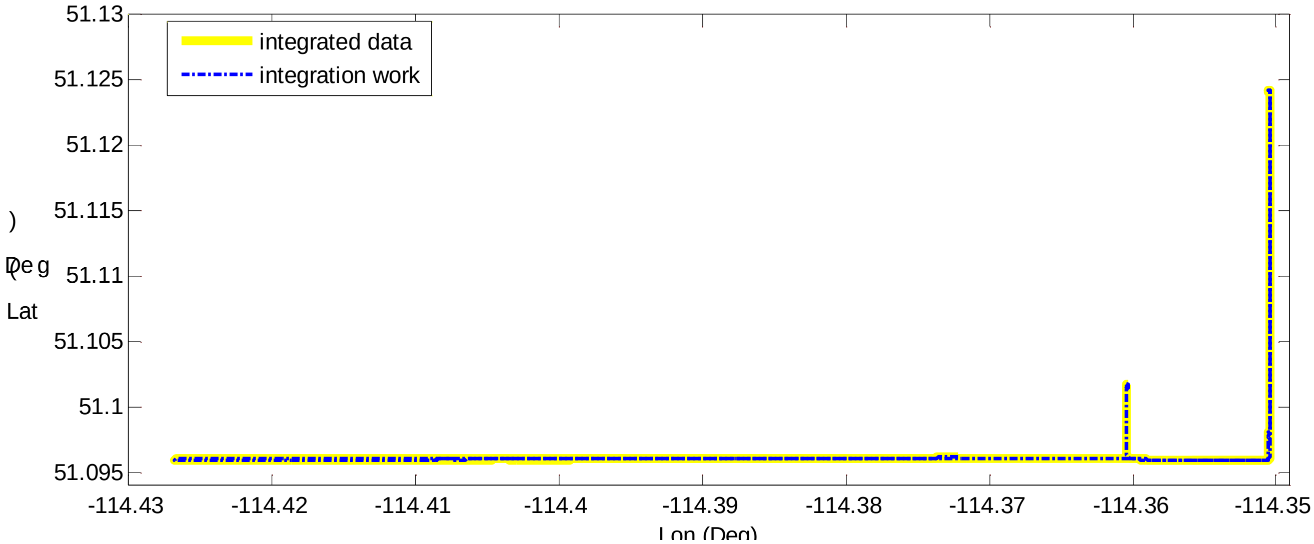


Figure (14): *Latitude and longitude from kalman filter*

Now, all algorithms (Quaternion INS model and kalman filter algorithm) in loosely coupled integration are ready to be used with the available low cost, poor accuracy IMU (MEMS ADXL202E) hardware fig. and b, and a (Mini Max) GPS receiver. Firstly, the dynamical characterization of the inertial sensor assemble has been carried out to figure out and verify crucial dynamical parameters including (scale factor, bias,…) using a single axes turn table and the associated data acquisition system as shown in figure 15.

Figure (15): *IMU (a and b) and GPS (c) kits*

Figure (16): *Turn table and control unit*

For example, the response for yaw gyroscope, mounted on a turning table in which the turning rates are pre-programmed, is shown in figure 17. Extensive tests have been conducted in various planes and positions.

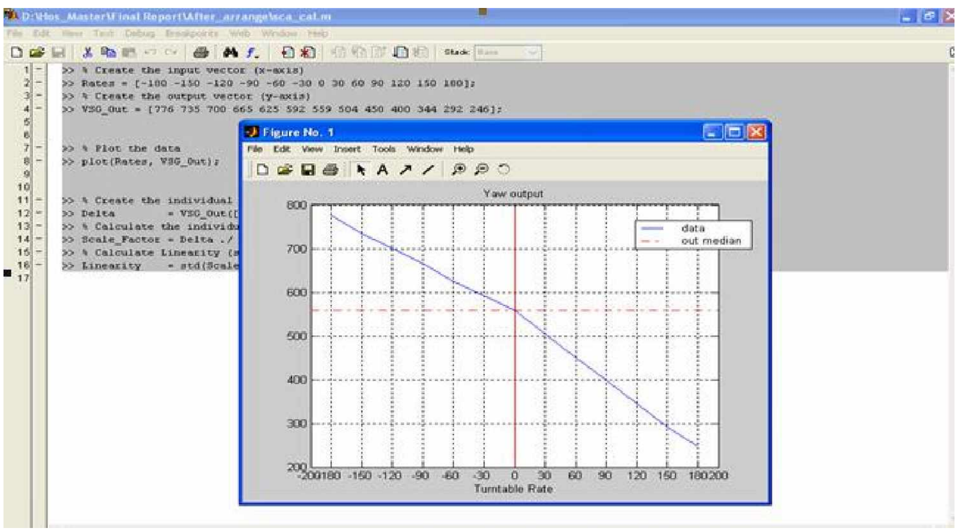


Figure (17): *Gyro scale factor test result*

Then, a real path is set in Alharam street to Elrmayaa city, the hardware (GPS and MEMS-based low cost IMU) is used to record the data during the motion of the vehicle. The proposed integrated navigation system is utilized to provide the navigation states and navigation parameters in which the results are illustrated in Figs. 18 – 27 as follows for analysis purpose. In addition, for better illustration for the set path Fig. 28 shows the trajectory from Google earth. The output from GPS only, INS only (Quat-INS) and the integration from both (GPS/Quat) are evaluated off line. An in progress work for online algorithm is under way.

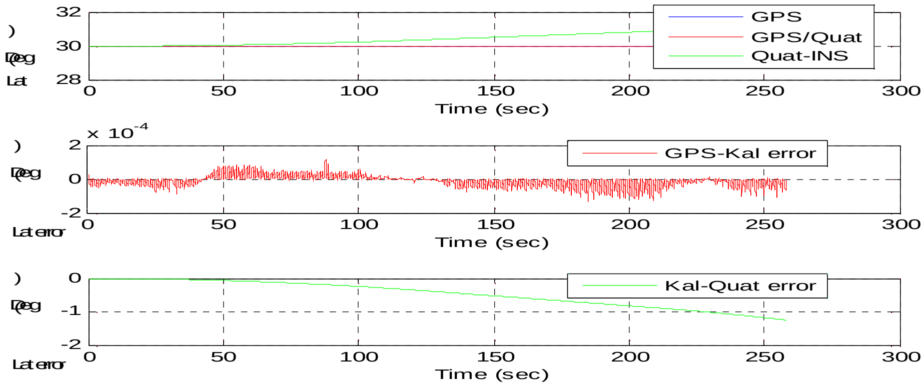


Figure (18): *Latitude of MEMS IMU, GPS, and Kalman filter*

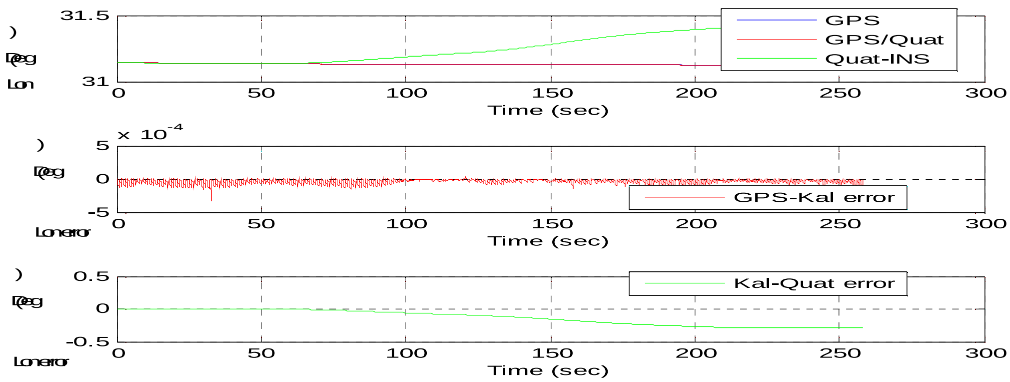


Figure (19): *Longitude of MEMS IMU, GPS, and Kalman filter*

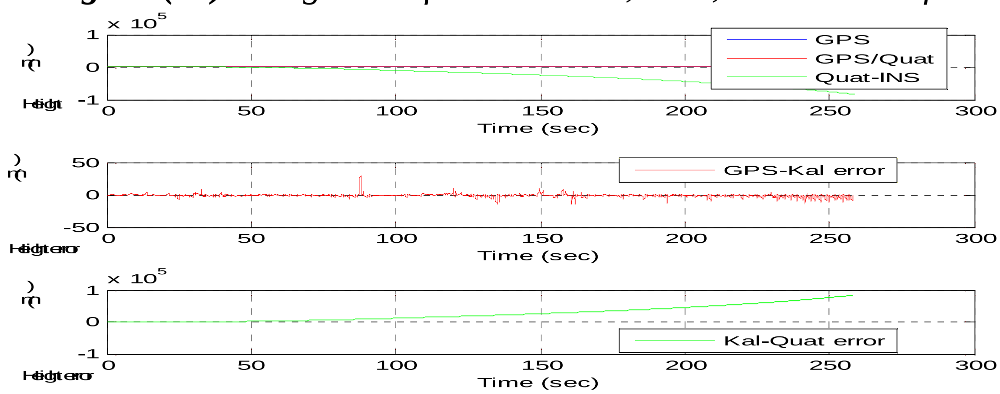


Figure (24): *Height of MEMS IMU, GPS, and Kalman filter*

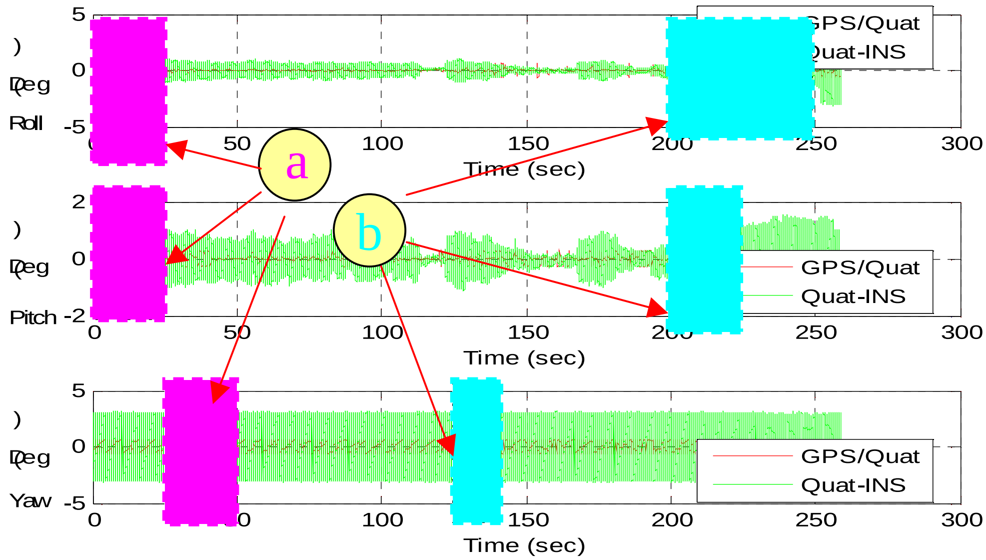


Figure (20): *Roll, pitch, and yaw*

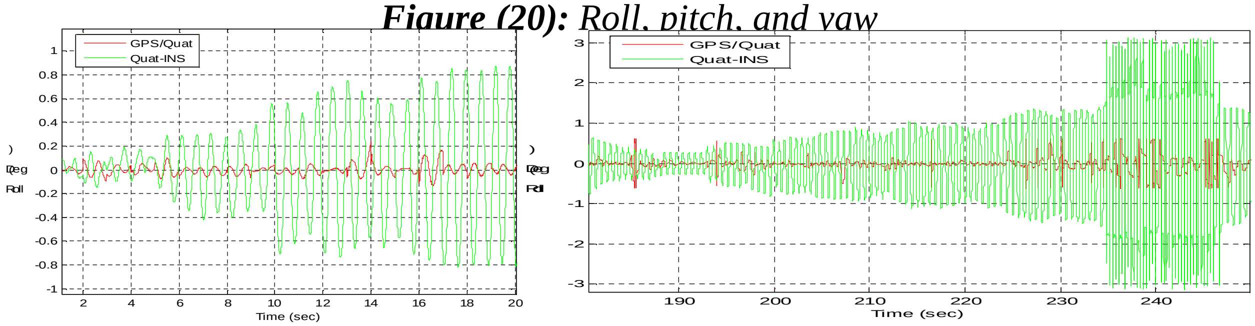


Figure (21): *Zoom in to roll a) time 0-20 sec b) time 200-250 sec*

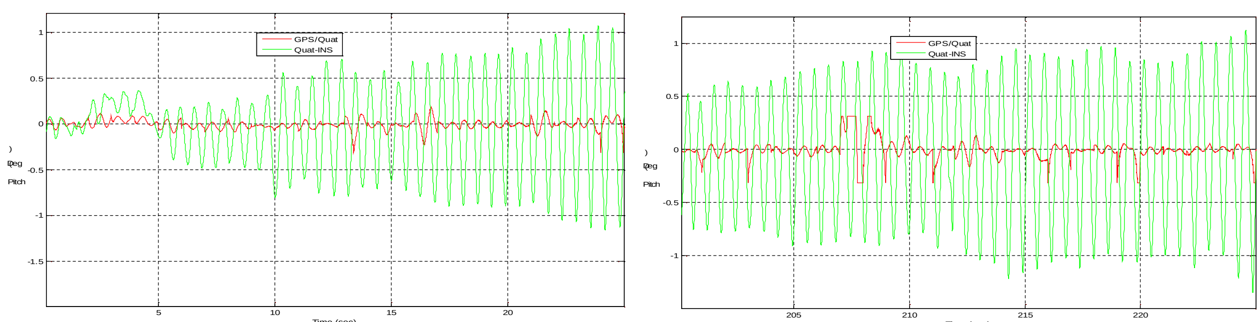


Figure (22): *Zoom in to pitch a) time 0-25 sec b) time 200-225 sec*

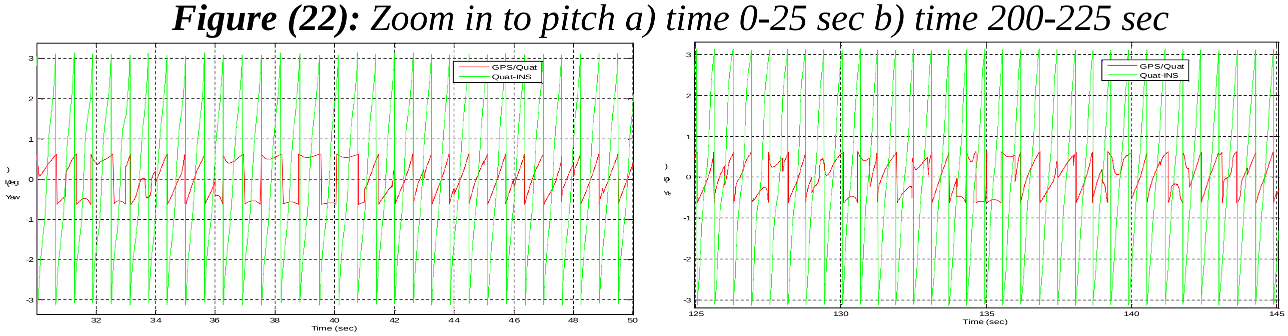


Figure (23): *Zoom in to yaw a) time 30-50 sec b) time 125-145 sec*

From figures 21, 22, and 23, it's clear that the kalman filter output is better and smoother than the output from the INS only algorithm, these output may be improved if an aided hardware is used like digital compass.

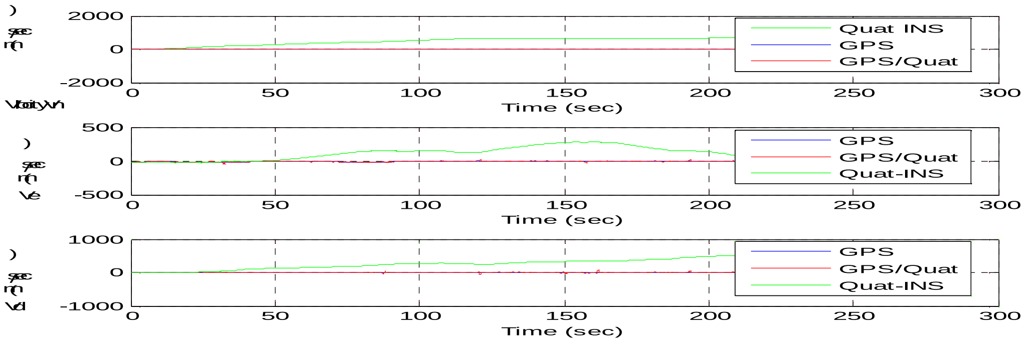


Figure (25): *Velocities (NED) of MEMS IMU, GPS, and Kalman filter*

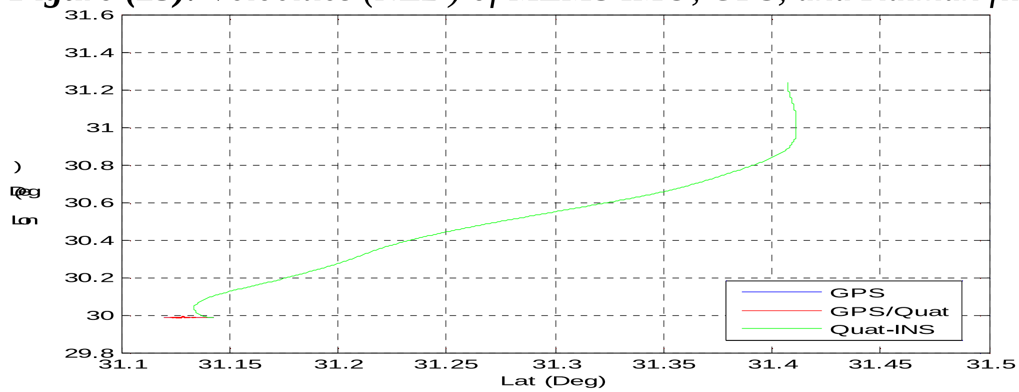


Figure (26): *Latitude and longitude from INS, GPS, and Kalman filter*

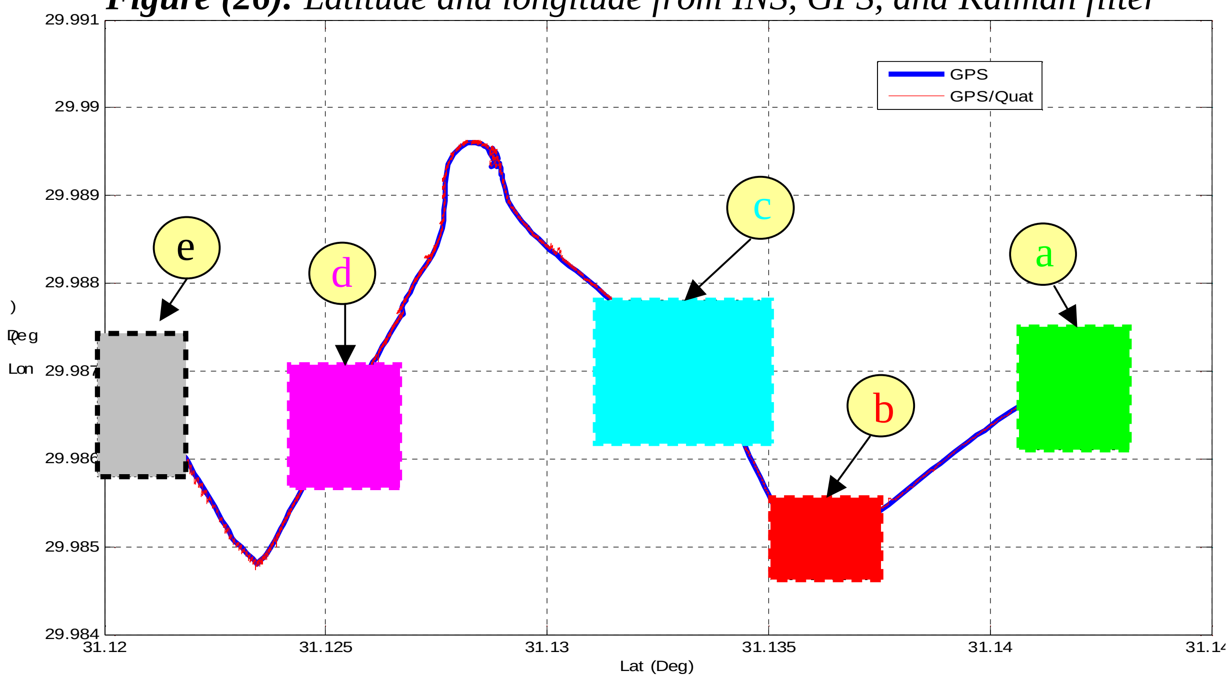


Figure (27): *Latitude and longitude from kalman filter*

Figure (28): *The trajectory from Google earth*

From Fig. 26, it is clear that there is a great deviation between the trajectory from the INS only algorithm and the trajectory from GPS and the integration model.

Figures (27) and (28) show the trajectory of the vehicle from the GPS receiver, the kalman filter integration algorithm and the real trajectory from the Google earth server respectively.

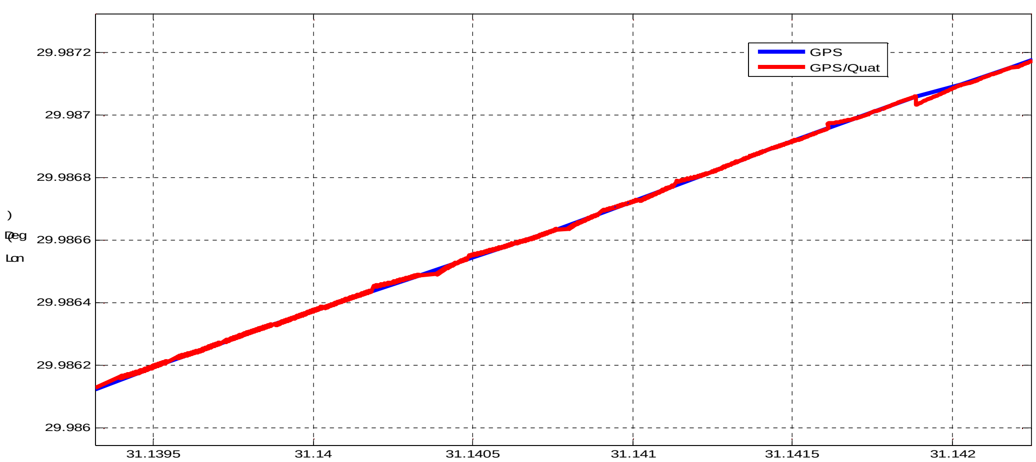


Figure (29): *Zoom in to section (a) Fig. 27*

The kalman filter is started with small perturbations then becomes smooth with the GPS output.

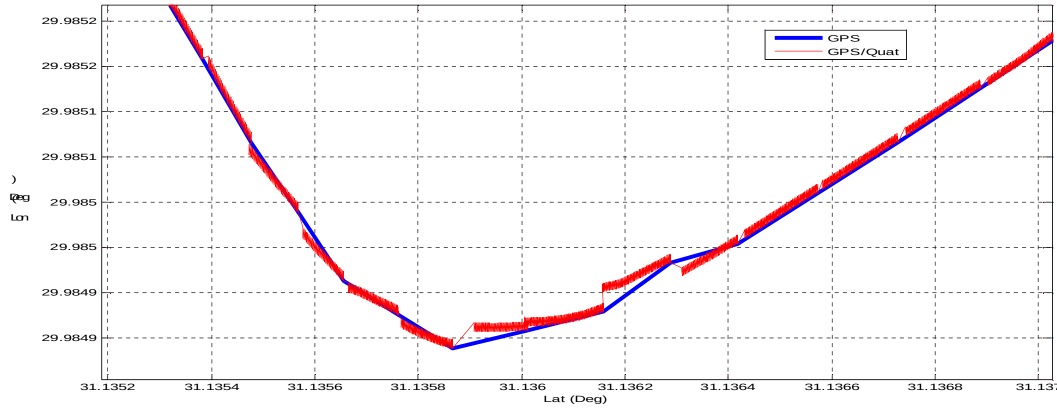


Figure (30): *Zoom in to section (b) Fig. 27*

Fig. (30) shows that the kalman filter follows the path during the curvature path.

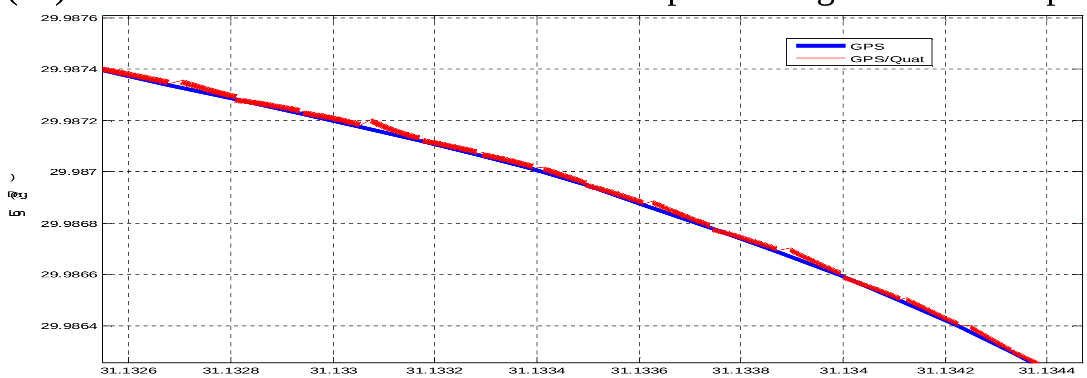


Figure (31): *Zoom in to section (22-c)*

Fig. (31) Shows that the kalman filter is still smooth after 120 sec of time running.

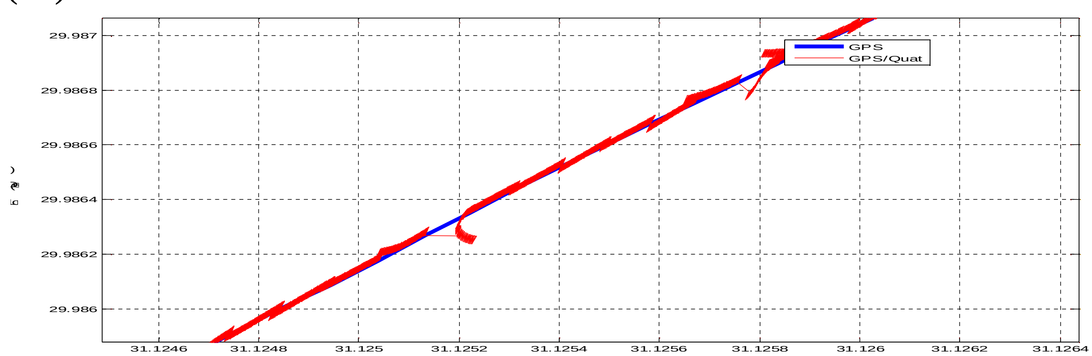


Figure (32): *Zoom in to section (22-d)*

Fig. (32) shows that the kalman filter started to deviate from the path but not continuously after 200 sec from starting work, this is done due to the rapid changing and accumulation of MEMS_IMU output errors.

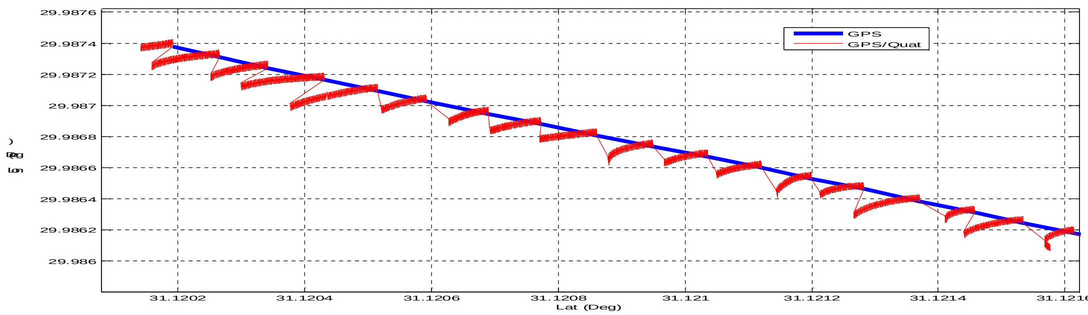


Figure (33): *Zoom in to section (22-e)*

Although the low cost IMU hardware accuracy is poor, the integration algorithm is still able to adequately follow the trajectory path. As a result, higher performance grade hardware while utilizing the achieved navigation algorithm may result in improving the overall performance of navigation states and navigation parameters.

5. Conclusions:

This paper addresses issues of reducing sources of errors and consequently increases the accuracy of the required measured quantities. To accomplish this, an experimental work has been done in this study for error modeling and characterization for physical low cost IMU. The sensor data required for estimating the error coefficients is obtained using the turntable associated with laboratory testing facilities and acquisition system. A rate test and multi-position test are carried out. A strap down inertial navigation (SDINS) algorithms has been carried out and verified using accurately real measurement data. Finally, an integrated GPS/INS system is developed, verified and tested in a field environment. A vehicle test was performed under varying operational conditions. Experimental results show an adequate performance for an in progress project. The performance analysis results are very relevant to system design and platform trajectory.

6. References

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***NOMENCLATURE*:**

ω_x : Angler velocity in x axis (body frame rad/sec)
ω_y : Angler velocity in y axis (body frame rad/sec)
ω_z : Angler velocity in z axis (body frame rad/sec)
φ : Position latitude in rad
λ : Position longitude in rad
h : Position altitude in meter
a_x : Acceleration in x axis in body frame in m/sec^2
a_Y : Acceleration in y axis in body frame in m/sec^2
a_z : Acceleration in z axis in body frame in m/sec^2
ϕ : Attitude roll in rad
θ : Attitude pitch in rad
ψ : Attitude yaw in rad
F_N : Acceleration in x axis (navigation frame in m/sec^2)
F_E : Acceleration in y axis (navigation frame in m/sec^2)
F_D : Acceleration in z axis (navigation frame in m/sec^2)
v_N : Velocity in x axis (navigation frame m/sec)
v_E : Velocity in y axis (navigation frame m/sec)
v_D : Velocity in z axis (navigation frame m/sec)
C_n^b : Transformation matrix (navigation to body frame)
C_b^n : Transformation matrix (body to navigation frame)
g^n : Gravitational acceleration (navigation frame m/sec^2)
ω_e : Angler velocity of earth in rad/sec