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Cairo, Egypt

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Pyramids in digital imaging

By

Rimon Elias*

Abstract:

This article surveys important concepts related to pyramidal architecture, hierarchical processing and multi-resolution techniques in imaging context. Types of processes including bottom-up and top-down related to such hierarchical structures are explained. Relationships consisting of intra-level and inter-level communication networks are discussed. We investigate regular and irregular pyramidal structures. This includes bin-pyramid; quad-pyramid (and overlapped quad-pyramid); dual-pyramid; hexagonal-pyramid; adaptive pyramid and disparity pyramid.

Keywords:

Pyramidal architecture; pyramids; hierarchical processing; hierarchy; multi-resolution processing; hierarchical structures; data structures; regular pyramids; irregular pyramids; bin-pyramid; quad-pyramid; overlapped quad-pyramid; dual-pyramid; hexagonal-pyramid; adaptive pyramid and disparity pyramid.

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* Faculty of Media Engineering and Technology, German University in Cairo, Egypt

1. Introduction:

Processing of visual information or images is characterized by intensive calculations. Many algorithms and techniques have been developed to ensure appropriate computational time for the appropriate amount of information or image size. The family of techniques that are concerned with such an issue is referred to as *hierarchical multi-resolution* techniques or *pyramidal architecture* techniques. Working on pyramids was introduced by Kelly and Tanimoto in the seventies of the last century [8,13]. A good introduction to multi-scale image understanding can be found in [3] and a complexity analysis can be found in [1].

This article is not meant to be an extensive research project; rather, it gives a very brief introduction to some of the issues related to multi-resolution or pyramidal architecture techniques and discusses the basics of such techniques. It is organized as follows. Section 2 gives an introduction to the concept of hierarchical multi-resolution. Section 3 discusses the basic types of pyramidal structures while Section 4 is a conclusion.

2. Hierarchical Multi-resolution

"Reduce the amount of the input data to speed up the execution of the algorithm." This concept refers to *hierarchical multi-resolution*. A hierarchical structure consists of a number of levels (or layers). Each layer represents the image at some resolution (or level of abstraction). Such a structure is referred to as a *pyramid*. The pyramid is fed with an input image at its lowest level that is called the *base* [6]. Higher levels are built up using information fed at the base. The highest level of the pyramid is called the *apex* where more global information is gathered. This hierarchical structure is depicted in Figure 1.

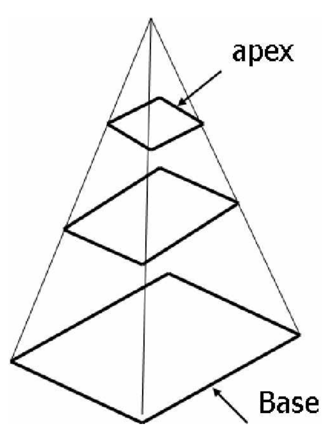


Figure (1): *A pyramid*

In general, a pyramid consists of a number of levels as mentioned previously. Each level consists of a number of *nodes* or *cells*. At the base of the pyramid, a cell represents one pixel that is connected to its neighbors either using 4-connectivity or 8-connectivity. At the higher levels, a cell should represent a region at the base (also known as a *receptive field* which is defined as the set of all descendents located at the base of the pyramid). A level can be represented as a 2D array of cells or in some cases as a graph (i.e., adjacency list or adjacency matrix). Links are constructed among cells in the same level to form what is called the *intra-level communication network* and between different levels comprising the *inter-level communication network*.

Note that a cell at the position $[x,y]$ of level h is referred to as $[x,y,h]$ where the level is referred to as I_h. Most of the time, the base, I_0, is fed with a square image with a side length of 2^n where 2^n is called the *diameter*. For example, a 256x256 pixel image has a diameter of 2^8 with pixels in the range of $[0, 2^n\text{-}1]$. At a given level h, the set of coordinates is indicated by the range $[0, 2^{n-h}\text{-}1]$. Thus, the set of coordinates at the apex is given by $[0, 2^{n-n}\text{-}1]$ or the position $[0,0,n]$.

The information is passed from the bottom to the top of the pyramid and in the opposite direction constituting two types of operations that are called *bottom-up* and *top-down* processes. The goal of the bottom-up process is to extract the most important features and/or patterns from the input image. The levels in this process get smaller with each creation. In the top-down process, local information at lower levels is used to refine the values extracted at the higher coarser levels resulting in smoother results.

3. Pyramidal Architecture Types

All pyramidal structures may fall into two main categories. These are regular and irregular pyramidal structures [6] (or pyramids for short). The main differences between both categories are mainly how to build the levels of the pyramid and what nodes are connected in the same level and among levels as well.

Many pyramids have been introduced in each of these categories. Bin-pyramid, quad-pyramid, overlapped quad-pyramid, dual-pyramid and hexagonal-pyramid are examples of regular pyramids. Custom-made pyramid, Stochastic pyramid, adaptive pyramid and disparity pyramid are examples of irregular pyramids. Interested reader can refer to [9,5,12,11,7,4].

3.1 Bin-Pyramid

As depicted in Figure 2, in a bin-pyramid, every two nodes (or cells) contribute to one parent in the upper level. To achieve a good convergence, the direction of choosing children is to alternate between rows and columns. In this pyramid, every cell has one parent and two children except for the cells at the base and apex levels.

The number of links in this pyramid is small at the cost of greater number of nodes which is equivalent to 2^{n+1}-1 if 2^n is the side length of the image inserted at the base.

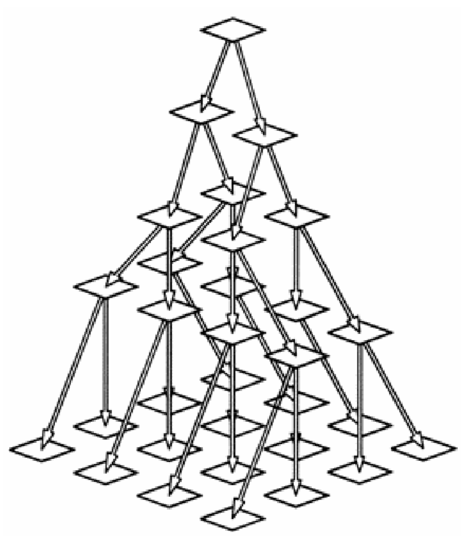
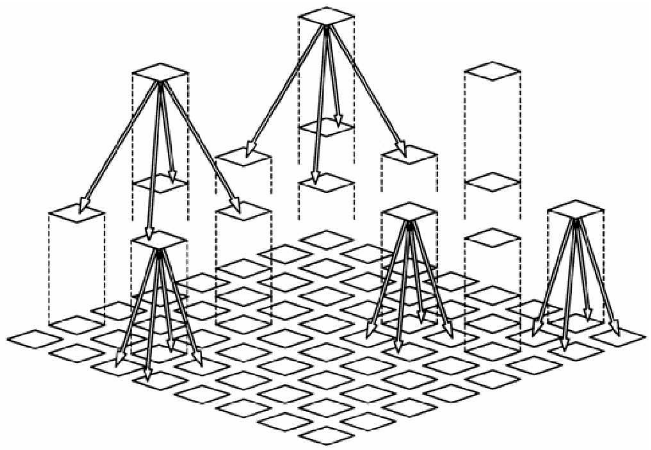


Figure (2): *Bin-pyramid*

3.2 Quad-Pyramid

The most common pyramidal architecture is the quad pyramid (as shown in Figure 3). In this pyramid, one cell survives to the next from a collection of four neighbors. Thus, a cell at a level (other than the base or the apex) should have exactly four children and one parent.



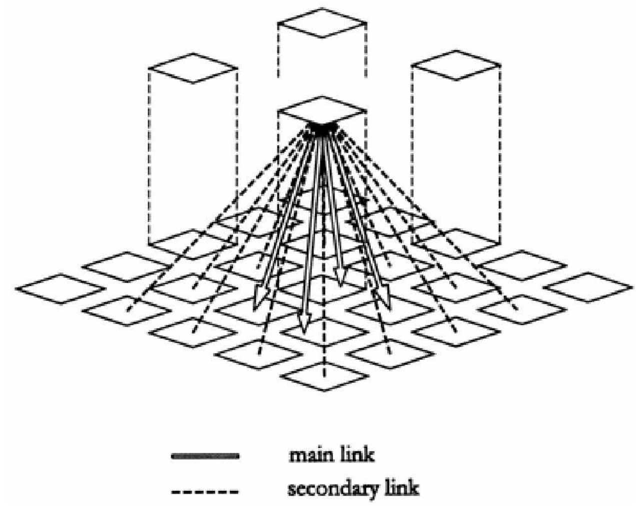


Figure (3): *Quad-pyramid*

Figure (4): *Overlapped Quad-pyramid*

In the quad-pyramid, the number of nodes is given by:

$$\# \text{Cells} = 1 + 2 \times 2 + 4 \times 4 + \ldots + 2^{n-1} \times 2^{n-1} + 2^n \times 2^n = \frac{4^{n+1} - 1}{3} \tag{1}$$

The location of the parent of any given cell [*x*,*y*,*h*] is given by:

$$\text{Parent}\,(x, y, h) = \left[\left\lfloor \frac{x}{2} \right\rfloor, \left\lfloor \frac{y}{2} \right\rfloor, h + 1 \right] \tag{2}$$

3.3 Overlapped Quad-Pyramid

The quad-pyramid can be altered to the overlapped quad-pyramid (Figure 4) if the neighbors of the four children are added to the calculation. In such a case, a parent would have 16 children and a child may have four parents. The locations of the parents of any given cell [*x*,*y*,*h*] are given by:

$$
\begin{aligned}
\text{Parent}_1(x, y, h) &= \left[\left\lfloor \frac{x}{2} \right\rfloor, \left\lfloor \frac{y}{2} \right\rfloor, h + 1\right] \\
\text{Parent}_2(x, y, h) &= \left[\left\lfloor \frac{x}{2} \right\rfloor + 2(x\%2) - 1, \left\lfloor \frac{y}{2} \right\rfloor, h + 1\right] \\
\text{Parent}_3(x, y, h) &= \left[\left\lfloor \frac{x}{2} \right\rfloor, \left\lfloor \frac{y}{2} \right\rfloor + 2(y\%2) - 1, h + 1\right] \\
\text{Parent}_4(x, y, h) &= \left[\left\lfloor \frac{x}{2} \right\rfloor + 2(x\%2) - 1, \left\lfloor \frac{y}{2} \right\rfloor + 2(y\%2) - 1, h + 1\right]
\end{aligned}
\qquad (3)
$$

3.4 Hexagonal-Pyramid

In the *hexagona1*-pyramid [5], a triangular tessellation is used. So a cell represents a triangular region. In this structure, the apex is made of 6 triangles arranged in a hexagon [6]. Each triangle within a level (other than the base and the apex has four children and a parent. Three levels of such a pyramid are depicted in Figure 5.

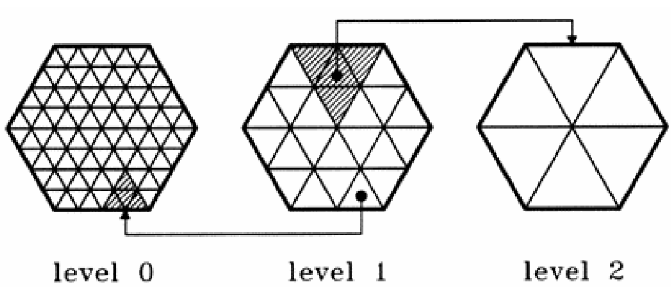
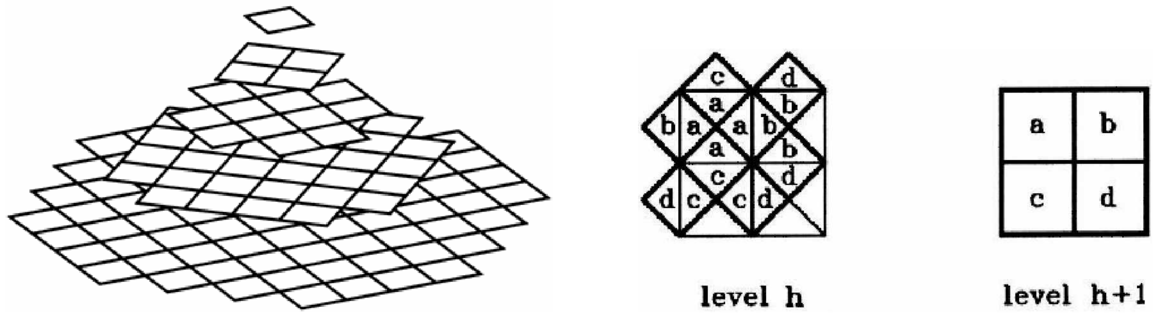


Figure (5): *Hexagona1-pyramid*

3.4 Dual-Pyramid

Figure 6 shows an example of the dual-pyramid [9]. In such a pyramid, the levels are rotated 45º alternatively. This is done in such a way that the diagonal of a square becomes an edge of a square in the next level.



(a) (b) (c)

Figure (6): *Dual-pyramid: (a) The shape of the structure (b) The thick small square represents cells at level h. The thin diagonals represent edges of the square on the next level h+1. (c) Level h+1.*

3.5 The Adaptive Pyramid

This adaptive pyramid [7] is an irregular data-dependent pyramid. The *decimation procedure* (i.e., the procedure by which the surviving cells are selected) is based on the following two rules:

1. Two neighbors cannot survive at the next level.
2. For each non-surviving cell, there exists at least one surviving cell in its neighborhood.

A cell can survive only if it is a local extreme of an *interest operator* which is the variance in this case. Two binary variables (e.g., *p* and *q*) are used to detect the surviving cells, so the cell will survive if *p* is set to 1. The links are established according to the gray level, so the non-surviving cell is linked to the least contrasted surviving cell in its support. An example of the decimation process is shown in Figure 7.

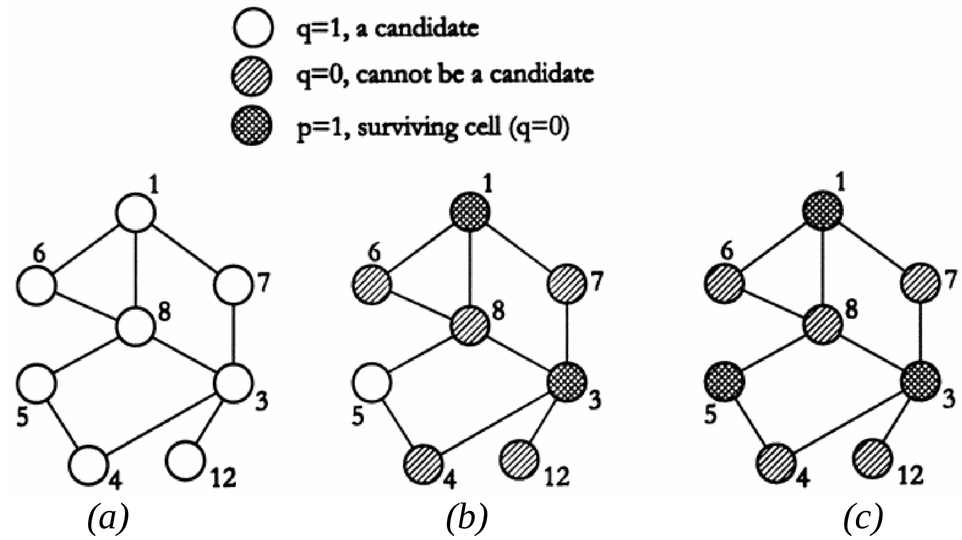


Figure (7): *The adaptive pyramid: (a) The variance values before the decimation process. (b) The surviving and non-surviving cells after the first iteration. (c) The surviving and non-surviving cells after the second iteration.*

3.6 The Disparity Pyramid

The disparity pyramid [4] is an irregular data-dependent pyramid. This pyramid is used with a stereo pair of images to estimate the disparity between them. Also, it can be used to extract objects with different gray levels from the scene. The sum of squared differences (SSD) calculation is done over a disparity range to get the difference in intensity between points in the two images. The following equation is applied at a point $[x,y]^T$ over a disparity range (dx_{min}, dx_{max}, dy_{min}, dy_{max}).

$$F_{xy}(dx, dy) = \sum_{i,j=-1}^{i,j<2} \left(I_1(x+i, y+j) - I_2(x+i+dx, y+j+dy)\right)^2 \tag{5}$$

where $dx \in [dx_{min}, dx_{max}]$ and $dy \in [dy_{min}, dy_{max}]$. The previous equation calculates the interest operator $F_{xy}(dx,dy)$ where the lower the value the higher the interest of this cell.

A cell can survive to the next level if it has the minimum difference $F_{xy}(dx,dy)$ over the disparity range (dx_{min}, dx_{max}, dy_{min}, dy_{max}). The same rules applied to the adaptive pyramid are used here as well. These rules are:

1. Two neighbors cannot survive at the next level.
2. For each non-surviving cell, there exists at least one surviving cell in its neighborhood.

A non-surviving cell in the lower level is linked to the surviving cell in its neighborhood. That surviving cell at the higher level acts as its parent. At the highest level of the pyramid, each cell represents a disparity vector.

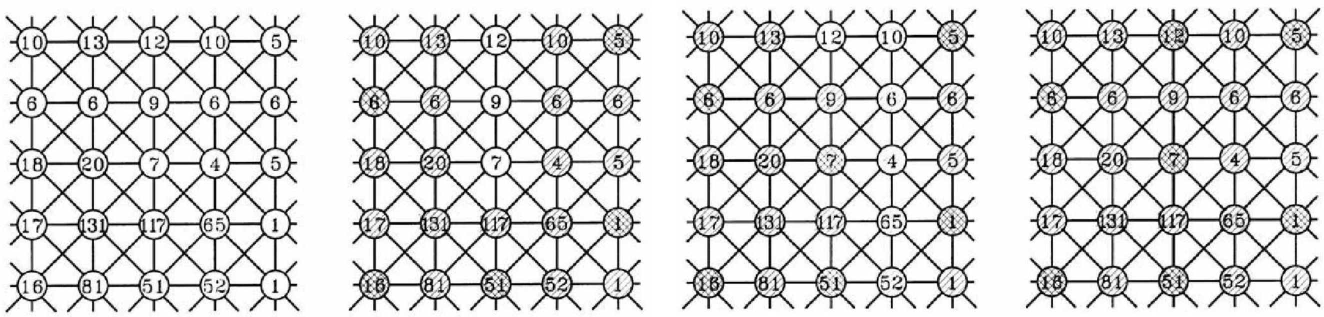


Figure (8): *The disparity pyramid: (a) Initial graph with F_{ijh} values. (b) First iteration: global minima are extracted (c) Second iteration: local minima are added (d) Third iteration: local minima are added.*

3.7 Advantages and Disadvantages

Problems with regular pyramids in comparison with irregular pyramids are discussed in [2]. A main disadvantage of using the regular pyramid is that the root at the top level cannot represent an irregular region; however, it becomes an approximation between the largest square contained in the region and the smallest square containing the region. Such rigidity may result in some distortions. On the other hand, an irregular region can be represented more accurately using an irregular pyramid. A comparison between the two main categories; regular and irregular, can be found in [10].

4. Conclusions:

This paper investigated briefly the concept of hierarchical multi-resolution in image analysis and computer vision. The notion of hierarchical architecture and pyramidal structures was explored. Two main pyramidal categories were discussed. Those are regular and irregular categories. Under these categories, some pyramids were explored. Among those are the bin-pyramid, the quad-pyramid (and overlapped quad-pyramid), the dual-pyramid, the hexagonal-pyramid, the adaptive pyramid and the disparity pyramid.

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