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COMPREHENSIVE STUDY of SOIL SUBGRADE REACTION USING DIFFERENT CONSTITUTIVE SOIL MODELS

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Abstract

Oftentimes, static analysis of a slab supported by vertical uncoupled Winkler springs is frequently utilized in the structural design of shallow foundations. Many structural designers typically simulate the soil beneath the footing and the raft as a collection of springs with stiffness coefficient estimated based on the modulus of subgrade reaction (ks). Therefore, various approaches for determining ks are compared and reviewed in this work.

The main objective of this study is to estimate the modulus of subgrade reaction (ks) of the shallow foundation rested on sandy layer adopting three constitutive soil models (Elastic, Mohr-Coulomb and Hardening soil) utilizing 3D Plaxis analysis. Two parametric studies have been implemented (isolated footing and raft) taking into consideration the effects of foundation thickness and soil constitutive model on ks.

The results outlined in this paper show that a thicker footing typically has a lower soil subgrade reaction than one that is thinner; and the estimation of ks based on empirical formulas is being convenient in case of isolated footing. On the other side, the distribution of ks is non-uniform through the raft for all adopted soil models analyses. ks (when considering soil plasticity) is lesser than ks estimated using elastic soil analysis. Using non-linear finite element analysis (considering the soil plasticity) is very important to get more accurate ks and do not depend only on the empirical formulas to evaluate ks of the raft.

Keywords

Constitutive Models; Finite Element; Subgrade Reaction; Shallow .Foundation; Soil- Structure Interaction

1. BACKGROUND

Building foundations present a geotechnical issue because the interaction between the soil and the structure is important to achieving the most cost-effective design that satisfies all safety and serviceability standards. To achieve a successful design, both geotechnical and structural engineers should work together. The topic of soil structure interaction has a great importance among many engineers specially, structural and geotechnical ones. This is attributed to the complex behavior of the soil when subjected to the effect of foundation.

Generally, the geotechnical engineer evaluates the soil stiffness and subgrade-reaction values then the structural engineer these values. An ideal and cost-effective foundation can be achieved with the help of a realistic model of how structure and soil interact. The main scientific challenge is incorporating the soil properties, particularly the soil stiffness, into the structure model.

Conventionally, the structure-soil interaction is typically modelled using two different approaches. The beam/plate approach resting on an elastic foundation is one method, and the continuum method using finite element analysis is another (FEA). Therefore, most engineers replace this interaction by a simpler methodology titled subgrade model. The Winkler model, which is well-known to almost all designers, is one of the most prevalent and basic models in this context. The Winkler foundation model idealizes soil as a series of springs that displace due to the load acting on them. The major disadvantage of the model is that it does not account for the interaction of the springs. The linear stress-strain behavior of the soil is also used to describe it.

The original Winkler model's spring coupling was taken into account by Filonenko-Borodich (1940) [1] by introducing the coupling effect to the elastic springs stiffness. Terzaghi (1955) [2] developed a set of equations to calculate the ks value using a 0.3 x 0.3 m square plate or a circular plate with a diameter of 0.3 m. The modulus of subgrade response (k) that is required is then produced by adjusting (k0.3) for the dimensions of the footing. Vesic in (1961) [3] analyzed an infinitely long beams, and described the subgrade as an elastic, homogeneous, and isotropic half-space. The Pseudo-Coupled Idea was first developed by Bowles (1982) [4] and the ACI Committee 336 (1988) [5] in an effort to preserve the clarity of Winkler's method while producing a more accurate description of the subgrade behavior. Liao (1995) [6] concluded that the soil modulus of elasticity, the stiffness of the beam, and the loading circumstances all had a significant impact on the value of ks at various points.

Daloglu and Vallabhan (2000) [7] proposed a method to estimate an equivalent value of the modulus of subgrade reaction (k) to improve Winkler's model. They concluded using higher values of ks at slab edges to obtain realistic results and the depth of soil layer has a significant effect on ks. The distribution of soil reaction pressures at the base of a footing supported by a multi-layered subgrade was assessed by Dey et al. in (2011) [8]. The results indicated that the variation of the subgrade modulus and consequently the generated contact pressures are highly affected by the type of loading. The results of this study show how significantly the subgrade coefficient's variability affects the mat's flexural response. According to Showdhary's (2012) [9] analysis, several consistent values of the modulus of subgrade response were used in the analysis. It was demonstrated that the generated bending moments and deflections closely matched those obtained from a thorough finite element study. The observed maximum bending moments increased by 20% as the subgrade response modulus increased. Larkela et al. (2013) [10] investigated the interaction between soil and structure using PLAXIS 3D, a finite element sofware, and demonstrated that the subgrade modulus is variable, particularly when the foundation is loaded with uniform pressure, in addition to accounting for the effect of soil plasticity in the estimation of the contact pressure. The coupling effect and shear interaction among Winkler's soil springs are taken into account in a novel analytical model that Lee et al. (2015) [11] created. They used Pasternak model, which takes into account the spring's shear interaction in the soil-structure interaction. Loukidis and Tamiolakis (2017) [12] improved on Liao's work from 1995 in order to produce the more accurate set of ks values. Instead of elastic springs, the soil is represented by a volume of 3D continuum elements. They outlined that the modulus of subgrade reaction is typically constant for 60% of the footprint of the foundation and increases significantly as it gets closer to the corners. Additionally, it was determined that using a consistent ks causes the peak positive moments to be underestimated and the peak negative moments to be overestimated. Mohamed Saad Eldin (2019) [13] examined the behavior of thin and thick plates lying on various sand soils. It is found that for loose sand, there is small changing in the settlement values in case various plate thicknesses. And no change in settlement for medium dense sand and no more variation in case dense sand. Ahelah A. Jawad, Raid R. Almuhanna, and Alaa M. Shaban (2020) [14] studied the maximum surface settlement and the subgrade reaction modulus from the static plate load test based on numerical analysis using 3D Plaxis. The findings show that the subgrade modulus increases as soil dry unit weight and degree of compaction increase. Conversely, a decrease in the subgrade reaction modulus is caused by an increase in water content. Sami W. Tabsh and Magdi El-Emam (2021) [15] concluded a significant relationship between the developed rigidity factor, the critical soil bearing pressure, and the raft's maximum internal bending moment. The rigidity of raft has no effect on the critical shear force. Each of raft thickness and span between columns have more critical effect than elasticity modulus of raft and soil subgrade reaction. Haitham H. Saeed (2022) [16] indicated that punching shear and bending moments of raft are underestimated when considering the foundation soil as linear elastic rather than considering it an elastoplastic material.

Table 1: Modulus of subgrade reaction formulas, ks

Source of formula	Suggested formula
Winkler (1867)	$ks = \frac{q}{\delta}$
Biot (1937)	$ks = \frac{0.95\,Es}{B(1-\upsilon_s^2)}\left[\frac{Es\,B^4}{(1-\upsilon_s^2)EI}\right]^{0.108}$
Terzaghi (1955)	$ks = ksp\left(\frac{B+B_P}{2B}\right)$
Vesic (1961)	$ks = \frac{0.65\,Es}{B(1-\upsilon_s^2)}\sqrt[12]{\frac{Es\,B^4}{EI}}$
Meyerhof and Baike (1965)	$ks = \frac{Es}{B(1-\upsilon_s^2)}$
Selvadurai (1984)	$ks = \frac{0.65\,Es}{B(1-\upsilon_s^2)}$
Bowles (1996)	$ks = 40\,SF\,qa$
Bowles (1998)	$ks = \frac{Es}{B(1-\upsilon_s^2)mI_sI_F}$
Daloglu et al. (2000)	$ks = \frac{0.78\,Es}{B(1-\upsilon_s^2)}\left[\frac{Es\,B^4}{EI}\right]^{0.0938}$
Liu (2000)	$ks = \frac{0.74\,Es}{B(1-\upsilon_s^2)}\left[\frac{Es\,B^4}{EI}\right]^{0.0903}$
Fischer et al. (2000)	$ks = \frac{0.82\,Es}{B(1-\upsilon_s^2)}\left[\frac{Es\,B^4}{EI}\right]^{0.0973}$
Yang (2006)	$ks = \frac{0.95\,Es}{B(1-\upsilon_s^2)}\left[\frac{Es\,B^4}{EI}\right]^{0.108}$
Henry (2007)	$ks = \frac{0.91\,Es}{B(1-\upsilon_s^2)}\left[\frac{Es\,B^4}{EI}\right]^{0.1043}$
Arul et al. (2008)	$ks = \frac{0.87\,Es}{B(1-\upsilon_s^2)}\left[\frac{Es\,B^4}{EI}\right]^{0.1008}$

And in closing, according to various studies the following Table 1 displays empirical relationships of modulus of soil subgrade reaction.

q = stress beneath foundation, δ = settlement related to q, Es = soil elasticity modulus, υ_s = Poisson's ratio, B = foundation width, EI = foundation flexural rigidity, Bp = width of plate test, ksp = subgrade reaction deduced from plate test, Sf = safety factor, qa = allowable bearing capacity, IS and IF = influence factors depend on the shape of footing and m takes 1, 2 and 4 for edges, sides and center of the foundation, respectively.

In this study, it is used the following Das (2012) [17] equation to estimate the elastic settlement S_e.

$$S_e = \Delta\sigma(\alpha B^{`})\frac{1-\upsilon_s{}^2}{E_s} I_s I_f \qquad (1)$$

Where: $\Delta\sigma$ = net applied pressure on the foundation, α = factor that depends on the location on the foundation where settlement is being calculated, $B^{`}$ = B/2 for center of foundation and B for corner of foundation, υ_s = Poisson's ratio of soil, E_s = average modulus of elasticity of the soil under the foundation measured from z = 0 to about z = 5B, I_s = shape factor and I_f = depth factor.

2. METHODOLOGY

Two case studies have been adopted, modelled, and investigated to make the scope more realistic. The models in these studies ought to shed more light on the advantages and disadvantages of the various soil models. The first one is an isolated footing and second is a raft.

The output results of subgrade reaction (ks) based on Plaxis and SAP 2000 analyses are compared with those obtained using empirical equations. Elastic model, Mohr-Coulomb model (MC model) and Hardening soil model (HS model) are used to simulate the soil in Plaxis analysis.

3. CASE STUDIES

3.1 Isolated Footing Analysis

This case study demonstrates the numerical analyses that are used to investigate the behavior of isolated footing (Dimensions = 2m*2m, load = 1200 kN, concrete elastic modulus Ec = 24250 MPa, thickness varies from 0.1 m to 2.0 m (to achieve various rigidities) and Poisson's ratio 0.15) founded on sandy soil (soil depth 20 m, soil elasticity modulus Es = 30 MPa, friction angle = 37^{o} and Poisson's ratio 0.3).

3.1.1 Effect of stress variation

Fig. 1 shows the stress-settlement relationship utilized elastic model, MC model, HS model and Das equation. It is deduced that the elastic model gives smallest settlement however, HS model gives highest values. Almost, at low stress levels (till 200 kPa) elastic model, MC model and Das analyses give very close values. Fig. 2 illustrates the relation between the stress and soil subgrade reaction. Both of elastic and Das analyses give constant relations and the elastic soil model gives the highest value. Obviously, ks decreases by increasing the stress in MC model, however, it decreases slightly in case HS model. It s obvious that when considering the plasticity of soil (as shown in cases of MC and HS models), the ks decreases with increasing stress beneath the footing because a greater stress causes the soil to deform more.

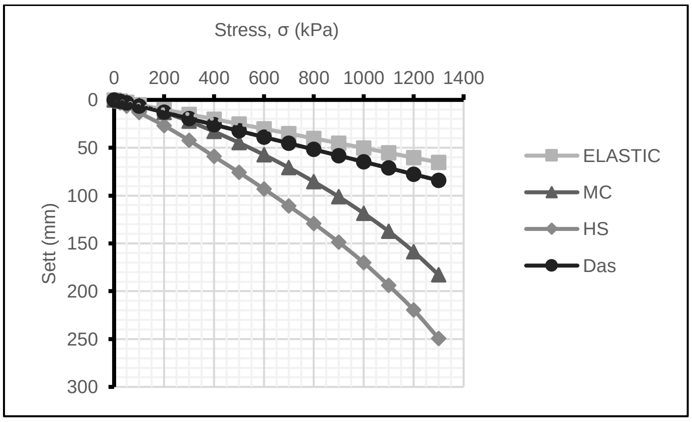

Fig. 1. Stress-settlement relationship

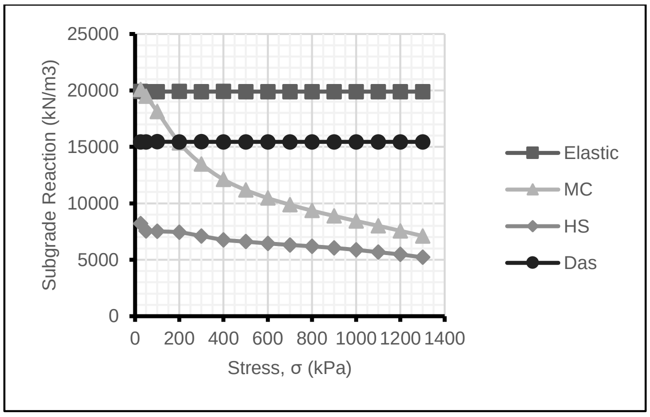

Fig. 2. Effect of stresses beneath footing on subgrade reaction

3.1.2 Effect of footing thickness

As illustrated in Fig. 3, it is obvious that ks decreases as footing thickness increases in cases of empirical equations Boit (1937), Vesic (1961) and Liu (2000). These relationships have almost same behavior and trend. The ks estimated using Boit (1937) gives the highest values for these empirical formulas. The elastic model gives the maximum ks, however, the hardening model gives the minimum and the results of MC model mostly intermediate all analyses. All of elastic, MC and HS models relationships show that ks decreases by increasing the thickness till reaching about 0.3 m (relatively flexible), then it goes to be almost constant.

It is obvious in Fig. 4 that the bending moment increases significantly with the increase of footing thickness till reaching about 0.3 m, and then it relatively goes constant by increasing footing rigidity. It is noted that all curves conventional method (BM at center of footing = 150 kN.m/m, ACI Committee 336 (1988) [5]).

have almost the same trend. The maximum bending moment results are obtained from elastic model; however, the minimum results are obtained from SAP analysis. Moreover, the results obtained from MC and HS models are very close and meet the bending moment obtained using

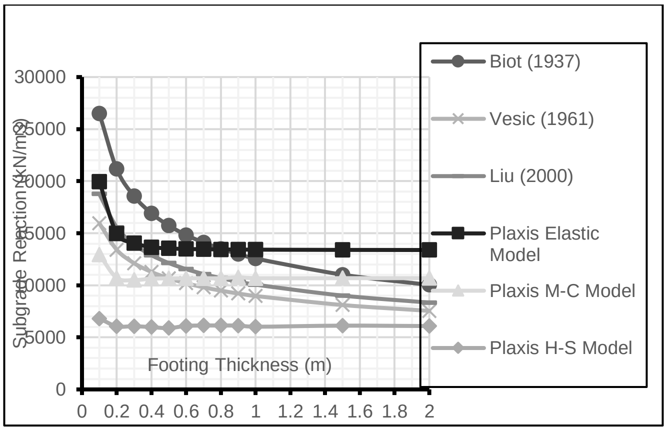

Fig. 3. Effect of footing thickness on subgrade reaction

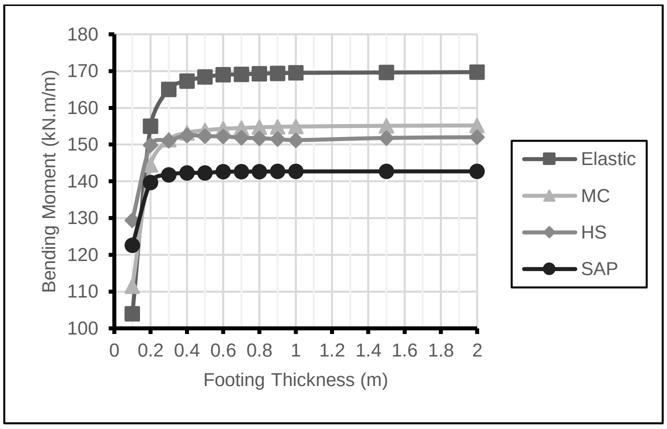

Fig. 4. Effect of footing thickness on bending moment

3.1.3 Effect of footing width

As illustrated in Fig. 5, increase in foundation width causes a decrease in subgrade reaction coefficient for cases of elastic, MC model and HS models. This fact is attributed to an increasing load area leads to the increasing of the settlement. It is noticed that all curves are being nonlinear and the elastic model curve gives the highest values. By increasing the width to be very large the curves mostly meet at same ks value. Also, Fig. 5 shows comparative study between ks of elastic, MC model, HS models and those estimated based on empirical equations. It is clear that all curves have almost the same trend. The ks of elastic, MC and HS Plaxis analyses are closer to Meyerhof-Baike (1965), Liu (2000) and Selvadurai (1984); respectively.

Generally, the relationship between footing width and soil subgrade reaction is inverse. This means that as the width of the footing increases, the soil subgrade reaction decreases. This is because a wider footing distributes the load over a larger area, which reduces the stress on the soil. As a result, the subgrade reaction is lower.

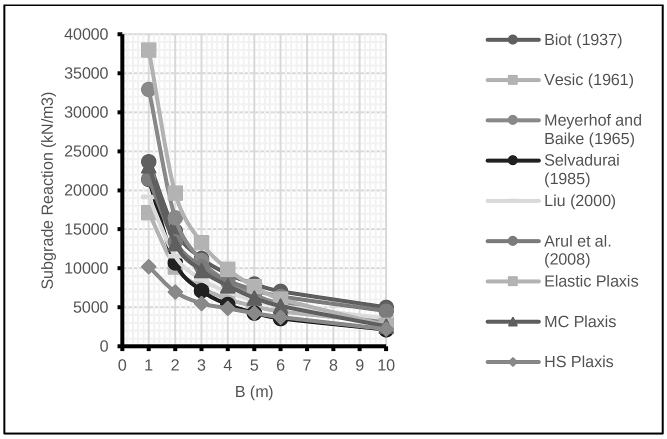

Fig. 5. Effect of footing width on subgrade reaction

3.2 Raft Analysis

Fig. 6 presents the raft dimensions and sections adopted in the numerical analyses (sec 1-1 is for inner columns and sec 2-2 is for outer). (Dimensions = 14m*14m, Pcol 1 = 6533 kN, Pcol 2 = 3267 kN, Pcol 3 = 1633 kN, elastic modulus Ec = 24250 MPa, thickness varies from 0.1 m to 2.0 m and Poisson's ratio 0.15) founded on sandy soil (soil depth 20 m, soil elasticity modulus Es = 30 MPa, friction angle = 37° and Poisson's ratio 0.3). Numerical analyses have been implemented to investigate the effect of raft thickness on the behavior of settlement, subgrade reaction and exhibited bending moment taking into consideration different soil models aforementioned. The subgrade reaction has been deduced at raft center Pt No 1, mid side Pt No 2 and corner Pt No 3.

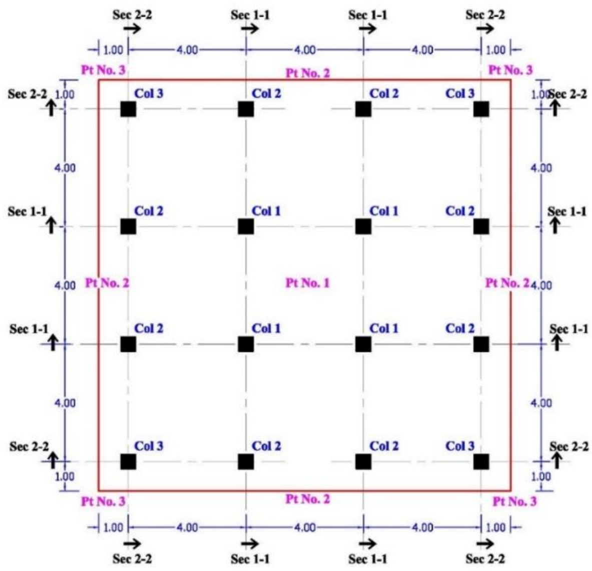

Fig. 6. Raft dimensions

Fig. 7 illustrates that the rate of decreasing in maximum settlement is significant by increasing the raft thickness until reaching traft = 1m; then, the settlement becomes almost constant. This leads to conclude that there is no effect of increasing raft rigidity on the maximum settlement. Also, it is noted that all curves have almost the same trend. Raft elastic settlement has been estimated according to Eq. 1 and it equals 97.51 mm. This value is very close to the settlement curves of MC, HS and SAP (at the raft thickness is higher than 1 m). In general, a thicker raft will have a lower maximum settlement than a thinner raft. As the raft thickness increases, the maximum settlement decreases at a decreasing rate. This is because the additional stiffness of the thicker raft does not have as great an effect on the settlement.

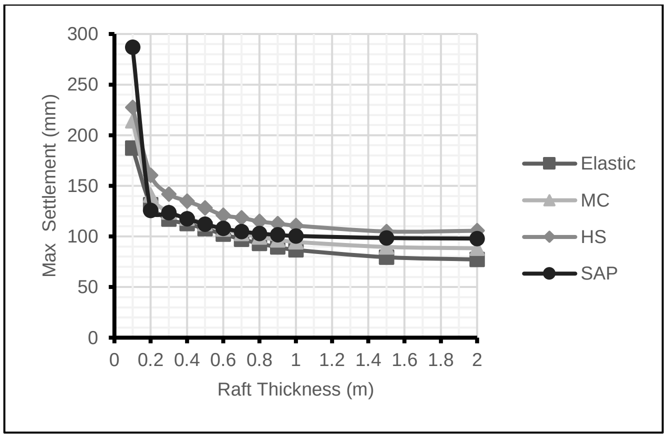

Fig. 7. Effect of raft thickness on maximum settlement

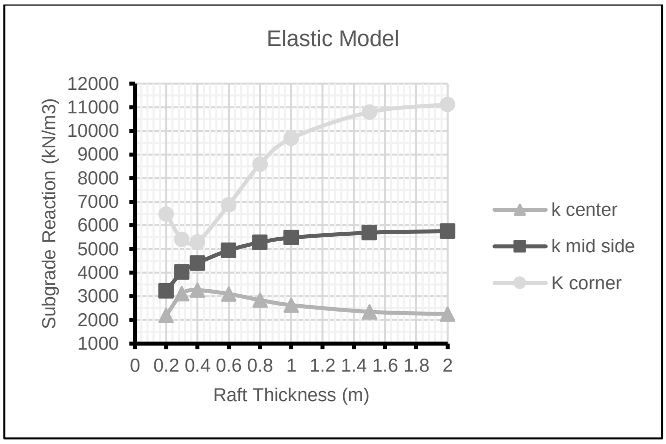

Fig. 8. Effect of raft thickness on subgrade reaction for Elastic Soil Model

Figs. 8, 9 and 10 reveal the effect of raft thickness on the subgrade reaction (ks at center, mid side and corner of raft) adopting elastic, MC and HS soil models; respectively. It is very clear, that the distribution of subgrade reaction is non-uniform through the raft for all soil models. The results indicate that (for all soil models) ks at corner > ks at mid side > ks at center, which supports the earlier results of other researchers. Generally, when considering soil plasticity (both in MC and HS models), the ks is lesser (for corner, mid side and center of raft).

It is obvious that ks curves at corner (for all soil models) have inflected point at traft = 0.4 m (the curves decrease with increasing the thickness till reaching 0.4m, after this value, they increase). However, in case raft center, ks goes to reach peak value (thickness = 0.4 m), then it decreases by increasing the thickness (to become going steady). In addition, for all soil models, the subgrade reaction ks modulus beneath the raft center is roughly equal. Further, ks (mid side raft) increases with increasing the raft thickness till reaching 1 m, then the rate of subgrade reaction goes steady.

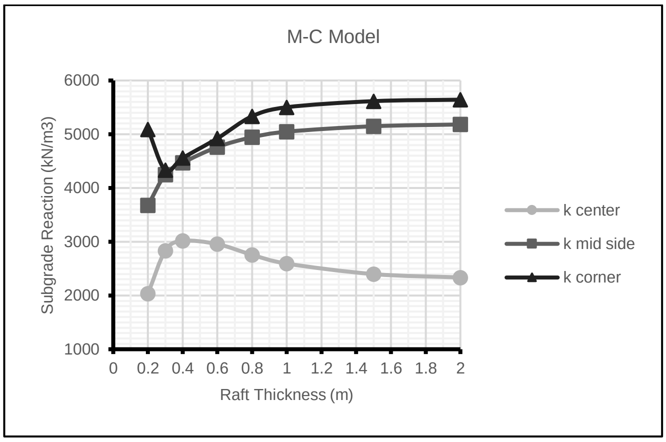

Fig. 9. Effect of raft thickness on subgrade reaction for MC Soil Model

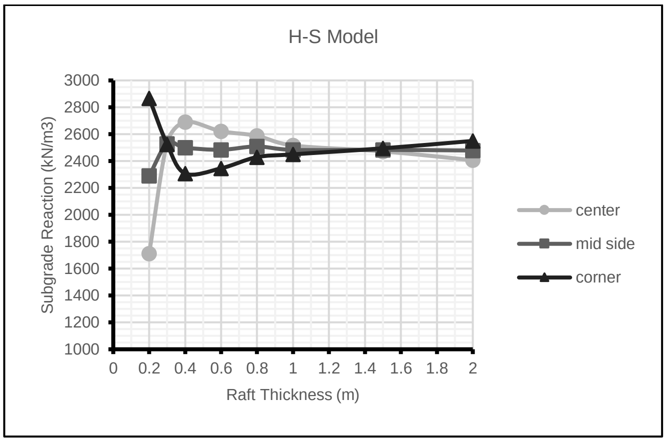

Fig. 10. Effect of raft thickness on subgrade reaction for HS Soil Model

Figs. 11, 12 and 13 illustrate the effect of raft thickness on the subgrade reaction at Col 1, Col 2 and Col 3 (they are close to center, mid side and corner of raft; respectively). Obviously, as shown inf Fig. 11 (for elastic, MC and HS soil models), the curves of ks for Col 1 are very close and matching with the curves of ks estimated upon empirical equations as shown in Fig. 14. Where, by increasing the thickness the ks decreases. Meanwhile, Fig. 12 (for Col 2) shows that ks increases by increasing the thickness till reaching 0.4 m, then the subgrade reaction rate subsequently becomes constant. It is noted that the results of ks at Col 1 and Col 2 are close to the ones estimated using empirical formulas (Fig. 14).

On the other side, Fig. 13 clarifies that the behavior of ks at Col 3 is very similar to the ks at raft corner. Where, ks curve at Col 3 (in all soil models) exhibits an inflected point when traft = 0.4 m, then, after reaching 0.4 m in thickness, the ks increases as the thickness increases.

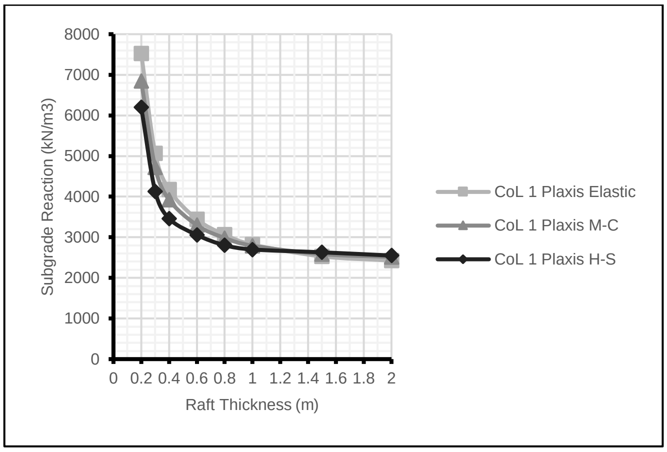

Fig. 11. Effect of raft thickness on subgrade reaction for Col 1

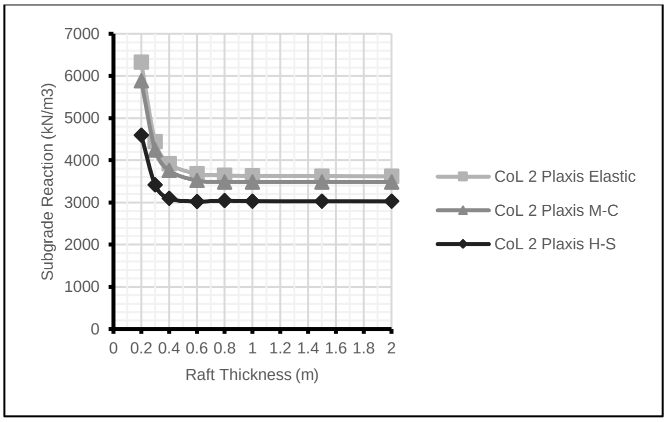

Fig. 12. Effect of raft thickness on subgrade reaction for Col 2

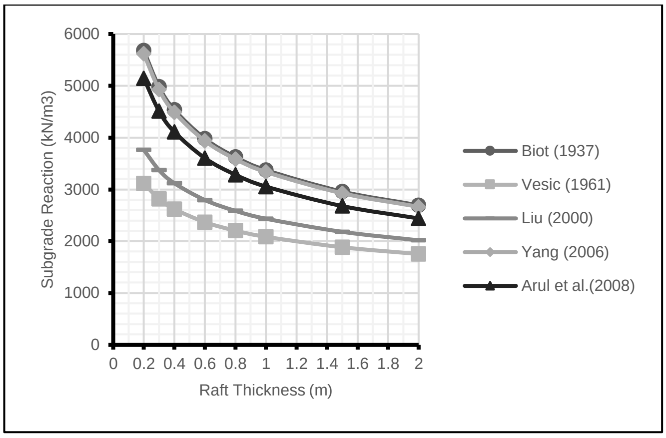

Fig. 13. Effect of raft thickness on subgrade reaction for Col 3

The following Fig. 14 shows the results of the effect of the raft thickness on the modulus of soil subgrade reaction estimated using various empirical formulas such as Biot (1937), Vesic (1961), Liu (2000), Yang (2006) and Arul et al. (2008). It demonstrates that there is an inverse relevance between the thickness and ks. These formulas take into consideration the soil properties such as modulus of elasticity, Poisson's ratio, raft width and raft flexural rigidity. Herein, the case study assumed that all of these mentioned parameters are constant except the raft thickness, which in turns effects on the rigidity

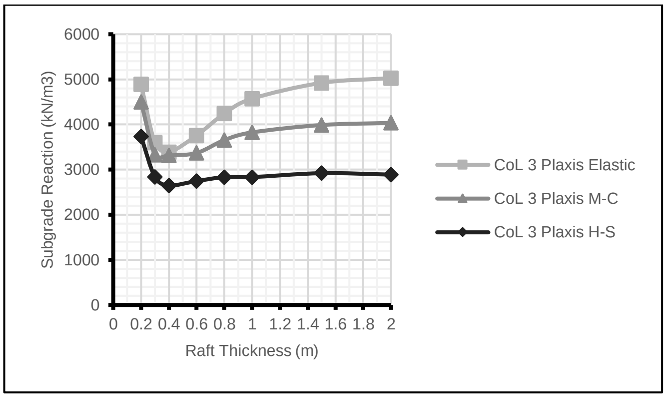

Fig. 14. Effect of raft thickness on subgrade reaction for raft using empirical methods

4. CONCLUSIONS

The main objective of the current study is to outline an in-depth study of soil subgrade reaction. Finite element analysis is utilized in this research (3D Plaxis and SAP

2000 softwares). Furthermore, estimation of soil subgrade reaction based on empirical formulas are adopted. Numerous of parametric analyses are implemented in case of isolated footing and raft foundation. The current study leads to the following conclusions:

4.1 For Isolated Footing

- In stress-settlement analysis, the results of the soil elastic model give the smallest settlement; however, HS model gives the highest values. Almost, at low stress levels (till 200 kPa) the results of elastic model, MC model and Das analyses give very close values.
- Both of soil elastic model and Das analyses give constant stress-subgrade reaction relationships. As well as, the elastic soil model gives the highest value of ks. However, the hardening model analysis gives the minimum results of ks.
- The curve of ks decreases by increasing the stress in case of MC model analysis, meanwhile, it decreases slightly in case HS model. Therefore, by considering the plasticity of soil (as shown in MC and HS models), the ks decreases with increasing stress beneath the footing.
- All of elastic, MC and HS soil models, the thickness-subgrade reaction relationships show that ks decreases by increasing the thickness till reaching about 0.2 m (relatively flexible), then it goes to be almost constant. Generally, A thicker footing typically has a lower soil subgrade reaction than one that is thinner.
- The bending moment increases significantly with the increase of the footing thickness till reaching about 0.3 m (relatively flexible), and then it goes constant by increasing footing rigidity. The maximum bending moment values are obtained from elastic soil model analysis; however, the minimum ones are obtained from SAP analysis. Moreover, the results obtained from MC and HS models are very close and meet the bending moment obtained using conventional method (BM at center of footing = 150 kN.m/m, ACI Committee 336 (1988) [5]).
- By increasing the width of the footing to be very large the curves of subgrade reaction mostly meet at almost same ks value. Generally, the relationship between footing width and soil subgrade reaction is inverse. This is because a wider footing distributes the load over a larger area, which reduces the stress on the soil and gives lower ks.
- It is clear that all curves of the effect of footing width on modulus of subgrade reaction have almost the same trend. The ks of elastic, MC and HS Plaxis analyses are closer to Meyerhof-Baike (1965), Liu (2000) and Selvadurai (1984); respectively.
- Finally, the estimation of the modulus of subgrade reaction based on empirical formulas is being convenient in case of isolated footing. And introduces to geotechnical and structural engineers reliable results matching to the finite element analysis.

4.2 For Raft

- The rate of decreasing in maximum settlement is significant by increasing the raft thickness until reaching traft = 1m; then, the settlement becomes almost constant. The maximum settlement outlined using MC, HS and SAP analyses is very close to the value estimated according to Das equation.
- It is very clear, that the distribution of subgrade reaction is non-uniform through the raft (at center, mid side and corner of raft) for all adopted soil models analyses.
- Generally, the ks (when considering soil plasticity, both in MC and HS models) is lesser than ks estimated using elastic soil analysis (for corner, mid side and center of raft).
- The ks estimated utilizing elastic model is inappropriate at raft corner, and gives overestimated values. Therefore, non-linear models such as MC and HS are suggested to get acceptable values of ks. The HS is the most reasonable model in case of sandy soil.

- It is obvious that ks curves at corner (for all soil models) have inflected point at traft = 0.4 m. The curves decrease with increasing the thickness till reaching 0.4m, after this value, it increases.
- In case of raft center, ks increases till peak value (at thickness = 0.4 m), then it decreases by increasing the thickness (to become going steady). In addition, for all soil models, the subgrade reaction ks modulus beneath the raft center is roughly equal.
- Obviously, for elastic, MC and HS soil models, the curves of ks at Col 1 are very close and matching with the curves estimated upon the empirical equations as shown in Fig. 14. Where, by increasing the thickness ks decreases.
- The analysis results at Col 2 show that ks increases by increasing the thickness till reaching 0.4 m, then the subgrade reaction rate subsequently becomes constant.
- The behavior of thickness-ks at Col 1 and Col 2 is very similar to the ones in case of the isolated footing (for all soil models).
- The behavior of ks at Col 3 is very similar to the ks at raft corner. Where, ks curve at Col 3 exhibits an inflected point when traft = 0.4 m, then, after this value the ks increases as the thickness increases (for all soil models).
- The modulus of soil subgrade reaction estimated using empirical formulas such as Biot (1937), Vesic (1961), Liu (2000), Yang (2006) and Arul et al. (2008) demonstrates that there is an inverse relevance between the thickness and ks without any inflected points.
- Empirical formulas give nearby results of ks to that estimated using finite element analyses in case of Col 1 and Col 2, however, they are inconvenient in case of Col 3, center, mid side and raft corner.
- In the closing, it is recommended to estimate the modulus of subgrade reaction using non-linear finite element analysis (considering the soil plasticity) and do not depend only on the empirical formulas to evaluate ks, which is considered commonly among the majority of engineers. Moreover, the estimation of ks shall to be adopted at different raft regions such as center, mid side, corner and some column locations. This division enhances the geotechnical and structural engineers to get more accurate raft design.

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