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Circular symmetric coupled Microstrip transmission lines

By

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Abstract:

The Circular Symmetric Coupled Microstrip Transmission Lines (CSCMTL) are introduced as a new kind of coupled microstrip transmission lines. A simple method is proposed to determine the capacitance matrix of CSCMTLs. In this method the laplace's equation is solved analytically using the Fourier series expansion. Finally, the properties of CSCMTLs and the validation of the proposed method are studied using an example.

Keywords:
Coupled Microstrip Transmission Lines, Capacitance Matrix, Circular Symmetric, Laplace's equation, Fourier series Expansion

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1. Introduction:

The Circular Symmetric Coupled Microstrip Transmission Lines (CSCMTL), as shown in Fig. 1, could be a new kind of microstrip transmission lines in the future. The geometry of the structure includes N similar circular strip lines on a cylinder with radius b. An internal cylinder with radius a is used as the ground and the space between it and the strips is filled by a dielectric with permittivity er. The CSCMTLs are less sensitive to external electromagnetic interference and also can be designed as large scale and low crosstalk interconnects. To analyze the multiconductor transmission lines such as CSCMTLs, it is necessary to know the per-unit-length capacitance and inductance matrices [1]. The capacitance matrix of lossless structures can be determined using conformal mapping transformations [2]-[3], variational methods [4]-[5], spectral domain techniques [6]-[7], method of moments and green's function [8]-[9] and solving Laplace's equation [10]-[11]. In this paper, the capacitance matrix of CSCMTLs is determined simply by solving Laplace's equation using fourier series method. The solutions are exact but they are expressed by means of infinite linear equations. Using these methods one can determine the voltage and current distribution on the strips, also. Finally, the properties of CSCMTLs and the validation of the proposed method are studied using an example.

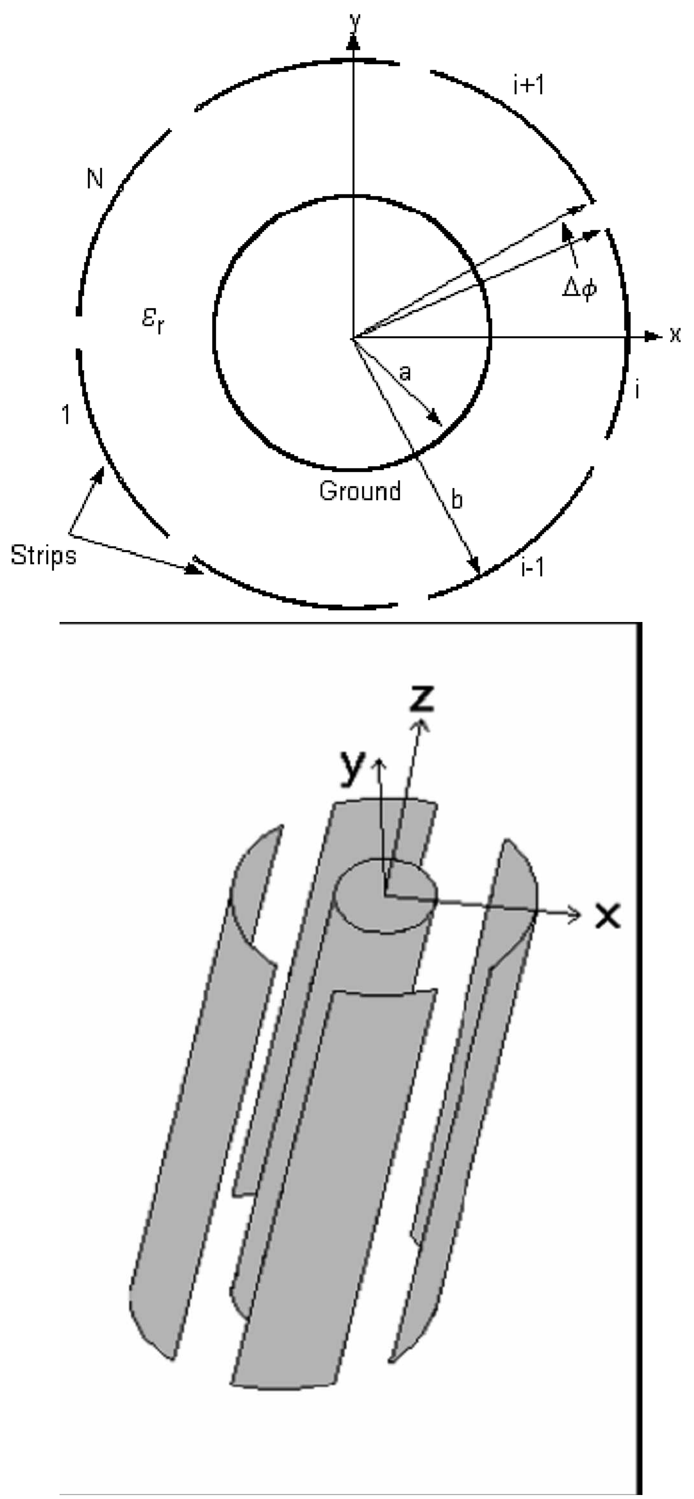

Figure (1): *The cross section and longitudinal view of a typical CSCMTL*

2. Analytic Solution of Laplace's Equation:

In this section the capacitance and inductance matrices of CSCMTLs are determined analytically. We consider three assumption: the principal propagation mode of the structure is quasi-TEM, the strips and ground lines have perfect conductivity and the substrate is lossless. Using the boundary conditions *V* (*a*,j)=*V* (¥,j) =0 , the voltage formula can be determined given by

$$V_1(r,\varphi) = \sum_{n=1}^{\infty} A_n \left(r^n - \frac{a^{2n}}{r^n}\right)\cos(n\varphi) + A_0 \ln\left(\frac{r}{a}\right) \qquad (1)$$

$a \le r \le b$ and

$$V_2(r,\varphi) = \sum_{n=1}^{\infty} \frac{B_n}{r^n}\cos(n\varphi) + B_0 \qquad (2)$$

for *r* ³ *b* . From the continuity of the voltage on the surface r = *b*, we will have:

$$B_n = (b^{2n} - a^{2n}) A_n \qquad (3)$$

and

$$B_0 = \ln(b/a) A_0 \qquad (4)$$

The surface charge on the boundary r = b becomes as follows:

$$\rho_s(\varphi) = \hat{a}_r.(\vec{D}_2 - \vec{D}_1) = \varepsilon_0\left(\varepsilon_r \frac{\partial V_1}{\partial r}\Big|_{r=b} - \frac{\partial V_2}{\partial r}\Big|_{r=b}\right) \qquad (5)$$

$$= \sum_{n=1}^{\infty} G_n A_n \cos(n\varphi) + G_0 A_0$$

where

$$G_n = \frac{\varepsilon_0}{b} n \frac{b^{2n} - a^{2n}}{b^n}\left(\frac{b^{2n} + a^{2n}}{b^{2n} - a^{2n}}\varepsilon_r + 1\right) \qquad (6)$$

$$G_0 = \frac{\varepsilon_0}{b}\varepsilon_r \qquad (7)$$

We assume that the voltages of the *i*-th strip and the other strips be *V*0 and zero, respectively. Also, we know that the surface charge is zero out of the strips. Therefore, to find the unknown coefficients *A*n, one can use the following relation

$$
\begin{aligned}
&\int_{\text{On the Strips}} V(b,\varphi)\cos(m\varphi)\,d\varphi + \int_{\text{Out of Strips}} \rho_s(\varphi)\cos(m\varphi)\,d\varphi \\
&= \int_{-\frac{\pi}{N}+\frac{\Delta\varphi}{2}}^{\frac{\pi}{N}-\frac{\Delta\varphi}{2}} V_0 \cos(m\varphi)\,d\varphi \qquad m = 0,1,2,\cdots
\end{aligned} \tag{8}
$$

Substituting (1) and (5) in (8), gives us:

$$
\begin{aligned}
&\sum_{n=1}^{\infty}\Big[A_n \frac{(b^{2n}-a^{2n})}{b^n} \sum_{k=0}^{N-1}\Big(\int_{(2k-1)\frac{\pi}{N}+\frac{\Delta\varphi}{2}}^{(2k+1)\frac{\pi}{N}-\frac{\Delta\varphi}{2}} \cos(m\varphi)\cos(n\varphi)\,d\varphi\Big)\Big] \\
&+ A_0 \ln(b/a) \sum_{k=0}^{N-1}\Big(\int_{(2k-1)\frac{\pi}{N}+\frac{\Delta\varphi}{2}}^{(2k+1)\frac{\pi}{N}-\frac{\Delta\varphi}{2}} \cos(m\varphi)\,d\varphi\Big) \\
&+ \sum_{n=1}^{\infty}\Big[A_n G_n \sum_{k=0}^{N-1}\Big(\int_{(2k+1)\frac{\pi}{N}-\frac{\Delta\varphi}{2}}^{(2k+1)\frac{\pi}{N}+\frac{\Delta\varphi}{2}} \cos(m\varphi)\cos(n\varphi)\,d\varphi\Big)\Big] \\
&+ A_0 G_0 \sum_{k=0}^{N-1}\Big(\int_{(2k+1)\frac{\pi}{N}-\frac{\Delta\varphi}{2}}^{(2k+1)\frac{\pi}{N}+\frac{\Delta\varphi}{2}} \cos(m\varphi)\,d\varphi\Big) = V_0 \int_{-\frac{\pi}{N}+\frac{\Delta\varphi}{2}}^{\frac{\pi}{N}-\frac{\Delta\varphi}{2}} \cos(m\varphi)\,d\varphi
\end{aligned} \tag{9}
$$

for m = 0, 1,. 2, … .
It is provable that

$$\sum_{k=0}^{N-1}\left(\int_{(2k+1)\frac{\pi}{N}-\frac{\Delta\varphi}{2}}^{(2k+1)\frac{\pi}{N}+\frac{\Delta\varphi}{2}}\cos(l\varphi)d\varphi\right) = N\Delta\varphi \,\mathrm{sinc}(l\frac{\Delta\varphi}{2\pi})(-1)^{l/N}\,\delta\big(\mathrm{rem}(l/N)\big) \tag{10}$$

and

$$\sum_{k=0}^{N-1}\left(\int_{(2k-1)\frac{\pi}{N}+\frac{\Delta\varphi}{2}}^{(2k+1)\frac{\pi}{N}-\frac{\Delta\varphi}{2}}\cos(l\varphi)d\varphi\right) = \begin{cases} -N\Delta\varphi \,\mathrm{sinc}(l\frac{\Delta\varphi}{2\pi})(-1)^{l/N}\,\delta\big(\mathrm{rem}(l/N)\big); & l\neq 0; \\ 2\pi - N\Delta\varphi; & l=0 \end{cases} \tag{11}$$

in which "rem(.)" represents the remainder of its argument.

Defining $r = N\Delta\varphi/(2\pi)\ h$ as the ratio of total separations between the strips to the peripheral of the circle r = b and after some manipulations using (9)-(11), two following relations are obtained for m = 1,. 2, 3, …

$$A_0\big(r_h G_0 + (1-r_h)\ln(b/a)\big) + r_h\sum_{n=1}^{\infty}\left(A_n(G_n - \frac{b^{2n}-a^{2n}}{b^n})\mathrm{sinc}(\frac{n}{N}r_h)(-1)^{n/N}\,\delta\big(\mathrm{rem}(n/N)\big)\right) = \frac{1}{N}(1-r_h)V_0 \tag{12}$$

and

$$
\begin{aligned}
&2r_h A_0 (G_0 - \ln(b/a))\operatorname{sinc}(\frac{m}{N} r_h)(-1)^{m/N}\,\delta(\mathrm{rem}(m/N)) \\
&+ A_m \frac{b^{2m} - a^{2m}}{b^m} \\
&+ r_h \sum_{n=1}^{\infty} \left[\begin{array}{l} A_n (G_n - \dfrac{b^{2n} - a^{2n}}{b^n}) \\ \times \left(\begin{array}{l} \operatorname{sinc}(\dfrac{m+n}{N} r_h)(-1)^{(m+n)/N}\,\delta\left(\mathrm{rem}(\dfrac{m+n}{N})\right) \\ + \operatorname{sinc}(\dfrac{m-n}{N} r_h)(-1)^{(m-n)/N}\,\delta\left(\mathrm{rem}(\dfrac{m-n}{N})\right) \end{array} \right) \end{array} \right] \\
&= \frac{2}{N}(1-r_h)\operatorname{sinc}(\frac{m}{N}(1-r_h))V_0
\end{aligned}
\qquad (13)
$$

Now, we can determine the unknown coefficients *A*n, assuming that *n*,*m* £ *M* . With this assumption, the relations (12) and (13) can be written in a matrix form as follows:

$$
\mathbf{B}_{(M+1)\times(M+1)} \mathbf{A}_{(M+1)\times 1} = \mathbf{V}_{(M+1)\times 1} \qquad (14)
$$

where $\mathbf{A} = [A_0, A_1, A_2, \ldots A_M]^T$ and the elements of $\mathbf{B}$

and the elements of B and V are Determined from (12)-(13). It is evident that as the number of harmonies, *M*, increases the unknown coefficients are obtained with lesser error. According to arguments of the sinc functions in equations (12) and (13), *M* must be several times of *N*/*rh*. In fact this is because as *N* increase or *rh* decreases, the slop of boundary functions and therefore the number of necessary harmonics increases.

3. Capacitance and Inductance Matrices:

After finding the unknown coefficients of voltage, we can now calculate the capacitance and inductance matrices of the circular microstrip lines. Each element of the matrix C, e.g. C(*j*,*i*), can be determined from total surface charge of the *j*-th strip, when the *i*-th strip is hold on the voltage *V*0 and the other strips are hold to zero voltage. Thus using (5), we have

$$\mathbf{C}(j,i)=\frac{1}{V_0}\int_{(j-i-0.5)\frac{2\pi}{N}+\frac{\Delta\varphi}{2}}^{(j-i+0.5)\frac{2\pi}{N}-\frac{\Delta\varphi}{2}}\rho_s(\varphi)b\,d\varphi=\frac{2\pi b}{V_0 N}(1-r_h)$$
$$\times\{G_0A_0+\sum_{n=1}^{\infty}[G_nA_n\cos(2\pi(j-i)\frac{n}{N})\mathrm{sinc}(\frac{n}{N}(1-r_h))]\} \quad (15)$$

for i and j = 1, 2, …, N. Also the inductance matrix L is obtained from [1]

$$\mathbf{L}=\frac{1}{c^2}\mathbf{C}_0^{-1} \quad (16)$$

where c is the velocity of the light and C0 is capacitance matrix assuming that e*r* = 1.

4. Example and Results:

In this section we calculate the capacitance and inductance matrices of a CSCMTL using the obtained relations. Consider a CSCMTL with the parameters N = 4, a = 0.1 mm, b = 1.0 mm, e r = 2.54, rh = 0.5 and V0 = 1. Figs. 2-4 show the coefficients An, and the functions V (b,j) and r (j) s , respectively. We see the satisfying of the boundary conditions on four 45o strips and between them. So one may satisfy about the validation of the obtained relations. Also, the capacitance and inductance matrices of the structure have been calculated as follows:

$$\mathbf{C}=\begin{bmatrix}41.57 & -11.99 & -3.07 & -11.99\\ -11.99 & 41.57 & -11.99 & -3.07\\ -3.07 & -11.99 & 41.57 & -11.99\\ -11.99 & -3.07 & -11.99 & 41.57\end{bmatrix}[pF/m]$$

$$\mathbf{L}=\begin{bmatrix}777.7 & 399.3 & 335.8 & 399.3\\ 399.3 & 777.7 & 399.3 & 335.8\\ 335.8 & 399.3 & 777.7 & 399.3\\ 399.3 & 335.8 & 399.3 & 777.7\end{bmatrix}[nH/m]$$

It is seen that these matrices are Reciprocal, Toeplitz and Cyclic. Figs. 5-6 show the effect of the parameters *a* and *rh* in the capacitance coefficients. We see that as the radius of the ground, *a*, increases, the self capacitances are increased and the induction capacitances are decreased and consequently the coupling or the crosstalks between the lines are decreased. Also as expected, as the ratio *rh* increases, all capacitance coefficients are decreased.

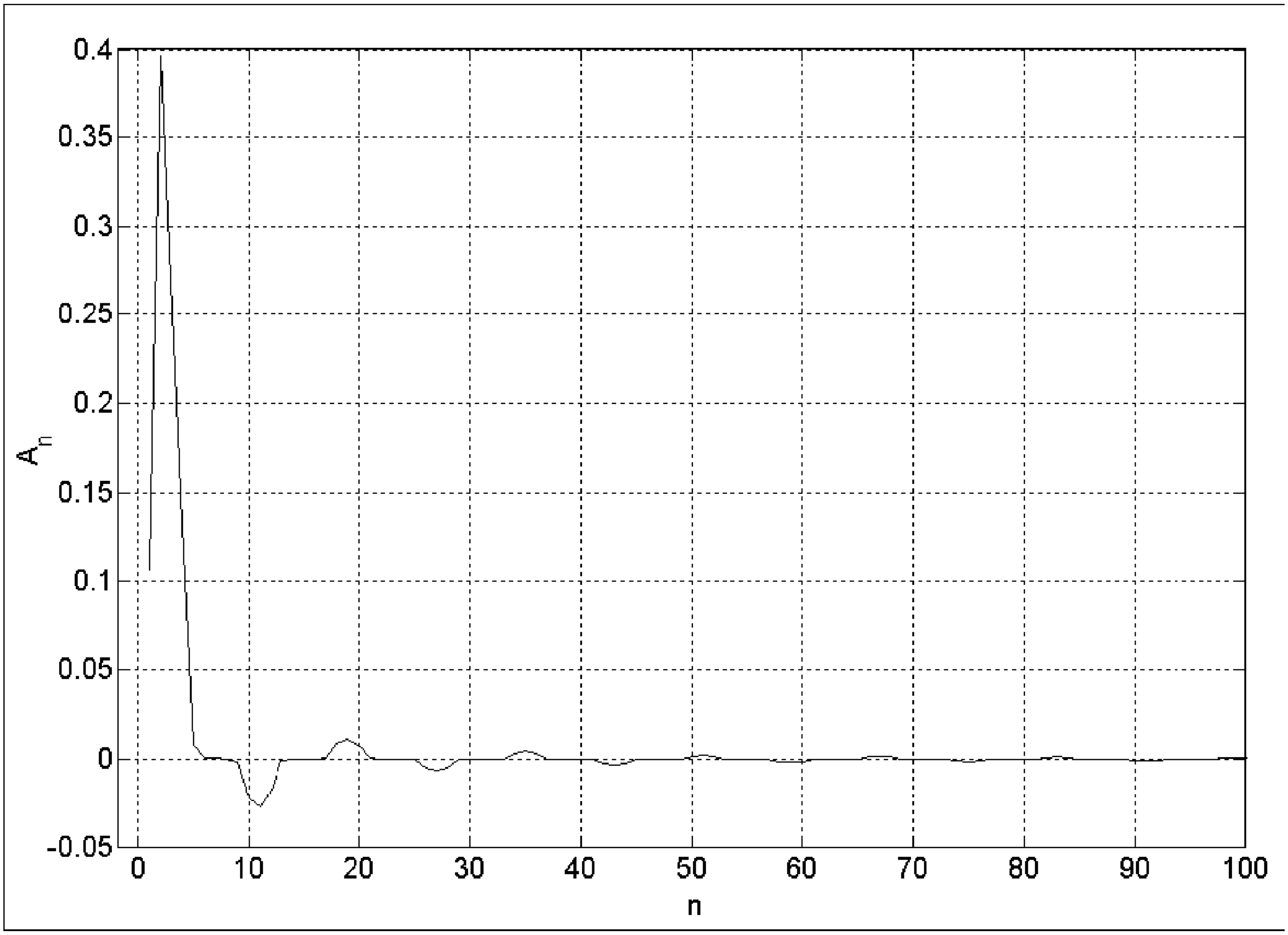

Figure (2): *The unknown coefficients An*

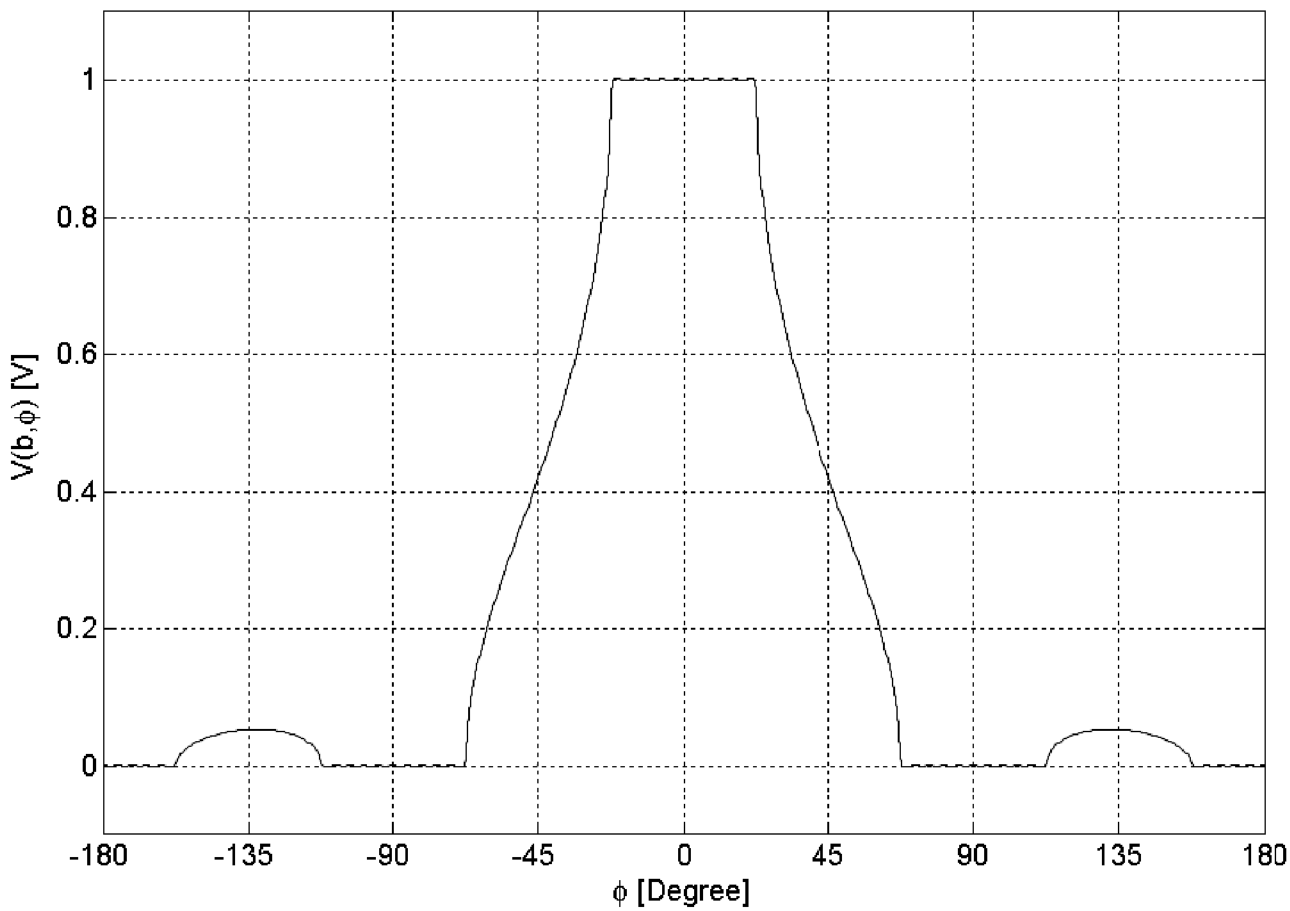

Figure (3): *The voltage of the boundary r = b*

5. Conclusions:

The Circular Symmetric Coupled Microstrip Transmission Lines (CSCMTL) were introduced as a new kind of coupled microstrip transmission lines. A simple method was proposed to determine the capacitance matrix of CSCMTLs. In this method the laplace's equation is solved analytically using the Fourier series expansion. The properties of CSCMTLs and the validation of the proposed method were studied using an example. The CSCMTLs are less sensitive to external electromagnetic interference and also can be designed as large scale and low crosstalk interconnects. It was seen that as the radius of the ground increases, the self capacitances are increased and the induction capacitances are decreased and consequently the coupling or the crosstalks between the lines are decreased.

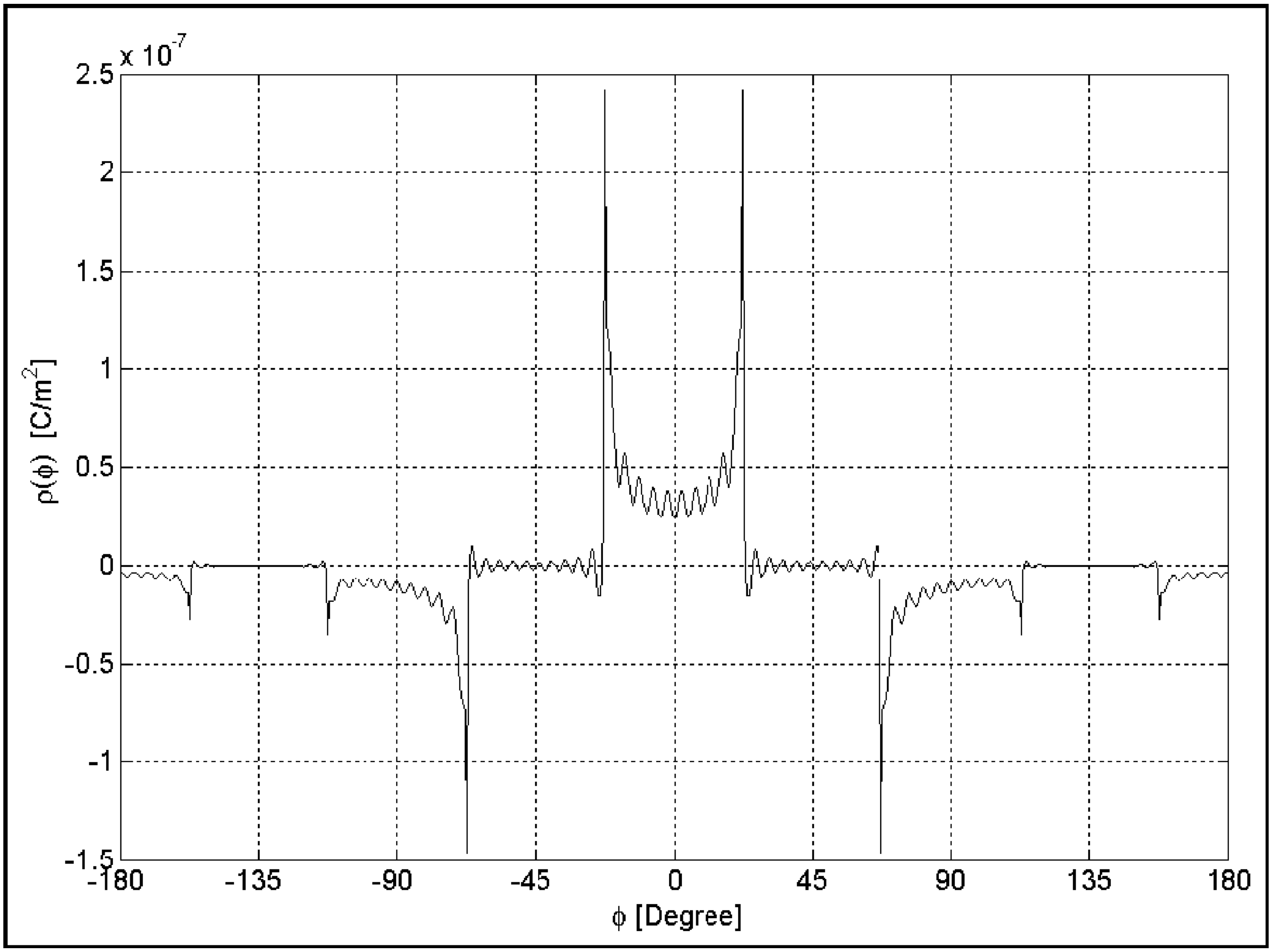

Figure (4): *The surface charge on the boundary r = b*

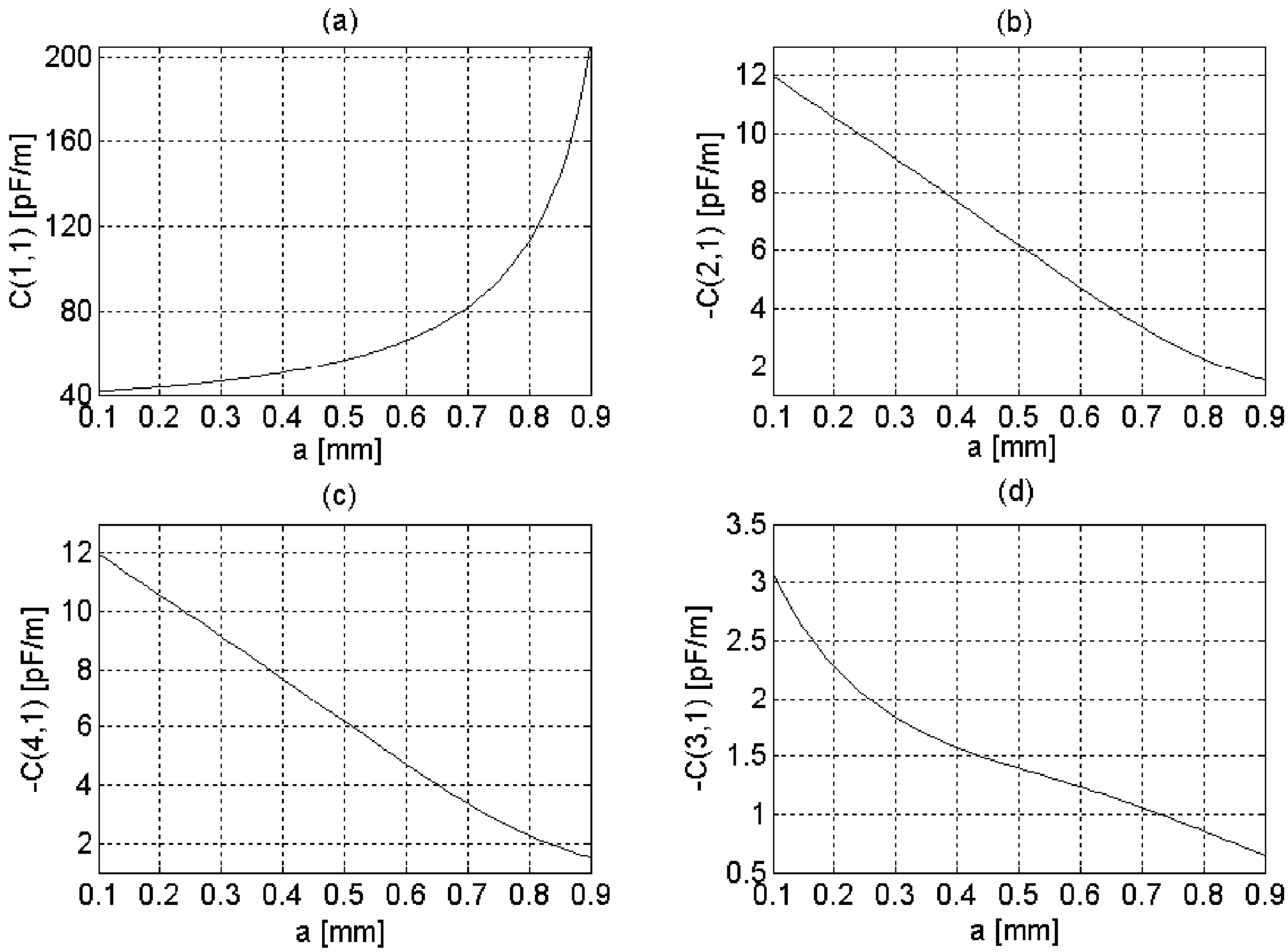

Figure (5): *The capacitance coefficients versus a for b = 1.0 mm and rh = 0.5*

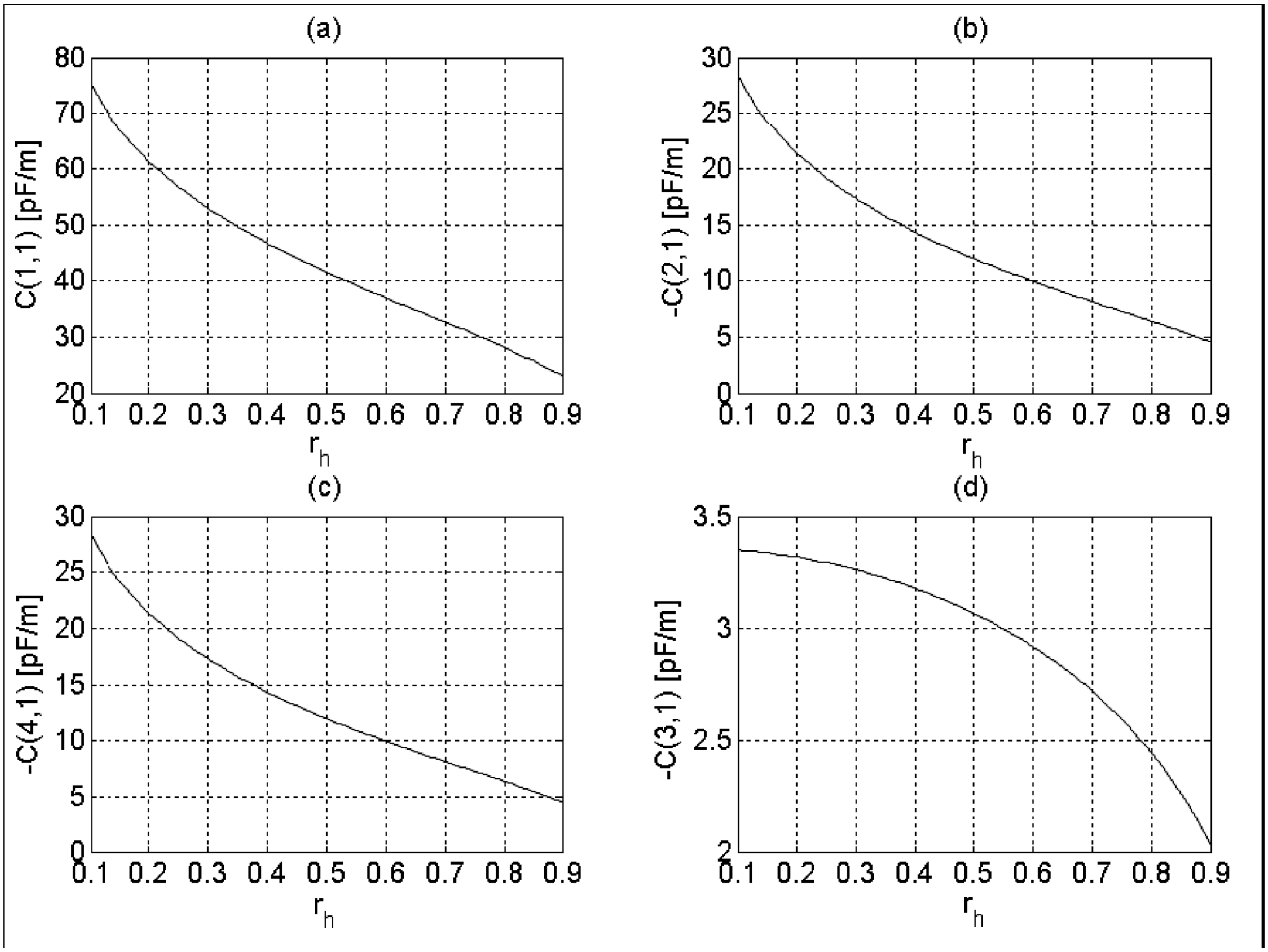

Figure (6): *The capacitance coefficients versus rh for a = 0.1 mm and b = 1.0 mm*

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