**Military Technical College**
**Kobry El-Kobbah,**
**Cairo, Egypt**

**6th International Conference**
**on Electrical Engineering**
**ICEENG 2008**

# Interaction of polarization mode dispersion and cross-phase modulation in wavelength division multiplexed optical fiber systems

*By*

M. S. Islam[*] S. P. Majumder[**] R. H. Akanda[*]

## *Abstract:*

Cross-phase modulation (XPM) changes the state-of-polarization (SOP) of the channels through nonlinear polarization rotation and induces nonlinear time dependent phase shift for polarization components that leads to amplitude modulation of the propagating waves in a wavelength division multiplexing (WDM) system. Due to the presence of birefringence, the angle between the SOP changes randomly and as a result polarization mode dispersion (PMD) causes XPM modulation amplitude fluctuation random in the perturbed channel. In this paper we analytically determine the probability density function of the random angle between the SOP of pump and probe, and evaluate the impact of polarization mode dispersion on XPM in terms of BER, channel spacing etc. for a two channel IM-DD WDM system at 10 Gb/s. We also show the dependence of SOP variance on PMD parameter and channel spacing.

## *Keywords:*

Image restoration, remote sensing and image blur models

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[*] Institute of Information and Communication Technology
[**] Department of Electrical and Electronic Engineering Bangladesh University of Engineering and Technology (BUET) Dhaka-1000, Bangladesh

## *1. Introduction:*

XPM is a nonlinear phenomena occurring in optical fibers when two or more optical fields are transmitted through a fiber simultaneously. XPM has an important impact on the performance of high-speed WDM in optical fiber communication systems [1]. In the last few years, many research works have been carried out on the interaction among fiber nonlinearity, polarization variation and polarization mode dispersion (PMD) [2-8]. Physically, PMD has its origin in the birefringence that is present in any optical fiber. Just like signal distortion due to chromatic dispersion and nonlinearity accumulate along the length of the communication link, so does the polarization and PMD. Polarization fluctuations and PMD has become increasingly important as the per-channel data rates have increased and now arguably the most important of the polarization effects. Q. Lin *et at* [4], developed a vector theory of XPM in optical fiber and used to investigate the effect of PMD on XPM crosstalk in a WDM system in terms of amplitude of probe fluctuation induced by a co-propagating pump channel and the results show that PMD reduces the difference in the average crosstalk level between cases of copolarized and orthogonally polarized channels. G. Zhang *et al* [5], experimentally reported that SPM can suppress PMD penalty and XPM-induced polarization scattering in dense WDM transmission systems can reduce the PMD impairment. R. Khosravani *et at* [6], numerically and experimentally showed that nonlinear polarization rotation can alter the polarization states of the bits that varies from one bit to the next in a way that is difficult to predict and in such cases, conventional PMD compensation becomes impossible.

In this paper we analytically determined the probability density function (pdf) of the random angle $\theta(z)$ between the pump and probe's state of polarization (SOP) fluctuations that produce intensity dependent pulse distortion on a propagating signal. Using the pdf we find impact of PMD on cross-phase modulation is in terms of bit error arte (BER**)** as a function of pump power, channel spacing.

## *2. Theoretical analysis:*

To describe the effect of PMD on XPM, we consider the pump-probe configuration and assume that the probe act as channel 2 and is in the form of a low-power continuous wave (CW), while the pump act as channel 1 and imposes the XPM-induced phase shift on channel 2. The probe is also assumed to be weak enough that the XPM, SPM and intrachannel PMD induced by it can be neglected. Although these effects broadens pulse in each channel, they barely affect the interchannel XPM interaction because the channel spacing typically is much larger than channel bandwidth and the evolution of the SOP of the channels is mainly governed by the birefringence. The XPM induces not only a time dependent phase shift in the probe channel, but also a nonlinear polarization rotation (NPR) of the probe channel. However, PMD changes the relative orientation between the pump and probe stokes vectors at a rate dictated by the magnitude of channel spacing and relative birefringence. It is noticed that the effectiveness of XPM depends not only on the group velocity mismatch $\delta\beta_1$, but also on the relative orientation of the pump and probe SOPs.

The pump field modulates the CW probe field induced by the combination of XPM and PMD. The crosstalk is measured by the XPM induced modulation amplitude of the probe signal. The normalized modulation amplitude $\tilde{\delta}_X(\omega)$ of the probe field is given by [4],

$$\tilde{\delta}_X(\omega) = -\int_0^L \varepsilon_2(z)\tilde{\boldsymbol{P}}_{\boldsymbol{0}}(z,\omega)\left[3+cos\theta(z)\right]\ \sin\left[\frac{1}{2}\int_z^L \omega^2\beta_2(z_1)dz_1\right]dz \qquad (1)$$

Where $\theta(z)=\hat{P}\,.\,\hat{S}(z)$ is the random angle between the pump and probe SOPs, $\tilde{\boldsymbol{P}}_{\boldsymbol{0}}(z,\omega)$ is the Fourier spectrum of the pump power at a distance $z$ inside the fiber, $\varepsilon_2 = 8/9\gamma_2$ is the effective nonlinear parameter and $\beta_2$ is the GVD coefficient of the pump. The effects of PMD enter in this equation through the angle $\theta(z)$. More precisely, PMD randomly changes the angle between $\hat{P}$ and $\hat{S}$ along the fiber and thus makes $\tilde{\delta}_X(\omega)$ a random quantity.

The average value of the modulation amplitude is obtained by averaging over random birefringence fluctuations, responsible for PMD, along the fiber length. We may consider $\left\langle \cos[\theta(z)]\right\rangle = cos\theta_0\ \exp(-\eta z)$, where $\eta = 1/L_d = D_p^2\Omega^2/3$, $\Omega = \omega_2-\omega_1$ is the channel spacing, $\theta_0$ is the value of $\theta$ at $z=0$, $\theta_0$ is the relative angle between the pump and probe SOPs at the input end of the fiber. Here we also assume that the effects of dispersion and nonlinearities do not significantly change the pulse shape of the pump channel along the fiber. Then, the average modulation amplitude, $\left\langle\tilde{\delta}_X(\omega)\right\rangle_P$ and average value of crosstalk variance, $\sigma_m^2$ at the end of a link of length ($L$) is given by [4],

$$\left\langle\tilde{\delta}_X(\omega)\right\rangle_P = \tilde{\boldsymbol{P}}_{\boldsymbol{0}}(0,\omega)\left\{3\left[\boldsymbol{I}(0,\omega)+\boldsymbol{I}^*(0,-\omega)\right]+\cos\theta_0\left[\boldsymbol{I}(\eta,\omega)+\boldsymbol{I}^*(\eta,-\omega)\right]\right\} \qquad (2)$$

$$\sigma_m^2 = \frac{B_0}{4\pi}\int_{-\infty}^{\infty} d\omega\left|\tilde{X}(\omega)\right|^2\left|\,3\left[\boldsymbol{I}(0,\omega)+\boldsymbol{I}^*(0,-\omega)\right]+\cos\theta_0\left[\boldsymbol{I}(\eta,\omega)+\boldsymbol{I}^*(\eta,-\omega)\right]\right|^2 \qquad (3)$$

We may assume that the state of polarization (SOP) of pump $\hat{P}$ remain constant and polarization angle between pump and probe $\theta(z)$ drive by the white noise process and can be written as,

$$\frac{d\theta}{dz} = \hat{S}(z);\ \text{Where, } \left\langle\hat{S}(z)\right\rangle = 0 \ \text{ and } \left\langle\hat{S}(z_1)\hat{S}(z_2)\right\rangle = \eta^2\delta(z_2-z_1) \qquad (4)$$

From equation (4) a diffusion equation for the probability distribution of $\theta(z)$ or simply $\theta$ can be obtained [9],

$$\frac{\partial p}{\partial z} = \frac{1}{2}\eta^2 \frac{\partial^2 p}{\partial \theta^2} \qquad (5)$$

Where $p(\theta, z)$ is $2\pi$ periodic in $\theta$ and $p(0,z) = \delta(\theta - \theta_0)$. By solving the equation (5) we get the probability density function $\theta$ and expressed as

$$p(\theta, z) = \frac{1}{2\pi} + \frac{1}{\pi}\sum_{m=1}^{\infty} \exp(-\frac{1}{2}\eta^2 m^2 z)\cos m(\theta - \theta_0) \qquad (6)$$

The source channel noise ratio (SCNR) can be written as,

$$SCNR = \frac{(R_d S_0)^2}{\sigma_m^2 + \sigma_n^2} \qquad (7)$$

Where $\sigma_m^2$ crosstalk variance due to PMD, $\sigma_n^2$ noise variance, $I_s = R_d S_0$, $R_d$ is the responsivity and $S_0$ is the probe power. Thus,

$$SCNR(for\ angle\quad \theta_0) = \frac{{I_s}^2}{\sigma_m^2(\theta_0) + \sigma_n^2} \qquad (8)$$

For a given value of $\theta_0$, the conditional BER can be expressed as

$$BER(\theta_0) = 0.5\, erfc\left(\frac{\sqrt{SCNR(for\ angle\, \theta_0)}}{\sqrt{2}}\right) \qquad (9)$$

Because of environmental change (*i.e.,* temperature, pressure, vibration, stress, twisting etc.), PMD fluctuates randomly. Generally, the impact of PMD fluctuations as well as the variation of angle, $\theta(z)$ between the pump and probe SOPs typically occurs on a time scale of milliseconds. Thus the average BER is given by,

$$BER = \int_{-\pi}^{\pi} BER(\theta_0)\; p(\theta, z) d\theta \qquad (10)$$

## 3. Results and discussion:

Following the analytical approach, the pdf of the random polarization angle between pump and probe is evaluated at various length of single mode fiber and shown in Fig.1. From the figure, it is found that for particular channel spacing the pdf is a delta function at $z = 0$ and becomes flat as the link length increases. Fig.2 shows the variance of pump and probe polarization SOPs versus fiber link length for different PMD co-efficient. It is noticed that as the PMD co-efficient increases the variance of SOPs also increases linearly. The average BER versus pump power is plotted in Fig.3. It is observed that interaction of PMD and XPM increases as the channel spacing decreases for a specific pump power and system suffers significant amount of BER. This significant dependence of XPM BER on channel spacing comes from the fact that the PMD diffusion length is inversely proportional to the square of the channel spacing. However, at increased pump power the BER increases and when pump power is about 16 dBm the BER curves makes a floor at $10^{-1}$ irrespective of channel spacing. The plots of average BER versus probe power is shown in Fig.4. It is seen that at high pump and probe power the BER performance of the fiber link deteriorates sharply. This happens due to the strong interaction of PMD and XPM on each other channel.

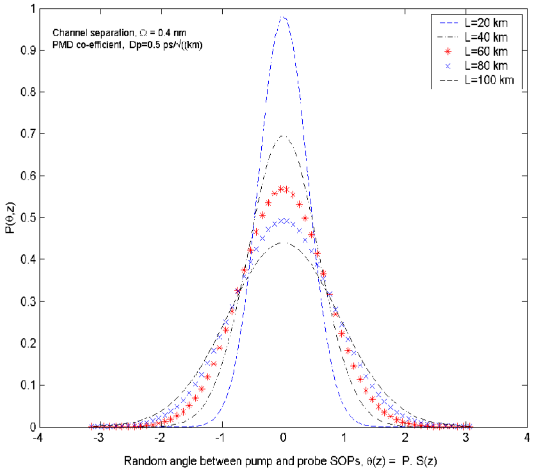

*Fig.1: Plots of pdf of* $\theta(z)$ *at different length of fiber link*

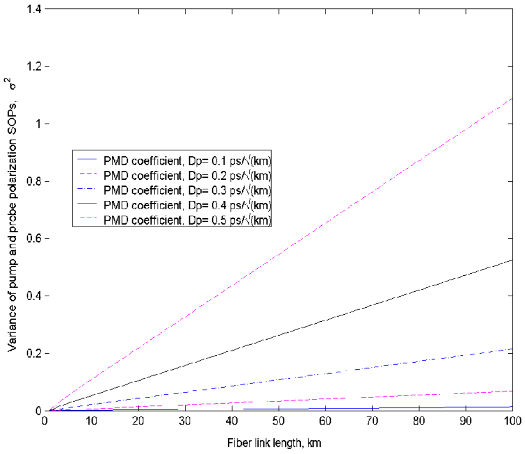

*Fig.2: P* *ber*
*link length for different PMD parameters.*

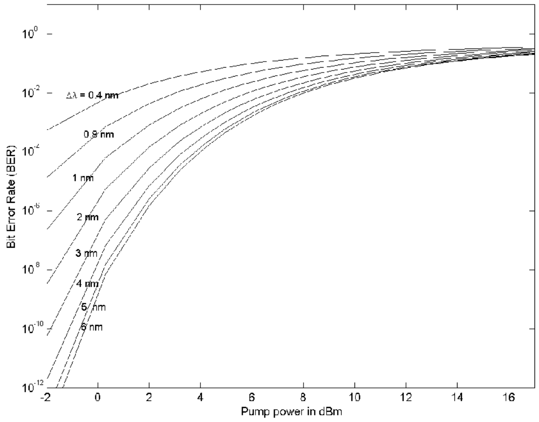

*Fig. 3: Plots of average BER vs pump power for different channel spacing*

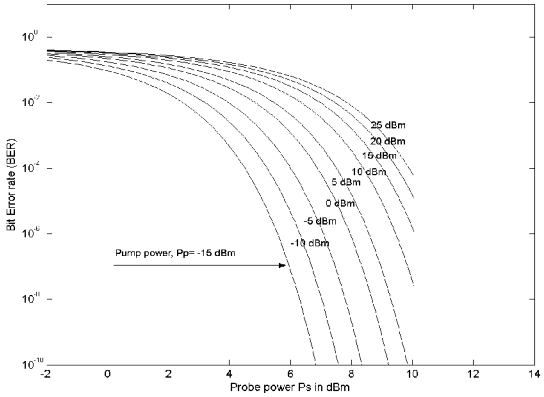

*Fig.4: Plots of average BER vs probe power for different pump power*

## 4. Conclusion:

An analytical approach is used to find the probability distribution of the random SOP angle between the pump and probe which cases the XPM induced modulation amplitude to be random. The strength of the XPM effect is strongly influenced by the evolution of light-polarization of the WDM carriers. Our results show that the XPM induced crosstalk becomes polarization independent when channel spacing is large or when the fiber has a relatively large value of PMD co-efficient. Thus at relatively high pump the combination of nonlinearity and PMD can limit the performance of a WDM fiber link significantly.

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