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Hot Quantum Electron, Ion, Nonmagnetized Dusty Plasma: Ion Sound Waves and Dispersion Relation

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The dispersion equation for isothermal ionic sound waves is derived and analyzed for collisionless nonmagnetized dusty plasma consisting of quantum gases of electrons, ions, and dust at hot temperatures, and an exact expression is obtained for the linear velocity of ionic sound. Analysis has been carried out by the method of the Bernoulli pseudopotential. The quantum effects include, Fermi degenerate pressure, and exchange correlation potential. The ranges of phase velocities of periodic ionic sound waves and soliton velocities are determined. It is shown that in the plasma under investigation, the soliton velocity cannot be lower than the linear velocity of ionic sound. A graph should plotted between the frequency and wave number, three region should obtained, solutions in the form of periodic ionic sound waves should be sought precisely in the range of velocities. The profiles of physical quantities in a periodic wave and in a soliton are constructed, as well as the dependences of the velocity of sound and the critical velocity on the ionic concentration in the plasma. It is shown that these velocities increase with the ion concentration and the term of dusty plasma as well as term of hot quantum electron and ion may effect on the dispersion relation. The application of this work has been pointed out for laboratory as well as for space dusty plasmas.

Keywords: *Extraction; REEs; Citric acid; D2EHPA; Separation.*

Introduction

The number of theoretical publications devoted to various collective processes in quantum effects at electron–ion–dust plasmas (henceforth referred to as *e*–*i*–d plasma has increased enormously in recent years. However, due to contamination, dust impurities may exist in quantum plasmas like the microelectronic devices or metallic nano structures. This interest is primarily due to the fact that such plasma is typical rather than exceptional in astrophysical conditions. For example, it is assumed that such plasma exist in the inner regions of accretion disks near black holes [1], in magnetospheres of neutron stars [2], in active galactic nuclei [3], and even in solar flares [4]. The difficulties in developing the nonlinear wave theory in degenerate plasmas at a nonzero temperature were overcome using the new method of the Bernoulli pseudo- potential [5] and an exact estimation of the Fermi–Dirac integral [6]; as a result, a nonlinear theory of isothermal electron plasma waves in a degenerate plasma at an arbitrary nonzero temperature has been developed [6].

The influence of quantum effects on the excitation of two instabilities, namely quantum dust acoustic and quantum dust-lower-hybrid waves due to the free streaming of ion/dust particles in uniformly magnetized dusty plasmas was investigated using a quantum hydrodynamic model pointed out for laboratory as well as for space dusty plasmas [7].

The kinetic quantum Zakharov equations in dusty plasmas that describe nonlinear coupling of high frequency Langmuir waves to low frequency plasma density variations, for cases of non-degenerate and degenerate plasma electrons was derived at [8].

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Possible Jeans instabilities of self-gravitating astrophysical quantum dusty plasma systems with electrostatic perturbations were investigated [9].

A generalized dielectric constant for unmagnetized quantum dusty plasma composed of electrons, ions, and charged dust particulates was described by using a quantum hydrodynamic (QHD) model with neglecting the electron inertial force in comparison with the electron pressure [10]. The authors of Refs.[11] and [12],[13],[14] have used quantum transport models for the electrons and ions, and derived modified dispersion relations for Langmuir and ion-acoustic waves at unmagnetized plasma. In nearly all these publications, the gas dynamic approach based on dynamic equations for gases was used [12]-[15], in which dusty plasma components are treated as cold; i.e., these components are at a nonzero temperature and obey one of the following equations of state for cold Fermi gases (depending on the dimensionality of the gas). However, in a number of recent publications [16–21], collective effects in a quantum dusty plasma are considered. For example, the existence of low temperature and high particle number density has been observed where the de Broglie wavelength of the plasma particles is comparable to the dimension of the system and so the quantum effects cannot be ignored.[16]. Quantum plasmas are studied mainly by two approaches, viz. quantum kinetic approach and quantum hydrodynamic (QHD) approach. The kinetic approach is needed to discuss the Landau damping [17] of waves in quantum plasmas. The most widely used approach for studying quantum plasmas is QHD approach. 18] was the first to give the mathematical derivation of QHD model. Due to the quantum tunneling effects, a new force in terms of the gradient of Bohm potential [19] appears in the momentum equation. The existence of DIA wave has been theoretically reported in metallic multi walled carbon nanotubes [20]. The various linear and nonlinear phenomena of quantum dust acoustic [21].

This study aims at developing a dispersion relation of sound waves in a hot nonmagnetized e-i-d plasma, in which the temperature of electron, ion and dust quantum – degenerate gases differ from zero temperature. In fact, this study is a continuation of [6,22, 23], in which the same basic concepts are used (three – liquid gas dynamics with massless electrons, ion and dust, exact nonintegral from the equations of state of hot Fermi gases, and the Bernoulli Pseudo-potential method) as in [22] which treated electron ,positron and ion plasma.

Basic Equations

Here, quantum plasma with electrons, ions, and dust particles is considered. The dust grains are negatively charged and do not move as they are highly massive. Owing to small mass, the electrons are supposed to be inertialess. The system of equations describing the dynamics is as follows, to describe the processes occurring in such plasma, the following 1D gas dynamic equations for the components: the continuity equation as in [22] and [24] will be used.

$$\frac{\partial n_{e,i,d}}{\partial t} + \frac{\partial (n_{e,i,d} V_{e,i,d})}{\partial t} = 0 \, , \tag{1}$$

The equation of the dynamics of ions and dust.

$$\frac{\partial V_{i,d}}{\partial t} + V_{i,d}\frac{\partial V_{i,d}}{\partial x} = -\begin{pmatrix} \frac{q_{i,d}}{m_{i,d}}\frac{\partial \varphi}{\partial x} \\ + \frac{1}{m_{i,d}\, n_{i,d}}\frac{\partial P_{i,d}}{\partial x} \end{pmatrix} + \frac{h^2}{16\pi^2 m_{i,d}{}^2}\frac{\partial}{\partial x}\{\frac{1}{n_{i,d}}\begin{pmatrix} \frac{\partial^2 n_{i,d}}{\partial x^2} \\ -\frac{1}{n_{i,d}}(\frac{\partial n_{i,d}}{\partial x})^2 \end{pmatrix}\} \, , \tag{2}$$

The equation of electron dynamics

$$\frac{\partial V_e}{\partial t} + V_e\frac{\partial V_e}{\partial x} = \begin{pmatrix} \frac{q_e}{m_e}\frac{\partial \varphi}{\partial x} \\ -\frac{1}{m_e\, n_e}\frac{\partial P_e}{\partial x} \end{pmatrix} + \frac{h^2}{16\pi^2 m_e{}^2}\frac{\partial}{\partial x}\{\frac{1}{n_e}\begin{pmatrix} \frac{\partial^2 n_e}{\partial x^2} \\ -\frac{1}{n_e}(\frac{\partial n_e}{\partial x})^2 \end{pmatrix}\}, \tag{3}$$

And the Poisson equation

$$\frac{\partial^2 \varphi}{\partial x^2} = 4\pi(n_e q_e + n_i q_i - n_{0d} z_{0d}) \, , \tag{4}$$

While in view of the quasi-neturality of the plasma, the following equality for the unperturbed values of concentrations can be written:

$$n_e q_e = n_i q_i - n_{0d} z_{0d} \, , \tag{5}$$

The terms in equations (3) and (4) related to the pressure gradient take into account the collective quantum mechanical interaction between particles (fermions), the last terms in these equations are also due to quantum effect namely the quantum wave nature of particles.

Disregarding the second quantum terms, equations of motions (2), (3) can be rewritten in the form:

$$\frac{\partial V_{i,d}}{\partial t} + V_{i,d}\frac{\partial V_{i,d}}{\partial x} = -\left(\begin{array}{c}\frac{q_{i,d}}{m_{i,d}}\frac{\partial \varphi}{\partial x} \\ +\frac{1}{m_{i,d}\, n_{i,d}}\frac{\partial P_{i,d}}{\partial x}\end{array}\right), \qquad (6)$$

$$\frac{\partial V_e}{\partial t} + V_e\frac{\partial V_e}{\partial x} = \left(\begin{array}{c}\frac{q_e}{m_e}\frac{\partial \varphi}{\partial x} \\ -\frac{1}{m_e\, n_e}\frac{\partial P_e}{\partial x}\end{array}\right), \qquad (7)$$

This system with the equation of state for a warm Fermi gas of electrons, ions and dust has been supplemented. It has the form of an implicit parametrically defined function and contains Fermi- Dirac integrals, which formerly assumed to be noncomputable. Nevertheless, following [23], these equations can be written in the nonintegral form:

$$n_{e,i,d}\left(\mu_{e,i,d}T_{e,i,d}\right) = -\left(\begin{array}{c}\frac{(m_{e,i,d}kT_{e,i,d})^2}{\sqrt{2}\pi^{3/2}\hbar^3} \\ Li_{\frac{3}{2}}(-\exp\left(\frac{\mu_{e,i,d}}{kT_{e,i,d}}\right))\end{array}\right), \qquad (8)$$

$$P_{e,i,d}\left(\mu_{e,i,d}T_{e,i,d}\right) = -\left(\begin{array}{c}\frac{(m_{e,i,d}kT_{e,i,d})^{5/2}}{\sqrt{2}\pi^{3/2}\hbar^3 m_{e,i,d}} \\ Li_{\frac{5}{2}}(-\exp\left(\frac{\mu_{e,i,d}}{kT_{e,i,d}}\right))\end{array}\right), \qquad (9)$$

Where $\mu_{e,i,d}$ is the chemical potential and $Li_v\,(\ldots???)$ are the polylogarthms [22],[23].

Linear theory of ionic sound waves

We derive the dispersion equation for ionic sound waves in the given model of the plasma. Let us assume a small harmonic wave perturbation to be a dependent variable of ion dynamics equations (1), (4), (6) and (7) relative to unperturbed values of these variables:

$$n_i = n_{i_0} + n_i^{\wr} e^{j(\mathrm{K}x-\omega t)}\ , \qquad (10)$$

$$V_i = V_i^{\wr} e^{j(\mathrm{K}x-\omega t)}, \qquad (11)$$

$$\varphi = \varphi^{\wr} e^{j(\mathrm{K}x-\omega t)}, \qquad (12)$$

$$\mu_i = \mu_{i_0} + \mu_i^{\wr} e^{j(\mathrm{K}x-\omega t)}\ , \qquad (13)$$

Where K and ω are the wavenumber and frequency of the perturbed quantities and the harmonic perturbation propagates along the x direction with the phase velocity, $V = \omega/\mathrm{K}$, and $j = \sqrt{-1}$.

For small perturbation with the unperturbed values n_{i_0} and μ_{i_0} formula (8) should take the form

$$n_i = n_{i_0} - \left(\begin{array}{c}\frac{(m_i kT_{i0})^{3/2}}{\sqrt{2}\pi^{3/2}m_i\hbar^3} \\ Li_{\frac{1}{2}}(-\exp\left(\frac{\mu_{i0}}{kT_{i0}}\right))\end{array}\right)\left(\frac{\mu_i^{\wr}}{kT_{i0}}\right)e^{j(\mathrm{K}x-\omega t)}\ , \qquad (14)$$

Comparing (10) and (14), we obtain

$$\mu_i^{\wr} = \frac{\sqrt{2}\pi^{3/2}m_i\hbar^3}{(m_i kT_{i0})^{3/2}}\, n_{i_0}kT_{i0}Li_{\frac{1}{2}}(-\exp\left(\frac{\mu_{i0}}{kT_{i0}}\right))\frac{n_i}{n_{i_0}}\ , \qquad (15)$$

Taking into account equation (15) and by substituting equations (10)- (13) into the initial equations of the problem, the following dispersion relation can be obtained as a result of the standard linearization procedure:

$$\frac{\omega_{i0}^2}{\mathrm{K}^2}/\left[\left(\frac{\omega}{\mathrm{K}}\right)^2 - V_{FDi}^2\right] = \frac{1}{\mathrm{K}^2}\left[\frac{1}{\lambda_{Dd}^2} + \frac{1}{\lambda_{De}^2}\right], \qquad (16)$$

Where $\omega_{i0}^2 = 4\pi q_i^2 n_{i_0}/m_i$ and the formulae for the Debye lengths of the electron and ion Fermi gases were found in [22] and [23].

$$V_{FDi}^2 = \omega_{i0}^2\lambda_{Di}^2 = \left(\frac{kT_{i0}}{m_i}\left[Li_{\frac{1}{2}}(-\exp\left(\frac{\mu_{i0}}{kT_{i0}}\right))\right]/\right.$$
$Li32(-\exp\mu i0kTi0)$,
(17)

Where V_{FDi}^2 is the square of the ion thermal velocity of the Fermi- Dirac gas. The formulae for the Debye lengths of the electron and ion Fermi gases were derived in [23]. In the present study, these formulae will be written and by the same way it’s derive for dust.

The square of the ionic Debye length is given by:

$$\lambda_{Di}^2 = \frac{Li_{\frac{3}{2}}(-exp\frac{\mu_{i0}}{kT_{i0}})kT_{i0}}{Li_{\frac{1}{2}}(-exp\frac{\mu_{i0}}{kT_{i0}})4\pi q_i{}^2\, n_{i_0}}\ , \qquad (18)$$

The square of the electron Debye length is

$$\lambda_{De}^2 = \frac{Li_{\frac{3}{2}}(-exp\frac{\mu_{e0}}{kT_{e0}})kT_{e0}}{Li_{\frac{1}{2}}(-exp\frac{\mu_{e0}}{kT_{e0}})4\pi q_e{}^2\, n_{e_0}}, \qquad (19)$$

And the square of the dust grain Debye length is

$$\lambda_{Dd}^2 = \frac{Li_{\frac{3}{2}}(-exp\frac{\mu_{d0}}{kT_{d0}})kT_{d0}}{Li_{\frac{1}{2}}(-exp\frac{\mu_{d0}}{kT_{d0}})4\pi(n_{0d}z_{0d})^2\, n_{0d}}\ , \qquad (20)$$

It appears that, Equation (16) describes the dispersion relation for ionic sound waves and allow us to find the range of periodic waves and ionic sound solutions.

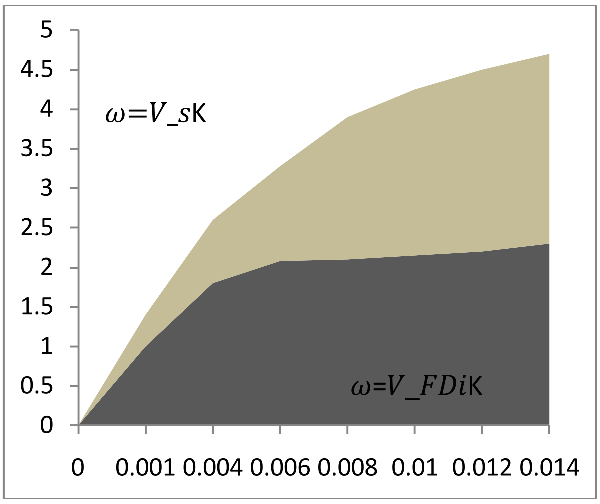

Figure 1 *ω* versus *K*

By plotting curve between (ω) $at\ y\ axis\ and$ (K)at x axis, an agreement with [22] was found with modification due to dusty plasma instead of positron and a shape typical of ionic sound and consists of three parts. The mid wave number and mid wavelength segments correspond to ion plasma oscillations with a group velocity substantially smaller than V_s and is followed by the higher wave number and short wavelength segment. While the second area between $\omega = V_s\mathrm{K}$ and $\omega = V_{FDi}\mathrm{K}$ are modified due to the presence of dust particles.

Note; some taken concepts related to the equations are rewritten at the index.

Conclusion

In this article, A collisionless nonmagnetized dusty plasma consisting of quantum gases of electrons, ions, and dust at nonzero temperatures is considered, the dispersion relation has been studied by applying Sagdeev's pseudopotential approach in an unmagnetized quantum dusty plasma together with the Poisson equation. The authors obtained and analyzed an exact solution to the initial equations. The dispersion equation for isothermal ionic sound waves is derived and analyzed, and the expression is obtained for the linear velocity of ionic sound. A graph plotted between the frequency and wave number, the ranges of phase velocities of periodic ionic sound waves and soliton velocities are determined. It is shown that in the plasma under investigation, Fermi pressure of electrons, and Bohm potential due to the quantum effects are taken into account. The quantum force increases and thus further increase in the group speed as well as the phase speed of the dust acoustic wave. Also it is found that the term of dusty plasma more affect in the dispersion and in the range of velocities from. V_{FDi} to V_s . The application of this work has been pointed out for laboratory as well as for space dusty plasmas.

Index

For one dimension (1D) the pressure $P = \frac{2\varepsilon_F}{5}\left(\frac{n}{n_0}\right)^{5/3}$ and for (3D) the pressure is
$P = \frac{2\varepsilon_F}{3}\left(\frac{n}{n_0}\right)^3$.
Where n is the concentration, n_0 is the initial concentration, and ε_F is the Fermi energy.
The authors used the following notations: electron mass and electron temperature, m_e, T_{0e} , ion mass and ion temperature, $m_{i,}T_{0i}$, and dust grain mass and dust grain temperature, $m_{d,}T_{0d}$ respectively.
Also, $\lambda_{D\ e,i,d} \gg \lambda_{dB\ e,i,d}$ de Broglie wavelength and, $\lambda_{dB\ e,i,d} \ll L$ the characteristic size of the system. $\hbar^2\omega_{0e,i,d} \ll kT_{e,i,d}$, where $\omega_{0e,i,d}$ are the Langmuir frequencies of the plasma components. We assume that $T_s = T_{0s}$ = constant for: $e, i, d = s$.

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