Journal of Physics: Conference Series

PAPER • OPEN ACCESS

Systematic derivation of the fundamental solutions for couple stress theory

To cite this article: A R El-metwaly *et al* 2023 *J. Phys.: Conf. Ser.* **2616** 012001

Systematic derivation of the fundamental solutions for couple stress theory

A R El-metwaly[1], M A Kamal [2], Y F Rashed[3], A S Ismail[1], and H Nasry[1]

[1] Engineering Mathematics Department, Military Technical College, Cairo, EGYPT
[2] Department of Basic Engineering Sciences, Shoubra-Faculty of Engineering, Benha University, EGYPT
[3] Department of Structural Engineering, Cairo University, Giza, EGYPT
E-mail: aly.rabie@mtc.edu.eg

Abstract. Classical elasticity theory failed to produce accurate results given in agreement with the experimental results. As this theory fails at stress concentration near the gaps or holes, or materials with significant microstructure contributions as soil, composite material, and polymer material generally in aerospace materials. In order to improve accuracy, the consistent couple stress theory is used. In the consistent couple stress theory, the microrotation and macro rotation are equal, where the macro rotation is half the curl of displacement, and the couple stress tensor is skew-symmetric. In this paper, the fundamental solutions for couple stress elasticity are derived in a systematic way via the Hörmander technique. The used technique is characterized by its ease and also develops the Galerkin tensor used in many applications, for example, the transformation of domain integral, computing the body force in the domain without discretization. All necessary kernels for displacements and tractions are derived and given in terms of a generalized Galerkin tensor for further use.

1. Introduction

According to the recent rapid microstructural development of materials, a new appeared as it was known as scale effects, which is a reflection of geometrical characteristics as well as microstructural characteristics of the material, the scale effect is associated with the ratio l/d, where l is length scale and d is the grain size. The traditional theories of elasticity and mechanics could not explain scale effects since it is incompatible with the length scale of materials, because it depends on the behavior of materials at the macro-scale (the strain tensor), where it neglects the microstructure size dependency, so it could not describe the behavior of composite materials. Therefore, the couple stress theory was developed by Mindlin and Tiersten[1] to keep pace with the development of composite materials which is used in a number of vital areas, for example, in aerospace industry[2]. The version in[1] suffers from the indeterminacy of the couple stress which was solved by Dargush[3] in the version called the consistent couple stress theory, where the couple stress tensor is proved to be skew symmetric. Hassanpour[4] gave a review on the micropolar elasticity theory and how it is simplified to the couple stress theory and then to the classical elasticity theory. Taig[5] used composite carbon fiber in the manufacture of aerospace structures not only for military aircraft production but also for civil aircraft, because of its high strength and stiffness in addition to the lighter weight. Kumar et al[6] employed high-temperature composite materials in the aerospace industry. Skrzat et al [7] used the couple stress theory to compute the effective elastic properties of a metal open-cell foam. Jun Lei et al [8] derived the general formulation for displacement and traction boundary integral equations of plane strain problems for couple stress with crack problems. The couple stress theory is also used in fluid, Hayat et al [9] used the couple stress theory to study the combined impacts of heat generation/absorption and convective condition in a three-dimensional magneto-hydrodynamic (MHD) flow which is used in many applications, including aerospace technology.

The fundamental solution of the micropolar elasticity is derived by N.Tosaka[10] who used Hörmander technique. The fundamental solution of the couple stress elasticity is derived by Dargush[11] using the decomposition method where the displacement is decomposed into dilatational and solenoidal components. Moreover it gives the final kernels without putting them as higher derivatives of scalar potential (such as the well-known Galerkin tensor in classical elasticity). It has to be noted that the fundamental solution for (2D) elasticity was derived by Rashed [12] using Hörmander technique and also he computed the fundamental solution for plates in bending by the same technique.

The importance of this method (Hörmander technique) is in the form of such solution in terms of generalized Galerkin tensor $\Phi(\mathrm{x}, \xi)$ which could be used in many fields, Cheng et al[13] used it in the direct boundary element method to transform the domain integral of the body force to the boundary by radial basis function for thermoelasticity and elasticity problems. Brebbia[14] generalize the Galerkin method and used it to solve the problem with body forces, centrifugal forces, and thermal effects. Brebbia and Rashed [15] presented general methods for processing domain integrals in boundary elements, such as by transforming to the boundary, like cell and Monte Carlo methods, as well as Green's Identity and Dual Reciprocity technique. Nasry [16] discover a new technique for transforming variables and coordinates from the original to the new. When using this technique, it is noted that it is simpler and faster to compute and more obvious than other techniques.
In this paper, the fundamental solutions for couple stress theory are derived via the Hörmander method where it is characterized by systematic and clear steps. On the other hand, the derived solutions, are presented as derivatives of the developed generalized Galerkin tensor, which could be used in many applications in future research.

2. Basic equations

Assume a solid occupies a region $\Omega \in R^2$ where linear isotropic couple stress theory is considered. The plane strain has three Degrees of freedom, two in-plane displacements u_α and one in-plane rotation u_3.
Figure 1 show the force stress and couple stress tensor. It is assumed that Greek indices take values (1,2), whereas Latin indices take values from 1 to 3, The notation for partial differentiation with respect to spatial coordinate x_α is $V_{,\alpha} = \frac{\partial(V)}{\partial x_\alpha}$.
As a result of these assumptions:

$$u_{\alpha,3} = 0 \quad \text{in } \Omega, \tag{1a}$$
$$e_{3\alpha} = e_{\alpha 3} = 0\,, \tag{1b}$$
$$\sigma_{3\alpha} = \sigma_{\alpha 3} = 0\,. \tag{1c}$$

where, $e_{\alpha\beta}$ is the strain tensor.

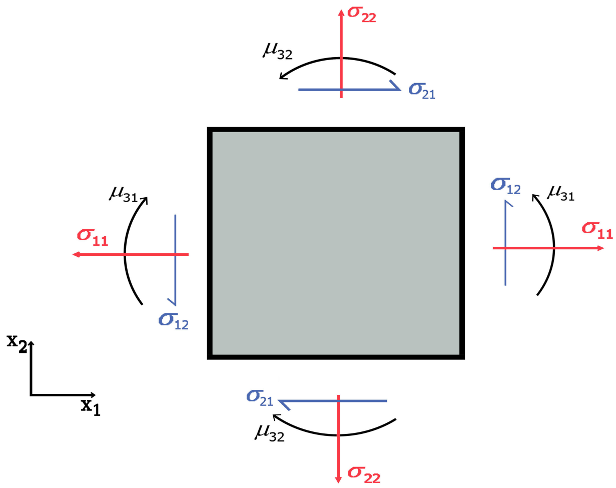

Figure. 1 Components of force stress and couple stress tensor.

The governing equations become:

$$\sigma_{\beta\alpha,\beta} + q_\alpha = 0, \tag{2a}$$
$$\mu_{\beta 3,\beta} + \varepsilon_{3\alpha\beta}\sigma_{\alpha\beta} = 0\,. \tag{2b}$$

where $\mu_{\beta 3}$ is the couple stress tensor, and $\varepsilon_{3\alpha\beta}$ is the permutation tensor (Levi-Civita), q_α the body force.
The generalized force stress tensor is:

$$\sigma_{\beta\alpha} = \lambda e_{\gamma\gamma}\delta_{\alpha\beta} + 2\mu e_{\alpha\beta} - 2\eta\varepsilon_{3\beta\alpha}\nabla^2 u_3 \tag{3}$$

where λ and μ are the usual classical Lamé elastic constants (μ is the shear modulus), in which $\lambda = \frac{2\mu\nu}{1-2\nu}$, where ν is Poisson's ratio. The symbol η is the modulus of isotropic couple stress theory.
The generalized strain tensor and rotation vector are:

$$e_{\alpha\beta} = \frac{1}{2}\left(u_{\alpha,\beta} + u_{\beta,\alpha}\right), \tag{4a}$$

$$u_3 = \frac{1}{2}\left(u_{2,1} - u_{1,2}\right). \tag{4b}$$

Substitute from Eqn. (4) into Eqn. (3), gives

$$\sigma_{\beta\alpha} = \lambda u_{\gamma,\gamma}\delta_{\alpha\beta} + \mu\left(u_{\alpha,\beta} + u_{\beta,\alpha}\right) - 2\eta\nabla^2\left[\frac{1}{2}\left(u_{\alpha,\beta} - u_{\beta,\alpha}\right)\right] \tag{5}$$

Differentiate Eqn. (5) with respect to the spatial coordinate x_β gives:

$$\sigma_{\beta\alpha,\beta} = \left(\lambda + \mu + \eta\nabla^2\right)u_{\beta,\alpha\beta} + \left(\mu - \eta\nabla^2\right)u_{\alpha,\beta\beta} \tag{6}$$

Substitute Eqn. (6) into Eqn. (2) and neglecting the body force, gives:

$$\left(\lambda + \mu + \eta\nabla^2\right)u_{\beta,\alpha\beta} + \left(\mu - \eta\nabla^2\right)u_{\alpha,\beta\beta} = 0 \tag{7}$$

The above equation can be rewritten as

$$L_{\alpha\beta}u_\beta = 0 \tag{8}$$

where the differential operator $L_{\alpha\beta}$ is given by

$$L_{\alpha\beta} = \begin{bmatrix} \left(\mu - \eta\nabla^2\right)\nabla^2 + \left(\lambda + \mu + \eta\nabla^2\right)\partial_1\partial_1 & \left(\lambda + \mu + \eta\nabla^2\right)\partial_1\partial_2 \\ \left(\lambda + \mu + \eta\nabla^2\right)\partial_1\partial_2 & \left(\mu - \eta\nabla^2\right)\nabla^2 + \left(\lambda + \mu + \eta\nabla^2\right)\partial_2\partial_2 \end{bmatrix} \tag{9}$$

where $\partial_\alpha = \frac{\partial}{\partial x_\alpha}$ and $\nabla^2 = \frac{\partial}{\partial x_\alpha}\frac{\partial}{\partial x_\alpha}$ implies two-dimensional Laplacian.

3. Hörmander technique

Hörmander technique [17] is the method used here to derive the fundamental solution as it is a general technique.
The fundamental solution for displacement $u_{\alpha\beta}(\mathrm{x}, \xi)$ is defined as follows:

$$L^{\text{adj}}_{\gamma\beta}\, u_{\alpha\beta}(\mathrm{x}, \xi) = -\delta(\mathrm{x}, \xi)\delta_{\gamma\alpha} \tag{10}$$

where $L^{\text{adj}}_{\alpha\beta}$ is the adjoint operator of the $L_{\alpha\beta}$ operator, $\delta(\mathrm{x}, \xi)$ is Dirac delta distribution, x is the field point, ξ is the source point, and $\delta_{\gamma\alpha}$ is Kronecker delta. Following Hörmander [17] the fundamental solution is defined as:

$$u_{\alpha\beta}(\mathrm{x}, \xi) = {}^{CO}\, L^{\text{adj}}_{\alpha\beta}\, \Phi(\mathrm{x}, \xi) \tag{11}$$

knowing that

$$L^{adj-1}_{\alpha\beta} = \frac{{}^{CO}\, L^{\text{adj}}_{\alpha\beta}}{\det\left[L^{adj}\right]} \tag{12}$$

where $\det\left[L^{adj}\right]$ is the determinate of L^{adj}, and $\Phi(\mathrm{x}, \xi)$ is an unknown scalar potential.
Substitute by Eqn. (12) and Eqn. (11) in Eqn. (10)

$$L^{\text{adj}}_{\gamma\beta}\, L^{adj-1}_{\alpha\beta} \det\left[L^{adj}\right]\Phi(\mathrm{x}, \xi) = -\delta(\mathrm{x}, \xi)\delta_{\gamma\alpha} \tag{13}$$

then simplify, to give:

$$\delta_{\gamma\alpha} \det\left[L^{adj}\right]\Phi(\mathrm{x}, \xi) = -\delta(\mathrm{x}, \xi)\delta_{\gamma\alpha} \tag{14}$$

Finally the scalar potential $\Phi(\mathrm{x}, \xi)$ could be computed from:

$$\det\left[L^{adj}\right]\Phi(\mathrm{x}, \xi) = -\delta(\mathrm{x}, \xi) \tag{15}$$

4. The proposed derivation

The fundamental solution for couple stress problem is derived as follows, the $\det\left[L^{adj}\right]$ could be obtained from Eqn. (9) as follows:

$$\det\left[L^{adj}\right] = \left(\mu - \eta\nabla^2\right)(2\mu+\lambda)\nabla^4 \tag{16}$$

Substitute into Eqn. (15), gives:

$$\Phi(\mathrm{x},\xi) = \frac{-1}{2\mu+\lambda}\left\{\frac{1}{\nabla^4\left(\mu-\eta\nabla^2\right)}\right\}\delta(\mathrm{x},\xi) \tag{17}$$

Using the partial fraction

$$\frac{1}{\nabla^4\left(\mu-\eta\nabla^2\right)} = \frac{a_1}{\nabla^2} + \frac{b_1}{\nabla^4} + \frac{c_1}{\mu-\eta\nabla^2} \tag{18}$$

where $a_1 = \frac{l^2}{\mu}$, $b_1 = \frac{1}{\mu}$, and $c_1 = l^4$, where l is the basic component of the small deformation size-dependent elasticity theory, which is known as the characteristic length of material, and $l^2 = \frac{\eta}{\mu}$.
Substitute Eqn. (18)into Eqn. (17) gives:

$$\Phi(\mathrm{x},\xi) = \frac{-1}{2\mu+\lambda}\left\{\frac{\left(l^2/\mu\right)}{\nabla^2} + \frac{(1/\mu)}{\nabla^4} + \frac{l^4}{\mu-\eta\nabla^2}\right\}\delta(\mathrm{x},\xi) \tag{19}$$

From [18], the following potentials could be computed:

$$\nabla^2\Phi_1(\mathrm{x},\xi) = -\delta(\mathrm{x},\xi) \tag{20}$$

$$\nabla^4\Phi_2(\mathrm{x},\xi) = -\delta(\mathrm{x},\xi) \tag{21}$$

$$(\frac{1}{l^2} - \nabla^2)\Phi_3(\mathrm{x},\xi) = -\delta(\mathrm{x},\xi) \tag{22}$$

$$\Phi(\mathrm{x},\xi) = \frac{1}{2\mu+\lambda}\left\{\frac{l^2}{\mu}\left(\Phi_1(\mathrm{x},\xi)\right) + \frac{1}{\mu}\left(\Phi_2(\mathrm{x},\xi)\right) + \frac{l^4}{\eta}\left(\Phi_3(\mathrm{x},\xi)\right)\right\} \tag{23}$$

Therefore, the unknown scalar potential for the couple stress theory is:

$$\Phi(\mathrm{x},\xi) = \frac{1}{2\mu+\lambda}\left\{\frac{l^2}{\mu}\left(\frac{-1}{2\pi}\ln r\right) + \frac{1}{\mu}\left(\frac{-1}{8\pi}r^2\ln r\right) + \frac{l^4}{\eta}\left(\frac{-1}{2\pi}K_0\left(\frac{r}{l}\right)\right)\right\} \tag{24}$$

It has to be noted that $\Phi(\mathrm{x},\xi)$ represents the generalized Galerkin tensor for the couple stress theory, and r is the distance between x, and ξ, and $K_n(\mathrm{x})$ is the modified Bessel function of second kind of order n.
The co-factor matrix of the adjoint operator in Eqn. (11) can be written in an indicial notation as follows:

$$^{CO}\,L^{\text{adj}}_{\alpha\beta} = (2\mu+\lambda)\nabla^2\delta_{\alpha\beta} - \left(\mu+\lambda+\eta\nabla^2\right)\partial_\alpha\partial_\beta \tag{25}$$

Substitute by Eqn. (25) in Eqn. (11), the generalized fundamental solution of displacement can be written as follows:

$$u_{\alpha\beta}(\mathrm{x},\xi) = \left[(2\mu+\lambda)\nabla^2\delta_{\alpha\beta} - \left(\mu+\lambda+\eta\nabla^2\right)\partial_\alpha\partial_\beta\right]\Phi(\mathrm{x},\xi) \tag{26}$$

Or

$$u_{\alpha\beta}(\mathrm{x},\xi) = (2\mu+\lambda)\delta_{\alpha\beta}\nabla^2\Phi(\mathrm{x},\xi) - (\mu+\lambda)\Phi_{,\alpha\beta}(\mathrm{x},\xi) - \eta\nabla^2\Phi_{,\alpha\beta}(\mathrm{x},\xi) \tag{27}$$

From Eqn. (24) , $\Phi(\mathrm{x},\xi)$ can be written as:

$$\Phi(\mathrm{x},\xi) = a\ln r + b\,r^2 lnr + c\,K_0\left(\frac{r}{l}\right) \tag{28}$$

where

$$a = \frac{-l^2}{2\pi\mu(2\mu+\lambda)}, \tag{29a}$$

$$b = \frac{-1}{8\pi\mu(2\mu+\lambda)}, \tag{29b}$$

$$c = \frac{-l^4}{2\pi\eta(2\mu+\lambda)}. \tag{29c}$$

Differentiating Eqn. (28) with respect to x_α, gives:

$$\Phi_{,\alpha}(\mathrm{x},\xi) = a\frac{r_{,\alpha}}{r} + b\left[r^2\frac{r_{,\alpha}}{r} + 2rr_{,\alpha}\ln r\right] + c\left[-K_1\left(\frac{r}{l}\right)\cdot\frac{r_{,\alpha}}{l}\right], \tag{30a}$$

$$= \frac{ar_{,\alpha}}{r} + b\left[rr_{,\alpha} + 2rr_{,\alpha}\ln r\right] - \frac{c}{l}\left[r_{,\alpha}K_1\left(\frac{r}{l}\right)\right]. \tag{30b}$$

Differentiate Eqn. (30) with respect to x_β, gives:

$$\begin{aligned}\Phi_{,\alpha\beta}(\mathrm{x},\xi) &= a\frac{rr_{,\alpha\beta} - r_{,\alpha}r_{,\beta}}{r^2} + brr_{,\alpha\beta}[1+2\ln r] + br_{,\beta}r_{,\alpha}[1+2\ln r]\\ &+brr_{,\alpha}\left[\frac{2r_{,\beta}}{r}\right] - \frac{c}{l}\left[\frac{r_{,\alpha}r_{,\beta}}{l}\left(-K_0\left(\frac{r}{l}\right) - \frac{l}{r}K_1\left(\frac{r}{l}\right)\right) + K_1\left(\frac{r}{l}\right)r_{,\alpha\beta}\right]\end{aligned} \tag{31}$$

where

$$r_{,\alpha\beta} = \frac{\delta_{\alpha\beta} - r_{,\alpha}r_{,\beta}}{r} \tag{32}$$

Simplifying Eqn. (31)

$$\begin{aligned}\Phi_{,\alpha\beta}(\mathrm{x},\xi) = \frac{a\delta_{\alpha\beta}}{r^2} - \frac{2ar_{,\alpha}r_{,\beta}}{r^2} + 2br_{,\alpha}r_{,\beta} + b[1+2\ln r]\delta_{\alpha\beta} + \frac{c}{l^2}r_{,\alpha}r_{,\beta}K_0\left(\frac{r}{l}\right)\\ - \frac{c}{rl}K_1\left(\frac{r}{l}\right)\delta_{\alpha\beta} + \frac{2c}{rl}K_1\left(\frac{r}{l}\right)r_{,\alpha}r_{,\beta}\end{aligned} \tag{33}$$

Put $\alpha = \beta$ the Laplacian of $\Phi(\mathrm{x},\xi)$ could be written as follows:

$$\nabla^2\Phi(x,\xi) = \Phi_{,\alpha\alpha}(x,\xi) = 4b[1+\ln r] + \frac{c}{l^2}K_0\left(\frac{r}{l}\right) \tag{34}$$

Differentiating Eqn. (34) with respect to x_α, gives:

$$\nabla^2\Phi_{,\alpha}(\mathrm{x},\xi) = 4b\frac{r_{,\alpha}}{r} - \frac{c}{l^2}K_1\left(\frac{r}{l}\right)\cdot\frac{r_{,\alpha}}{l} = 4b\frac{r_{,\alpha}}{r} - \frac{c}{l^3}r_{,\alpha}K_1\left(\frac{r}{l}\right) \tag{35}$$

Differentiating Eqn. (35) with respect to x_β, gives:

$$\nabla^2\Phi_{,\alpha\beta}(\mathrm{x},\xi) = 4b\frac{rr_{,\alpha\beta} - r_{,\alpha}r_{,\beta}}{r^2} - \frac{c}{l^3}\left[\frac{r_{,\alpha}r_{,\beta}}{l}\left(-K_0\left(\frac{r}{l}\right) - \frac{l}{r}K_1\left(\frac{r}{l}\right)\right) + K_1\left(\frac{r}{l}\right)r_{,\alpha\beta}\right] \tag{36}$$

Substituting Eqn. (30) to (36) into Eqn. (27) , gives:

$$\begin{aligned}u_{\alpha\beta}(\mathrm{x},\xi) &= (2\mu+\lambda)\delta_{\alpha\beta}\left[4b(1+\ln r) + \frac{c}{l^2}K_0\left(\frac{r}{l}\right)\right]\\ &- (\mu+\lambda)\left[\frac{a\delta_{\alpha\beta}}{r^2} - \frac{2ar_{,\alpha}r_{,\beta}}{r^2} + 2br_{,\alpha}r_{,\beta} + b(1+2\ln r)\delta_{\alpha\beta}\right.\\ &\left.+\frac{c}{l^2}r_{,\alpha}r_{,\beta}K_0\left(\frac{r}{l}\right) - \frac{c}{rl}K_1\left(\frac{r}{l}\right)\delta_{\alpha\beta} + \frac{2c}{rl}K_1\left(\frac{r}{l}\right)r_{,\alpha}r_{,\beta}\right]\\ &- \eta\left[\frac{4b\delta_{\alpha\beta}}{r^2} - \frac{8br_{,\alpha}r_{,\beta}}{r^2} + \frac{cr_{,\alpha}r_{,\beta}}{l^4}K_0\left(\frac{r}{l}\right) - \frac{cK_1\left(\frac{r}{l}\right)}{l^3r}\delta_{\alpha\beta} + \frac{2c}{l^3r}K_1\left(\frac{r}{l}\right)r_{,\alpha}r_{,\beta}\right]\end{aligned} \tag{37}$$

Then the final expression for the fundamental solution of displacement is:

$$\begin{aligned}u_{\alpha\beta}(\mathrm{x},\xi) =&\delta_{\alpha\beta}\left[\frac{-1}{2\pi\mu}K_0\left(\frac{r}{l}\right) - \frac{l}{2\pi\mu}\frac{1}{r}K_1\left(\frac{r}{l}\right) + \frac{l^2}{2\pi\mu}\frac{1}{r^2} - \frac{3-4\nu}{8\pi\mu(1-\nu)}\ln r - \frac{7-8\nu}{16\pi\mu(1-\nu)}\right]\\ &+ r_{,\alpha}r_{,\beta}\left[\frac{1}{2\pi\mu}K_0\left(\frac{r}{l}\right) + \frac{l}{\pi\mu}\frac{1}{r}K_1\left(\frac{r}{l}\right) - \frac{l^2}{\pi\mu}\frac{1}{r^2} + \frac{1}{8\pi\mu(1-\nu)}\right]\end{aligned} \tag{38}$$

Substitute by Eqn. (38) in Eqn. (5), then the stress kernel is:

$$\begin{aligned}\sigma_{\alpha\gamma\beta}(\mathrm{x},\xi) =&\frac{\delta_{\gamma\beta}r_{,\alpha}}{4\pi}\left[\frac{4K_0\left(\frac{r}{l}\right)}{r}+\frac{8lK_1\left(\frac{r}{l}\right)}{r^2}-\frac{8l^2}{r^3}-\frac{1-2\nu}{(1-\nu)r}\right]\\&+\frac{\delta_{\alpha\gamma}r_{,\beta}}{4\pi}\left[\frac{4K_1\left(\frac{r}{l}\right)}{l}+\frac{4k_0\left(\frac{r}{l}\right)}{r}+\frac{8lK_1\left(\frac{r}{l}\right)}{r^2}-\frac{8l^2}{r^3}-\frac{1-2\nu}{(1-\nu)r}\right]\\&+\frac{\delta_{\alpha\beta}r_{,\gamma}}{4\pi}\left[\frac{4K_0\left(\frac{r}{l}\right)}{r}+\frac{8lK_1\left(\frac{r}{l}\right)}{r^2}-\frac{8l^2}{r^3}+\frac{1-2\nu}{(1-\nu)r}\right]\\&+\frac{r_{,\alpha}r_{,\beta}r_{,\gamma}}{4\pi}\left[\frac{-16K_0\left(\frac{r}{l}\right)}{r}-\frac{32lK_1\left(\frac{r}{l}\right)}{r^2}-\frac{4K_1\left(\frac{r}{l}\right)}{l}+\frac{32l^2}{r^3}-\frac{2}{(1-\nu)r}\right]\end{aligned}\tag{39}$$

The traction kernel is:

$$T_{\gamma\beta}(\mathrm{x},\xi)=\sigma_{\alpha\gamma\beta}(\mathrm{x},\xi)n_\alpha(\mathrm{x})\tag{40}$$

Substitute by Eqn. (39) into Eqn. (40), then the fundamental solution for the traction is given by:

$$\begin{aligned}T_{\gamma\beta}(\mathrm{x},\xi) =&\frac{\delta_{\gamma\beta}r_{,n}}{4\pi}\left[\frac{4K_0\left(\frac{r}{l}\right)}{r}+\frac{8lK_1\left(\frac{r}{l}\right)}{r^2}-\frac{8l^2}{r^3}-\frac{1-2\nu}{(1-\nu)r}\right]\\&+\frac{r_{,\beta}n_\gamma}{4\pi}\left[\frac{4K_1\left(\frac{r}{l}\right)}{l}+\frac{4k_0\left(\frac{r}{l}\right)}{r}+\frac{8lK_1\left(\frac{r}{l}\right)}{r^2}-\frac{8l^2}{r^3}-\frac{1-2\nu}{(1-\nu)r}\right]\\&+\frac{r_{,\gamma}n_\beta}{4\pi}\left[\frac{4K_0\left(\frac{r}{l}\right)}{r}+\frac{8lK_1\left(\frac{r}{l}\right)}{r^2}-\frac{8l^2}{r^3}+\frac{1-2\nu}{(1-\nu)r}\right]\\&+\frac{r_{,n}r_{,\beta}r_{,\gamma}}{4\pi}\left[\frac{-16K_0\left(\frac{r}{l}\right)}{r}-\frac{32lK_1\left(\frac{r}{l}\right)}{r^2}-\frac{4K_1\left(\frac{r}{l}\right)}{l}+\frac{32l^2}{r^3}-\frac{2}{(1-\nu)r}\right]\end{aligned}\tag{41}$$

5. Conclusions

The couple stress theory was derived to treat failures in classical theory of elasticity, which appeared in composite materials, and crack problems. The fundamental solution for the couple stress theory was derived by other techniques, which sometimes have long-way derivations. This paper derived the fundamental solution for the couple stress theory in another way by using the Hörmander technique. The main advantage of this derivation is that the desired kernels are computed in terms of a generalized Galerkin tensor, which is made available for further use in many engineering fields, for example using the generalized Galerkin tensor when transferring the domain integral to the boundary.

References

[1] Mindlin, R. and Tiersten, H., "Effects of couple-stresses in linear elasticity," *Archive for Rational Mechanics and Analysis*, Vol. **11**, 1962, pp. 415–448.

[2] Shafiei, Z., Sarrami-Foroushani, S., Azhari, F., and Azhari, M., "Application of modified couple-stress theory to stability and free vibration analysis of single and multi-layered graphene sheets," *Aerospace Science and Technology*, Vol. **98**, 2020, pp. 105652.

[3] Hadjesfandiari, A. R. and Dargush, G. F., "Couple stress theory for solids," *International Journal of Solids and Structures*, Vol. **48**, No. 18, 2011, pp. 2496–2510.

[4] Hassanpour, S. and Heppler, G., "Step-by-step simplification of the micropolar elasticity theory to the couple-stress and classical elasticity theories," *ASME International Mechanical Engineering Congress and Exposition*, Vol. **46583**, American Society of Mechanical Engineers, 2014, p. V009T12A042.

[5] Taig, I., "Principles of design of a carbon fibre composite aircraft wing," *Philosophical Transactions of the Royal Society of London. Series A, Mathematical and Physical Sciences*, Vol. **294**, No. 1411, 1980, pp. 565–575.

[6] Kumar, S., Reddy, K. M., Kumar, A., and Devi, G. R., "Development and characterization of polymer–ceramic continuous fiber reinforced functionally graded composites for aerospace application," *Aerospace Science and Technology*, Vol. **26**, No. 1, 2013, pp. 185–191.

[7] Skrzat, A. and Eremeyev, V. A., "On the effective properties of foams in the framework of the couple stress theory," *Continuum Mechanics and Thermodynamics*, Vol. **32**, No. 6, 2020, pp. 1779–1801.

[8] Lei, J., Wei, X., Ding, P., and Zhang, C., "General displacement and traction BEM for plane couple-stress problems," *Engineering Analysis with Boundary Elements*, Vol. **140**, 2022, pp. 59–69.

[9] Hayat, T., Muhammad, T., Shehzad, S. A., and Alsaedi, A., "Simultaneous effects of magnetic field and convective condition in three-dimensional flow of couple stress nanofluid with heat generation/absorption," *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, Vol. **39**, No. 4, 2017, pp. 1165–1176.

[10] Yamashita, S. and Tosaka, N., "Boundary Element Method for Couple Stress Theory of Elasticity," *Boundary Element Methods*, Springer, 1992, pp. 383–392.

[11] Hadjesfandiari, A. R. and Dargush, G. F., "Fundamental solutions for isotropic size-dependent couple stress elasticity," *International Journal of Solids and Structures*, Vol. **50**, No. 9, 2013, pp. 1253–1265.

[12] Rashed, Y. F., "Tutorial 5: fundamental solutions: II-matrix operators," *Boundary Element Communications*, Vol. **13**, 2002, pp. 35–45.

[13] Cheng, A.-D., Chen, C., Golberg, M., and Rashed, Y., "BEM for theomoelasticity and elasticity with body force—a revisit," *Engineering Analysis with Boundary Elements*, Vol. **25**, No. 4-5, 2001, pp. 377–387.

[14] Brebbia, C. A., Telles, J. C. F., and Wrobel, L. C., *Boundary element techniques: theory and applications in engineering*, Springer Science & Business Media, 2012.

[15] Brebbia, C. and Rashed, Y., "On the treatment of domain integrals in BEM," *Transformation of Domain Effects to the Boundary", YF Rashed, CA Brebbia (ed), WIT Press*, 2003, pp. 1–22.

[16] Nasry, H., "Coordinate Transformation In Unmanned Systems Using Clifford Algebra," *Proceedings of the 5th International Conference on Mechatronics and Robotics Engineering*, 2019, pp. 167–170.

[17] Hörmander, L., *Linear partial differential operators: 4th printing*, Springer, 1976.

[18] Rashed, Y. F., "Tutorial 4: Fundamental solutions: I-simple and compound operators," *Boundary Elements Communications: An International Journal*, Vol. **13**, No. 1, 2002, pp. 38–46.