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CubeSat attitude control via linear quadratic regulator (LQR)

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Abstract. The interest in space-related activities has grown recently on a global scale. The determination and control of attitude are necessary all space duties. As it affects the satellites mission accuracy, many researches are related to it. Attitude control systems (ACS) design and modelling are represented in this paper. The mathematical models for CubeSat and reaction wheels that act as actuator and the proposed optimal control system are introduced. The proposed controller is applied to control and stabilize the CubeSat through a set of reaction wheels. The simulation results show the superior results of the proposed controller compared with traditional control systems in the presence of external disturbances and white noise. The withdraws of each control system are presented through the simulation results. The main contribution in this paper is solving the attitude control problem for a CubeSat using LQR approach in the presences of disturbances and noise.

1. Introduction

Micro satellites, also referred to as CubeSat, have recently captured the interest of satellite experts because to their excellent performance in a variety of missions. The term "CubeSat" refers to inexpensive tiny satellites that assist companies in gaining access to space. These satellites stand out for tiny size, adaptable design, and straightforward internal component layout[1].

A series of reaction wheels is what causes the CubeSat to move. The majority of CubeSat actuators now utilize reaction wheels systems for motion. By increasing the rotation of the wheel in one direction, reaction wheels cause the satellite to rotate in the other way. To provide the system with the appropriate torque, these wheels modify the flywheels' rotational speeds. The motors get saturated at their maximum rotational speed as a result of the reaction wheels building up momentum [2]. An orthogonal setup to manage the CubeSat in three dimensions is one of the well-known reaction wheel layouts [3].

The CubeSat attitude tracking control system's design depends heavily on ACS. ACS is expensive and/or enormous in contrast to a CubeSat's size. In ACS, passive control strategies are employed to save energy and cut costs. To ensure improved precision and accuracy in autonomous orbit determination and control, active control approaches are used [4, 5].

On the majority of space vehicles, the primary actuators for attitude control are controlled through reaction wheels. The torque needed for the satellite to perform attitude movements is delivered by the response wheel, which is a non-biased momentum wheel. Reaction wheels set are widely applied for the stabilization of majority of space vehicles. Each reaction wheel lays on major axis parallel to the body axes. Moreover, reaction wheels set can be applied to overcome the effect of cyclic disturbances torque that decelerate the satellite motion [6].

Due to the system's response wheels, ACS is precise and damped. For the attitude determination of the satellite, real time attitude reading of the satellite is compared with a pre-planned waypoint such that the difference in measurement values are defined as the satellite's attitude error. The ACS aims to offer a correcting torque that gets rid of this mistake. However, the correction cycle will continue permanently due to the sensitivity of the measurement and the effect of external disturbances. Through the literature, one should notice that ACS plays an important role in the attitude control of the satellite guaranteeing precision and maneuverability. Additionally, the reaction wheels system offers continuous and smooth control with the least amount of disturbance torques [7, 8].

In this paper, LQR controller has been designed and introduced for the stabilization and control of the CubeSat using a set of reaction wheels located through the three main axes. The proposed control approach shows a superior improvement on the performance of the CubeSat compared with conventional control techniques in the presences of external disturbances and noise.

The layout of this work is as follows: Section 2 present the mathematical modeling of the CubeSat while Section 3 present the control approach design. The simulation results validating the introduced control approach accompanied with a comparative study with other conventional controllers are presented in Section 4. Finally, the work is concluded and future work is presented in Section 5.

2. CUBESAT MODELING

Based on Newton's second law, a mathematical model representing the linearized CubeSat attitude dynamics is shown in equations (1-3) [9].

$$\frac{d}{dt}\omega_x = -I_{rwx}I_{tbx}\frac{d}{dt}\alpha_x \tag{1}$$

$$\frac{d}{dt}\omega_y = -I_{rwy}I_{tby}\frac{d}{dt}\alpha_y \tag{2}$$

$$\frac{d}{dt}\omega_z = -I_{rwz}I_{tbz}\frac{d}{dt}\alpha_z \tag{3}$$

On the other hand, the mathematical model of the reaction wheel DC motor is represented in equations (4-5) [9].

$$\frac{d}{dt}i = \frac{V_s}{L} - \frac{R}{L}*i - \frac{Ke}{L}*\Omega \tag{4}$$

$$\frac{d}{dt}\Omega = \frac{K_t}{I_{rw}}i - \frac{b}{I_{rw}}\Omega \tag{5}$$

Equation (6) and Equation (7) presents the state space representation of the linearized CubeSat attitude dynamics accompanied with the mathematical model of the reaction wheel DC motor. The CubeSat state space System is represented by 12 states as shown in Equation (6) where $\Omega_x, \Omega_y, \Omega_z$ are the angular velocity of reaction wheel along x, y, z respectively. i_x, i_y, i_z are the current of reaction wheel along x, y, z respectively. $\omega_x, \omega_y, \omega_z$ are CubeSat angular velocity along x, y, z respectively. $\alpha_x, \alpha_y, \alpha_z$ are CubeSat angular orientation along x, y, z respectively. The control inputs V_x, V_y, V_z are the voltage of reaction wheel along x, y, z respectively.

Substituting in (1,2,3,24,5) gives the state equation:

$$\begin{bmatrix} \dot{\Omega}_x \\ \dot{\Omega}_y \\ \dot{\Omega}_z \\ \dot{i}_x \\ \dot{i}_y \\ \dot{i}_z \\ \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} -0.0718 & 0 & 0 & 17.32 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.0718 & 0 & 0 & 17.32 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.0718 & 0 & 0 & 17.32 & 0 & 0 & 0 & 0 & 0 & 0 \\ -22.1 & 0 & 0 & -530 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -22.1 & 0 & 0 & -530 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -22.1 & 0 & 0 & -530 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.00006 & 0 & 0 & -0.015 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.00006 & 0 & 0 & -0.015 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.0001 & 0 & 0 & -0.03 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \\ i_x \\ i_y \\ i_z \\ \omega_x \\ \omega_y \\ \omega_z \\ \alpha_x \\ \alpha_y \\ \alpha_z \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3104 & 0 & 0 \\ 0 & 3104 & 0 \\ 0 & 0 & 3104 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} \tag{6}$$

The output equation:

$$\begin{bmatrix} \dot{\Omega}_x \\ \dot{\Omega}_y \\ \dot{\Omega}_z \\ \dot{i}_x \\ \dot{i}_y \\ \dot{i}_z \\ \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \\ \alpha_x \\ \alpha_y \\ \alpha_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \\ i_x \\ i_y \\ i_z \\ \omega_x \\ \omega_y \\ \omega_z \\ \alpha_x \\ \alpha_y \\ \alpha_z \end{bmatrix} \tag{7}$$

The CubeSat system is controllable and observable where the output states can estimate by sensors to present the attitude of the CubeSat.

3. CONTROL DESIGN

Through this section, the designed control technique applied for CubeSat attitude determination will be discussed. In [9], traditional control approaches based on proportional-integral- derivative control system were applied for CubeSat attitude determination. However, the introduced control approaches in [9] failed to handle high disturbances and noise. Linear quadratic regulator (LQR) is an optimal control approach has the ability to handle disturbances and noise.

3.1 LQR Controller

A constant-gain infinite-horizon LQR was specifically chosen because it has a lower computational cost than both time-varying-gain infinite-horizon and finite horizon LQR [10]. A set of linear differential equations are necessary for the LQR design. Describing the dynamics of the system. Consequently, the motion equations were linearized with respect to the ideal mindset, the state that the Ideally, the system will not waste any time. The outcome was linearized motion equations [Eqs. (6,7)] decrease to a fairly intricate representation of the matrix with terms that depend on the orbital position and velocity is a periodic function of time on its own [11].

For the given state-space model:

$$\dot{x} = Ax + Bu \tag{8}$$

the "K" gain matrix and the control vector signal u are determining through equation (9)

$$u = -Kx \tag{9}$$

Minimizes the performance index J which is the LQR cost function

$$J = \int_0^\infty (x^T Q x + u^T R u) dt \tag{10}$$

Where, R is the quadratic control weight matrix, and Q is the quadratic state weight matrix that penalize the control energy and the states, respectively.

The LQR feedback gain matrix (K) premultiplies the state vector to yield the control effort is given by

$$K = R^{-1} B^T P \tag{11}$$

Where, P is a symmetric positive definite matrix which is the solution of the Algebraic Riccati equation (ARE).

$$PA + A^T P + Q - PBR^{-1} B^T P = 0 \tag{12}$$

In order to minimize equation (10), Equation (12) has to be solved. Moreover, equations (9-10) are applied to solve for Q, R and K. The relation between the control effort and matrix R is inverse proportional, in other words, decreasing the control effort increase the matrix R values. More focus will be put on maintaining a smaller state error the larger the values in the Q matrix, and vice versa. The diagonal components of the Q matrix weight the squares of the states [12] and the diagonal entries of the R matrix weight the quadratic products of various control inputs, while off diagonal entries weight the squares of the control inputs (magnetic moment components).

In the design of the LQR, Q and R matrices were chosen to be diagonal matrices for minimization. All proportional states, derivative states, and input control components were all weighted identically, resulting in the employment of just two weight values and a single weight value to describe Q and R matrices, respectively [12].

When the desired performance is attained, the generated MATLAB code is adjusted to select the appropriate Q and R matrices. The algebraic Riccati equation can be solved using a variety of techniques. A direct MATLAB command is applied to calculate K given as

K = lqr (A, B, Q, R)

The values of R and Q matrix in the system understudy are as follows:

$$\mathrm{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathrm{Q} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10^6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10^6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10^6 \end{bmatrix} \tag{13}$$

While the values of A and B given in equation (6)
The feedback gain matrix, K is

$$\mathrm{K}_{3\times 12} = [a \quad b \quad c] \tag{14}$$

where

$$\mathrm{a} = \begin{bmatrix} -0.07 & -4\times 10^{-16} & -3\times 10^{-14} & -0.8 \\ -5\times 10^{-14} & -0.07 & -3\times 10^{-14} & -8\times 10^{-17} \\ 3\times 10^{-14} & -6\times 10^{-13} & -0.04 & -5.7\times 10^{-17} \end{bmatrix} \tag{15}$$

$$\mathrm{b} = \begin{bmatrix} -2\times 10^{-15} & -5\times 10^{-15} & 1\times 10^{3} & 10^{-12} \\ -0.84 & -1\times 10^{-15} & -6\times 10^{-11} & 10^{3} \\ -5.8\times 10^{-15} & -0.84 & 4\times 10^{-11} & 7\times 10^{-10} \end{bmatrix} \tag{16}$$

$$\mathrm{c} = \begin{bmatrix} 2\times 10^{-11} & 10^{3} & 6\times 10^{-13} & -1\times 10^{-11} \\ -6\times 10^{-13} & -5\times 10^{-12} & 10^{3} & 8\times 10^{-11} \\ 698 & 8\times 10^{-11} & 10^{-10} & 10^{3} \end{bmatrix} \tag{17}$$

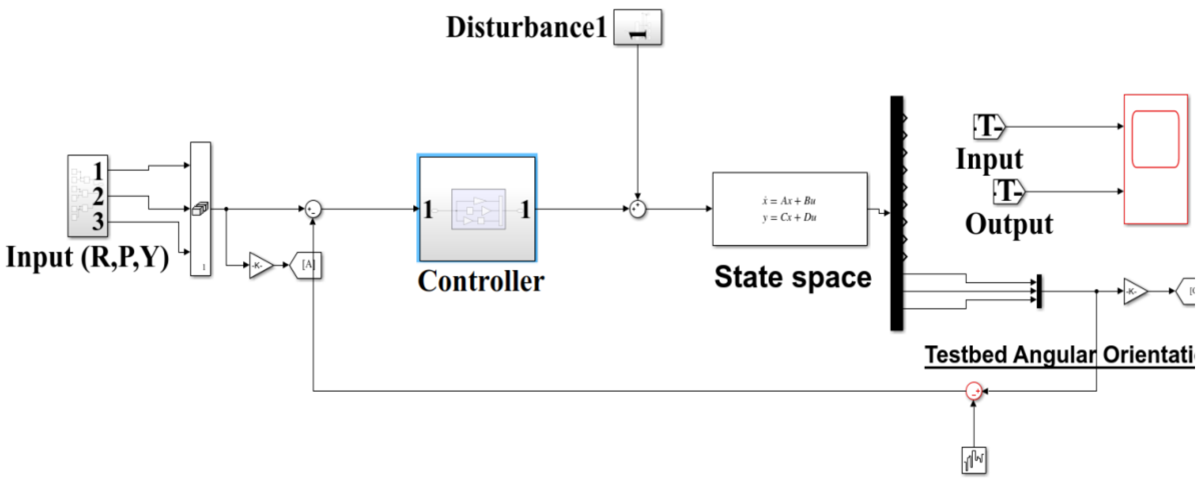

Figure 1. CubeSat Satellite Model affected by External Disturbances and White Noise.

4. SIMULATION RESULTS

The validation of the proposed LQR control system will be presented through this section. By applying the LQR control approach introduced in Section 3 to the mathematical linearized CubeSat model presented in Section 2, the problem of attitude determination of the CubeSat is solved. It is required that the motion of the CubeSat system is along the main three axes (roll – pitch – yaw).

The main goal in the design of the LQR approach is to sustain the desired attitude in the three axes using a single controller. The constrains on the system performance can be summarized in table 1 as follows:

Table 1. CubeSat Performance Characteristics.

Desired Rise Time	5 secs
Desired Overshoot	10%
Desired Roll Angle	0^o to 15^o
Desired Pitch Angle	0^o to 25^o
Desired Yaw Angle	0^o to 35^o

These constraints on the system performance is accompanied by the presence of input disturbance 10% from the reference value and 0.001 white noise. The scenario of validation consists from three parts along the main three axes (roll – pitch – yaw).

4.1. First Scenario: Attitude Determination Along Roll Axis

CubeSat motion along the roll axis is controlled by designed LQR. Respecting the constrains in table 1, the simulation results shows the success of the designed LQR to maintain the desired performance in the presence of external disturbances and white noise. Figure 2 presents the superior performance of the designed LQR on the traditional control approaches in the presences of external disturbances and white noise affecting the measured values. The CubeSat system performance is characterized by its stability and the success of the LQR controller to minimize the effect of the external disturbances and noise. Moreover, Figure 3 presents the system performance from along 100 sec starting from t= 75 sec till t = 175 sec where the desired roll angle varies from 10^o to 15^o and return back to 5^o presenting the strengthen of the controller.

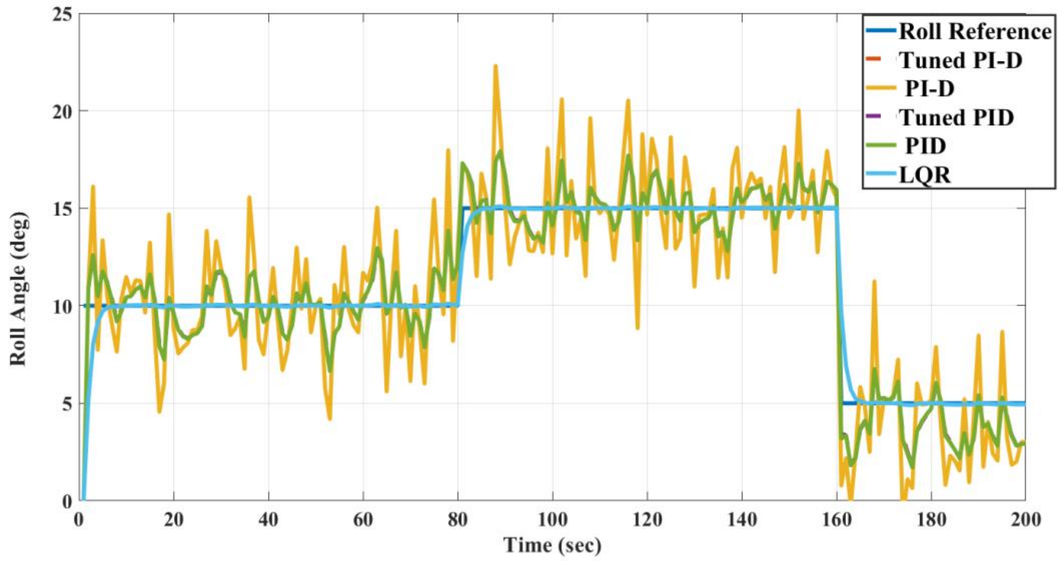

Figure 2. Desired angle along the roll axis under the effect of external disturbances and measurement Noise.

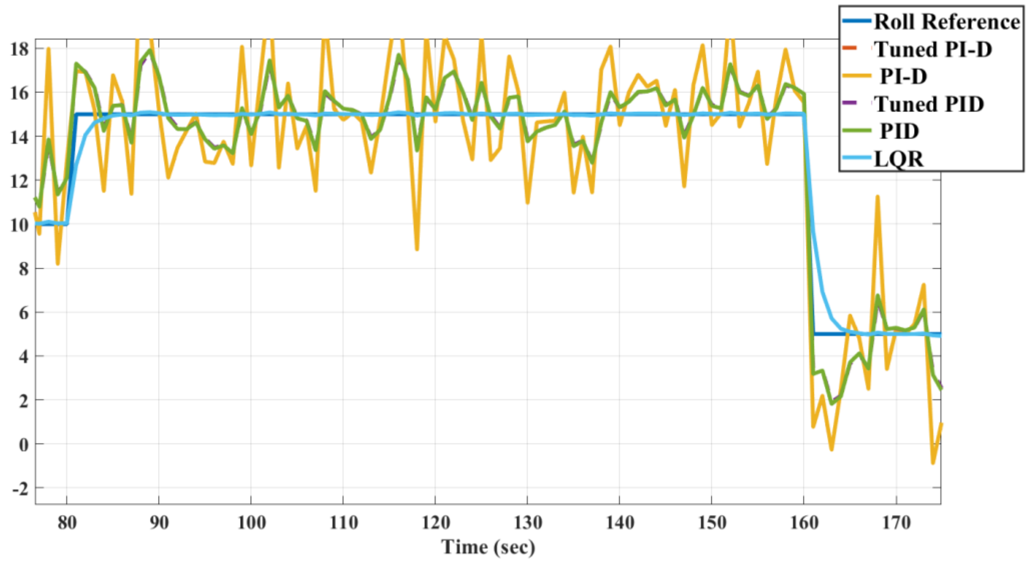

Figure 3. Desired roll angle form t=75 sec till t = 175 sec.

4.2. Second Case: Attitude Determination Along Pitch Axis

Similar to the first scenario, the proposed LQR control system was applied to solve the CubeSat attitude determination problem. Figure 4 present the success of the designed LQR approach to converge to the desired pitch angle under the effect of external disturbances and noise. Moreover, Figure 5 shows the system performance along 100 secs through which the desired pitch angle varies from 0^{o} to 25^{o} and return back 15^{o}. The proposed control approaches are applied to maintain a desired pitch angle varying from 0^{o} to 25^{o}. LQR approach guarantees stability and robustness of the system even with the effect of external disturbances and measurement noise.

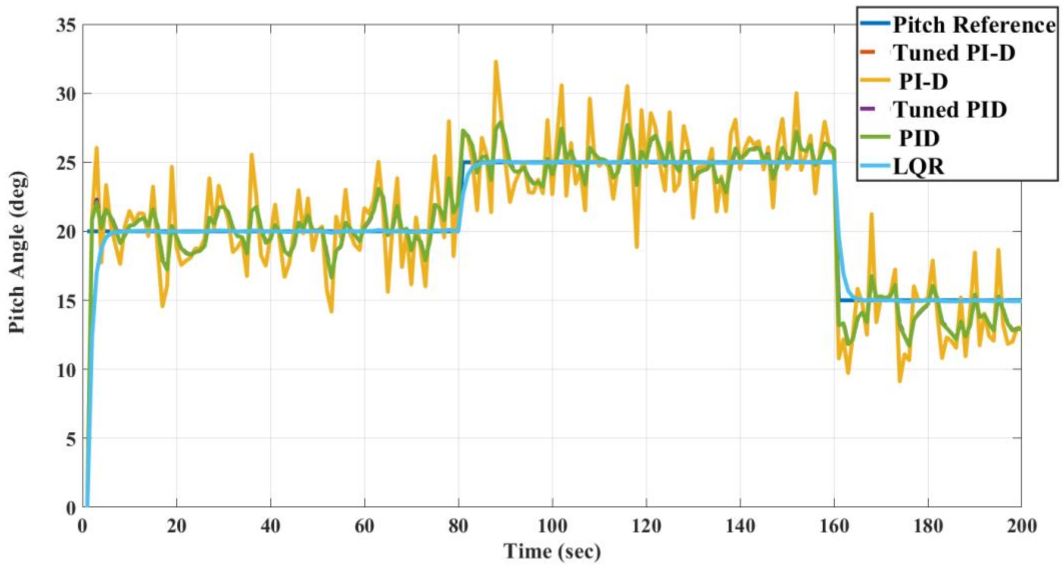

Figure 4. Desired pitch angle under the effect of external disturbances and measurement noise.

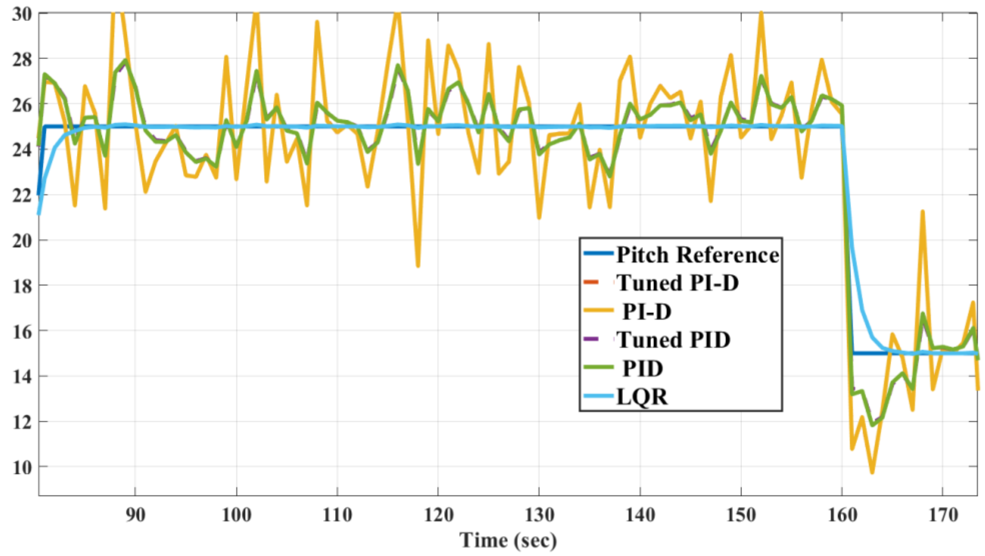

Figure 5. Desired pitch angle along 100 secs.

4.3. Third Case: Attitude Determination Along Yaw Axis

The last step in the validation of the proposed LQR approach is to present its ability to control the CubeSat motion along the yaw axis. The simulation results presented in figure 6 and figure 7 shows the success of the introduced controller to respect the desired yaw angle and convergence of the system to the desired values. The optimum LQR controller has a better performance compared with conventional control approaches.

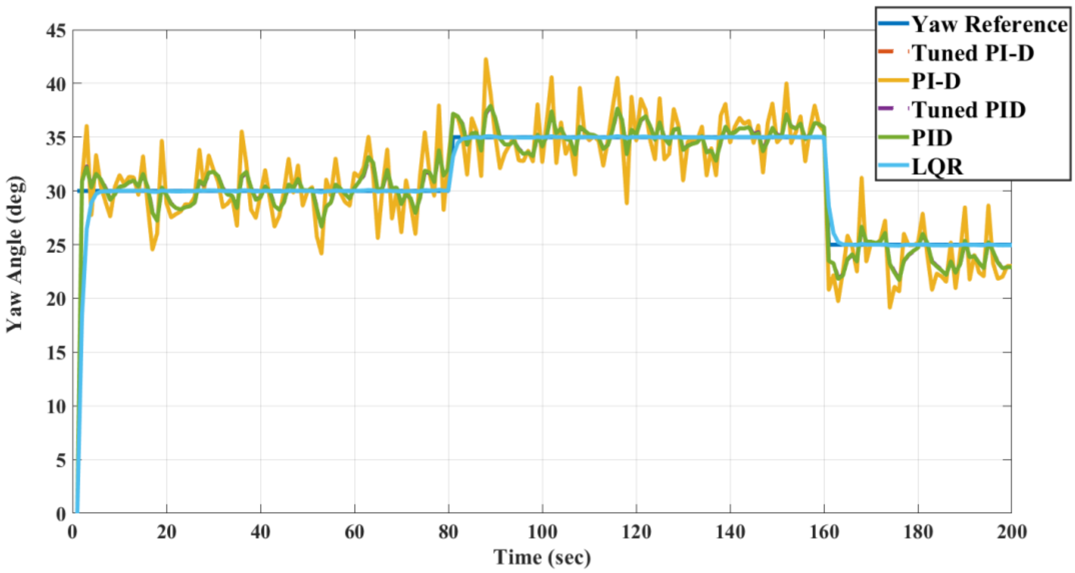

Figure 6. Desired yaw angle performance under the effect of external disturbances and measurement noise.

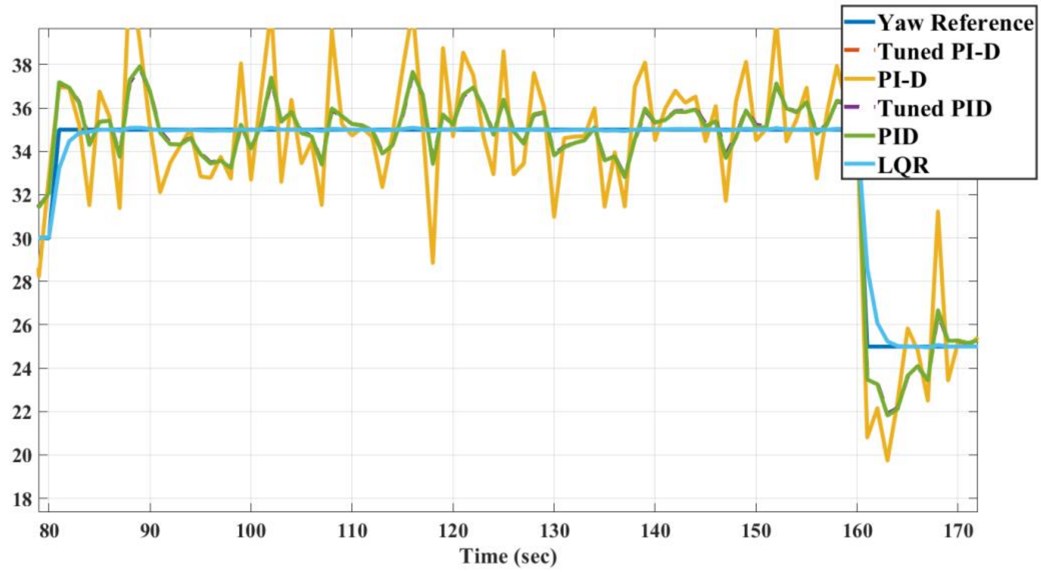

Figure 7. Variation of the yaw angle to show the strengthen of the proposed LQR.

5. Conclusion

In this work, attitude determination of a micro satellite (CubeSat) was solved using a proposed LQR controller under the effect of external disturbances and measurement noise. A comparison between variety of controller approaches as PID, Modified PI-D, genetically tuned of PID and PI-D and LQR was applied. The simulation results show the success of the designed controllers to maintain the CubeSat attitude to the desired values. The PID, tuned PID, PI-D and tuned PI-D response has more fluctuation about steady state value and more affecting with noise and disturbance so, the system not be able to stabilize and the response diverge from the desired value. So, the controllers fail to do its mission to maintain the satellite in stable motion when affecting by disturbance and noise. On the contrary, LQR controller has suitable settling time and it could converge to the desired value because the succeed in minimize the noise and reject the disturbance that affecting on the satellite.

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