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A land vehicle's INS/GNSS integrated navigation system using left invariant extended kalman filter

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A land vehicle's INS/GNSS integrated navigation system using left invariant extended kalman filter

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Abstract. Land vehicles need high-precision navigational systems in which multi-sensor integration may be provided. Moreover, land vehicles regularly use Global Navigation Satellite Systems (GNSS) to estimate their position. Unfortunately, several locations, such as tunnels and inside parking garages, where GNSS signals cannot be detected. Several types of research have been conducted to improve positioning information using multi-sensor integration. Then, the vehicle needs another system for finding its location in GNSS-denied conditions, such as Inertial Navigation System (INS). Despite the accuracy of INS in short-time period use, inertial navigation systems (INS) are liable to drifts of their positioning solution due to the inertial sensor errors that are inherent to them; therefore, this problem leads to errors accumulation over time then integration techniques are used to eliminate the resulting errors. Moreover, many filters are used in the process of integration, such as the Extended Kalman Filter (EKF), Unscented Kalman Filter (UKF), Particular Filter (PF) and Invariant Extended Kalman Filter (IEKF). Moreover, this work introduces the left-invariant extended Kalman filter (LIEKF) as a navigation filter for a loosely coupled integration to eliminate positioning errors. Furthermore, the LIEKF is based on the symmetry-preserving observer theory, which claims that the estimation error depends on the theory of a Lie group matrix, and the proposed system INS/GPS-based LIEKF converges to constant values, unlike the traditional INS/GPS. Moreover, the proposed system INS/GPS-based LIEKF depends on State-estimate-independent Jacobians, and the LIEKF is more efficient and has better performance due to results such as the 2D position RMS error due to the INS/GPS-based EKF is $19.43m$. However, the 2D position RMS error due to the INS/GPS-based LIEKF is $3.32m$ with 83% improvement. Moreover, the 2D position errors were enhanced using the INS/GPS-based LIEKF system compared to the INS/GPS-based EKF system.

1. Introduction

The prime target in land vehicle navigation is providing continuous and precise navigation solutions in all environmental conditions. Moreover, the most common source of navigation solutions for autonomous vehicles is the GNSS because of its solution long-term stability [1]. However, there are several situations where the GNSS can't provide either accurate or any solution. Particularly in tunnels and high buildings where a line of sight between at least four GNSS satellites and the receiver's antenna a degraded. Therefore, there is an important need to use a backup system such as INS, Radar or speedometer [2, 3].

The INS solution is immune to signal interference, but its quality depends mainly on the Inertial Measuring Unit (IMU) grade. However, IMUs are vulnerable to various error sources, categorized into deterministic and stochastic errors. Therefore, the deterministic errors are mitigated by calibration, while the stochastic errors can't be calibrated but can be modeled. Despite that, the INS solution drifts over time. Because its mechanization utilizes Newton's laws of motion to provide position, velocity, and attitude (PVA) [4]. However, for low-cost IMUs, there are unbounded errors affecting its raw measurements deriving the INS mechanization [5,6].

Therefore, GNSS and INS do not satisfy the navigation solution requirements. Thus, integration between two systems is applied to provide benefits from each system's advantages and eliminate each system's drawbacks [7]. EKF has been traditionally used for nonlinear estimation by using Taylor Series Expansion. EKF performance is good in the first order approximation, but in high order approximation, EKF has fallen and has bad performance. Moreover, the unscented Kalman filter (UKF) solves this problem of high-order approximation [8].

The state estimation process represents the main difference between the EKF and UKF. In the EKF, the state estimation is estimated depending on Gaussian random variables, which are estimated through the first-order linearization of the nonlinear system, then provides errors in posterior mean and covariance of the transformed Gaussian random variables, which affects the filter performance and will diverge, unlike the UKF which plays an important role to solve this problem by using a sampling approach. Moreover, these sample points completely capture Gaussian random variables' correct mean and covariance. When estimated through the true nonlinear system, this accurately captures the posterior mean and covariance to the second order (Taylor Expansion) for any nonlinearity [9].

Moreover, the mean and covariance of the state's estimation are calculated to second- order or better, unlike the first order in the EKF, provides for accurate implementation of the error estimation equations, which is the basis for both the EKF and UKF. Moreover, the EKF achieves only first-order accuracy. Consequently, the UKF does not depend on Jacobian or Hessian calculations. Nevertheless, the process of UKF is more complex than the EKF, and it needs more time, unlike the process of the EKF [10].

Furthermore, Artificial Intelligence (AI) techniques, such as Conventional Neural Networks (CNN), Fuzzy Inference systems (FIS), Adaptive Neuro-Fuzzy Inference Systems (ANFIS), and Fuzzy Clustering Techniques, play an important role in providing error estimation for the integrated INS/GNSS systems [11]. Consequently, an accurate, reliable, and continuous navigation solution is achieved. Unfortunately, the training process takes a long time and has high processing costs [12, 13].

The Left Invariant Extended Kalman Filter (LIEKF) is based on the symmetry-preserving observer theory, in which the error estimation depends on a Lie group matrix. Moreover, the main advantage of LIEKF over EKF is that the state linearization and measurement models are independent of the current estimation of the state, then this leads to a state independent of Jacobians at any linearization point, and the covariance matrix of the LIEKF is computed using the Riccati equation [14].

This paper introduces the LIEKF as a navigation filter for the INS/GNSS integration scheme in which its observable state variables converge within an area of attraction. Moreover, it is independent of the system's trajectory and relies on the IMU dynamics. In addition, this paper shows how the specified system can be used as a process model for the LIEKF as it satisfies the property of log-linear error dynamics [15].

This paper's contributions are summarized as follows:

- Design of Left Invariant EKF for the GPS measurement model and the IMU process model.
- Typical EKF is applied and evaluated using a real dataset.
- Comparison between the performance of two filters using a real dataset, including dynamics and checking the error calculations.

The remainder of this paper is divided into the following sections. Section 2 provides the methodology of using Invariant EKF for inertial navigation and the INS/GPS Integration using LIEKF. Section 3 provides Experimental Work and the specification of the utilized sensors. Section 4 provides all work results and explains how the proposed system enhances the positioning errors. Section 5 provides the conclusion and future work.

2. Methodology

2.1. Left Invariant Extended Kalman Filter (LIEKF)

The states of LIEKF are not dependent on Jacobian linearization like the traditional EKF. Moreover, the invariant error can be estimated by using the Lie group [16].

2.1.1. State Representation To estimate $(\mathbf{P}_t, \mathbf{V}_t, \mathbf{A}_t)$ of the vehicle, the system model states are represented by equation (1). Moreover, for autonomous vehicles positioning, the required states only position $\mathbf{P}_t$ and attitude $\mathbf{A}_t$ states are estimated.

$$\mathbf{X}_t = \begin{bmatrix} \mathbf{P}_t & \mathbf{V}_t & \mathbf{A}_t \\ 0 & 1 & \mathbf{0}_{1\times 3} \\ 1 & 0 & \mathbf{0}_{1\times 3} \end{bmatrix} \tag{1}$$

Where, the system model is denoted by $\mathbf{X}_t$, the position $\mathbf{P}_t$ states (latitude, longitude, and altitude(φ, λ, h)), the velocity $\mathbf{V}_t$ states (V_E, V_N, V_U), and the attitude $\mathbf{A}_t$ states (roll, pitch, and yaw (r, p, y)) are in the navigational frame. Moreover, the position and the altitude will be estimated.

This system model equation can be represented in a reduced form as follows:

$$\mathbf{X}_t = [\varphi,\ \lambda,\ h,\ V_E,\ V_N,\ V_U,\ r,\ p,\ y]^T \tag{2}$$

The estimated update of the system model states is shown in equation (3).

$$\hat{\mathbf{X}}_{t_k}^{+} = \hat{\mathbf{X}}_{t_k} \exp\left(\mathbf{L}_{t_k}\left(\hat{\mathbf{X}}_{t_k}^{-1}\mathbf{Y}_{t_k} - \mathbf{b}\right)\right) \tag{3}$$

Where, b is the measurement model's design matrix.
The measurement model is represented by equation (4).

$$\mathbf{Y}_{t_k} = \hat{\mathbf{X}}_{t_k}\mathbf{b} + \mathbf{V}_{t_k} \tag{4}$$

Where, $\mathbf{V}_{t_k}$ is the measurement error.

The invariant error $\eta_{t_k}^{l}$ is represented by equation (5) as follows [17]:

$$\eta_{t_k}^{l+} = \eta_{t_k}^{l} \exp\left(\mathbf{G}_{t_k}\left(\left(\eta_{t_k}^{l}\right)^{-1}\mathbf{b} - \mathbf{b} + \hat{\mathbf{X}}_{t_k}^{-1}\mathbf{V}_{t_k}\right)\right) \tag{5}$$

The covariance update equations of the LIEKF are shown in equation (6).

$$\mathbf{P}_{t_k}^{+} = \left(\mathbf{I} - \mathbf{L}_{t_k}\mathbf{H}\right)\mathbf{P}_{t_k}\left(\mathbf{I} - \mathbf{L}_{t_k}\mathbf{H}\right)^T + \mathbf{G}_{t_k}\hat{\mathbf{N}}_{t_k}\mathbf{G}_{t_k}^{T} \tag{6}$$

Where, $\mathbf{G}_{t_k}$ is the filter gain and $\mathbf{H}$ is the linearized measurement model design matrix and

$$\hat{\mathbf{N}}_{t_k} = \hat{\mathbf{X}}_{t_k}^{-1} Cov\left[\mathbf{V}_{t_k}\right]\hat{\mathbf{X}}_{t_k}^{-T}\ ,\ \ \mathbf{G}_{t_k} = \mathbf{P}_{t_k}\mathbf{H}^T\mathbf{S}^{-1},\ \ \mathbf{S} = \mathbf{H}\mathbf{P}_{t_k}\mathbf{H}^T + \hat{\mathbf{N}}_{t_k} \tag{7}$$

2.1.2. Measurement Model The measurement vector $\mathbf{Y}_{t_k}$ is represented as follows [18]:

$$\mathbf{Y}_{t_k} = \mathbf{X}_t\mathbf{H} + \mathbf{V}_{t_k} \tag{8}$$

where,

$$\mathbf{Y}_{t_k} = \begin{bmatrix} \varphi_{INS} - \varphi_{GPS} \\ \lambda_{INS} - \lambda_{GPS} \\ h_{INS} - h_{GPS} \end{bmatrix} \tag{9}$$

And, $\mathbf{H}$ is the measurement model linearized design matrix and is given as shown in equation (10):

$$\mathbf{H} = \begin{bmatrix} \mathbf{I}_{3\times3} & \mathbf{0}_{3\times6} \end{bmatrix} \tag{10}$$

The expanded form of the measurement model is given as in equation (11).

$$\begin{bmatrix} \varphi_{INS} - \varphi_{GPS} \\ \lambda_{INS} - \lambda_{GPS} \\ h_{INS} - h_{GPS} \end{bmatrix} = \begin{bmatrix} \varphi \\ \lambda \\ h \\ V_E \\ V_N \\ V_U \\ r \\ p \\ y \end{bmatrix}^T \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T + \mathbf{V}_{t_k} \tag{11}$$

Where, the measurements covariance matrix R_k is given as in equation (12).

$$R_k = \begin{bmatrix} \sigma_\varphi^2 & 0 & 0 \\ 0 & \sigma_\lambda^2 & 0 \\ 0 & 0 & \sigma_h^2 \end{bmatrix} \tag{12}$$

2.2. INS-GPS Integration Using LIEKF

The fusion or integration of the two systems provides a more effective solution than any of them alone, despite the INS and GPS's short- and long-term stability, respectively. A loosely coupled system creates the integration strategy as shown in figure 1. Moreover, for this kind of integration, a GPS solution is essential for the filter to function correctly, and the INS system offers a continuous PVA solution with unbounded overtime drift. Therefore, aiding information is necessary for preventing errors and improving solutions, and then location readings from the GPS are used to update the INS. However, the EKF, in general, is not an optimal estimator used for nonlinear systems but will fail in highly nonlinear order approximation because its states estimate are applied by Jacobians. However, Left Invariant Extended Kalman Filter states are not the typical Jacobian linearization along a trajectory because the Invariant error depends on the Lie group matrix [19].

The main advantage of LIEKF over EKF is that the state linearization and measurement models are independent of the current estimation of the state, leading to state-independent Jacobians at any linearization point; then, in this work, we will apply LIEKF as our navigation filter; after the process of estimation of the LIEKF, we extract our states position, velocity and attitude and, then obtain the corrected overall navigation solutions [20].

3. Experimental Work

Three major sensors are utilized, such as a reference GNSS NOVATEL-SPAN ProPak6, KVH 1750 IMU, and a MEMS-grade VTI-IMU [21]. Moreover, the sensor's specifications are shown in Table 1.

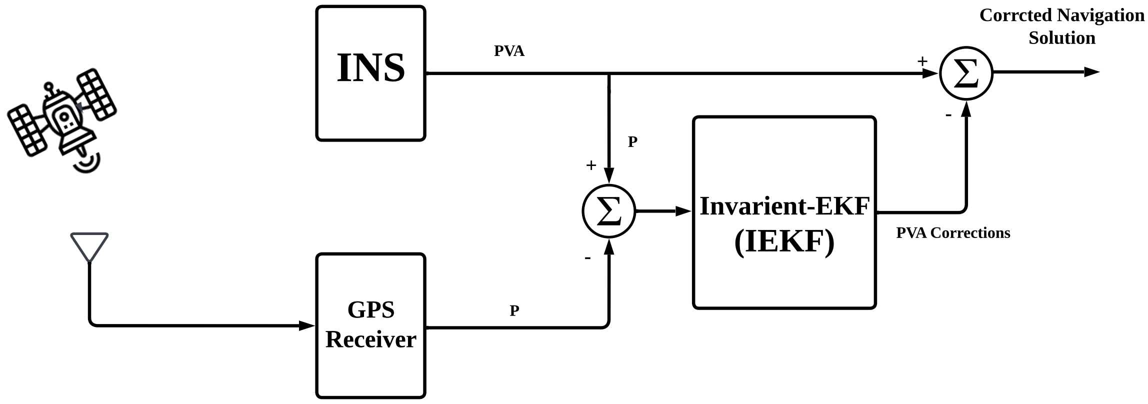

Figure 1. The INS/GPS-based LIEKF Integrated Navigation System Block Diagram

Table 1. Sensors' specifications Comparison

	GNSS NOVATEL ProPak6	KVH-1750 IMU	VTI-IMU
2D-position accuracy	$\leq 1.5m$		
Velocity accuracy	$\leq 0.03m/s$		
Time accuracy	20 ns		
Data rate	can reach 100 Hz adjusted to be 1Hz.	can reach 1000 Hz adjusted to 100 Hz	20 Hz
Gyroscopes Bias Instability		$\leq 0.05°/hr, 1\sigma$	$1.5deg/sec$
Angle Random Walk		$\leq 0.75°/hr/\sqrt{Hz}$	
Accelerometers Bias Instability		$0.01mg^{-1\sigma}$	
Scale Factor Linearity		0.008	$\leq 2\%$
Velocity Random Walk		0.024 $mg/\sqrt{Hz}$	0.86 $deg/\sqrt{Hz}$

4. Results and Discussion

The LIEKF filter is applied to a loosely coupled INS/GPS integration for autonomous vehicle navigation. Moreover, the results show a significant improvement in the positioning information provided to the vehicle compared to the traditional EKF.

The proposed INS/GPS-based LIEKF integrated navigation system showed better performance during several dynamics, such as straight driving, turns, and consecutive turns at various speeds. figure 2 shows the overall trajectory overlaid onto a digital map based on the reference from a tightly coupled INS/GPS integration between the Novatel Propack6 GPS receiver and the KVH-1750 IMU. Moreover, the performance of the two applied filters is tested using two different trajectory parts. The first part is shown in figure 3, which contains several dynamics, such as straight driving and a right turn. In this part, the INS/GPS-based EKF

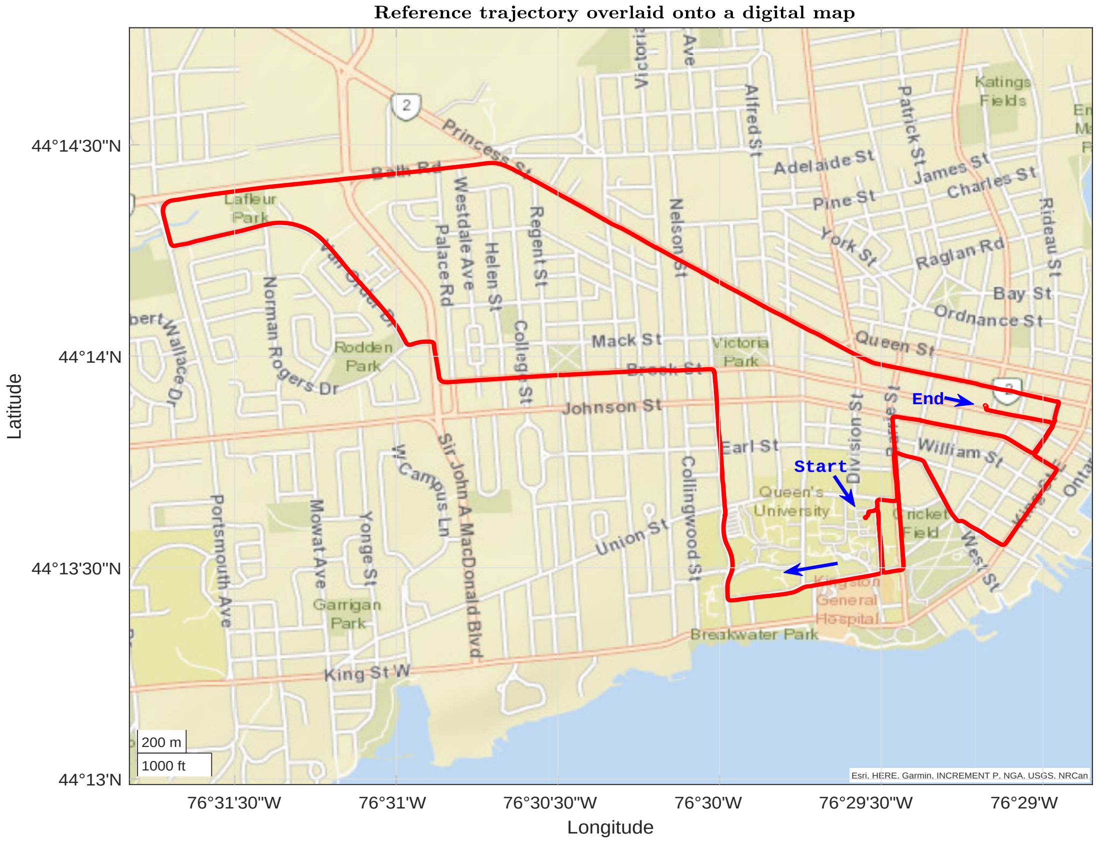

Figure 2. The vehicle's trajectory overlaid onto a digital map

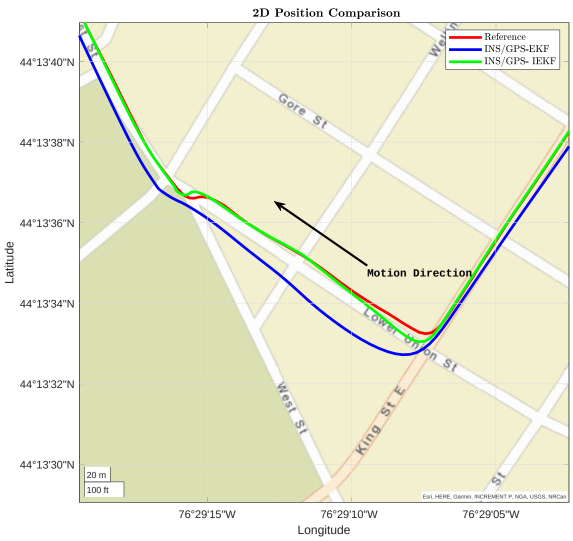

Figure 3. First dynamic section for Reference, EKF and LIEKF

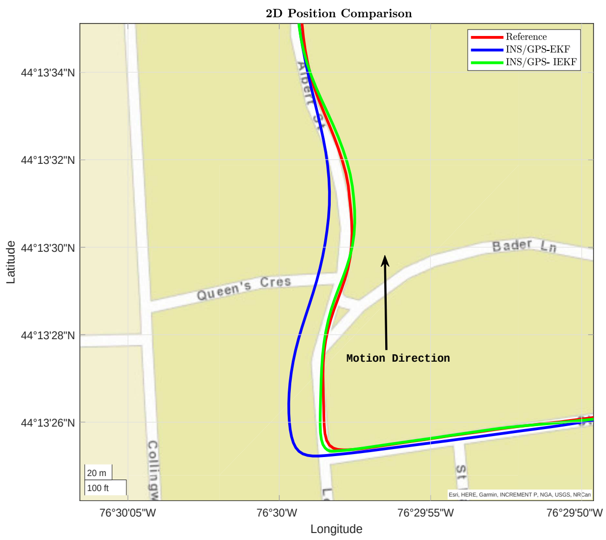

Figure 4. Second dynamic Section for Reference, EKF and LIEKF

diverges, unlike the INS/GPS-based LIEKF, which converges with the reference.

The second part is shown in figure 4 involves straight driving, a right turn and a slight right turn; then, the proposed INS/GPS-based LIEKF outperformed the traditional INS/GPS-based

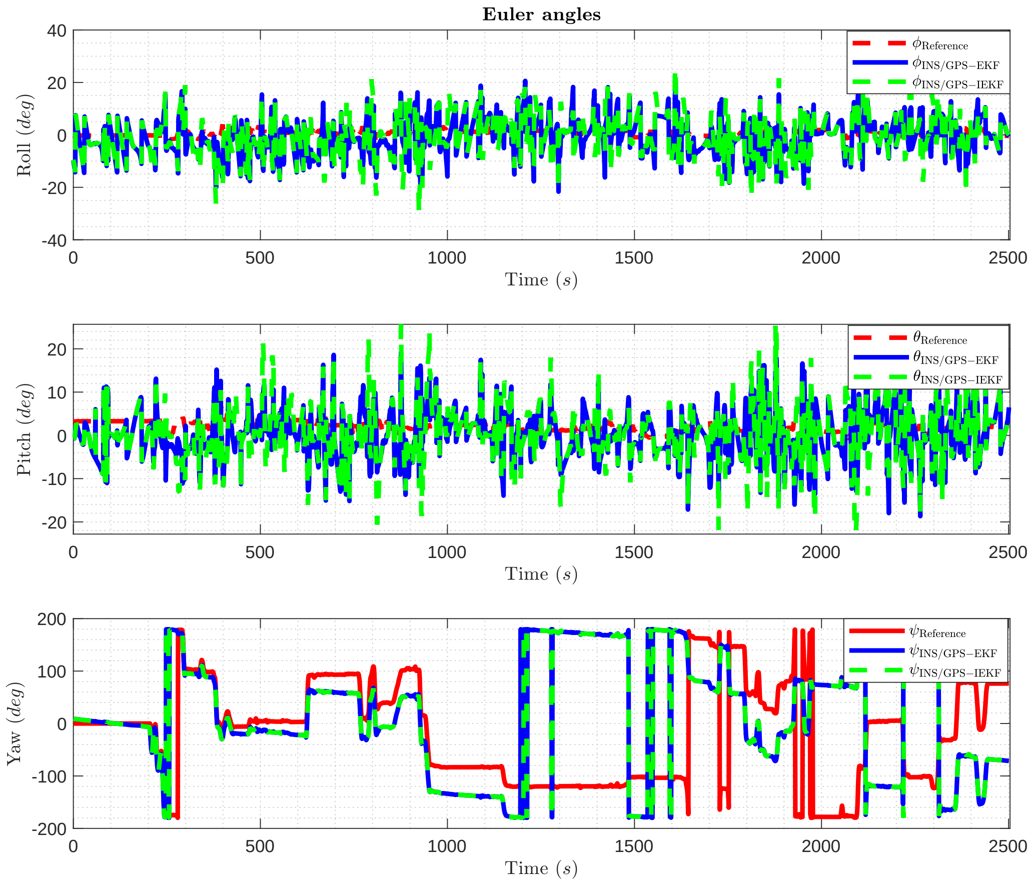

Figure 5. Eular Angels Comparison

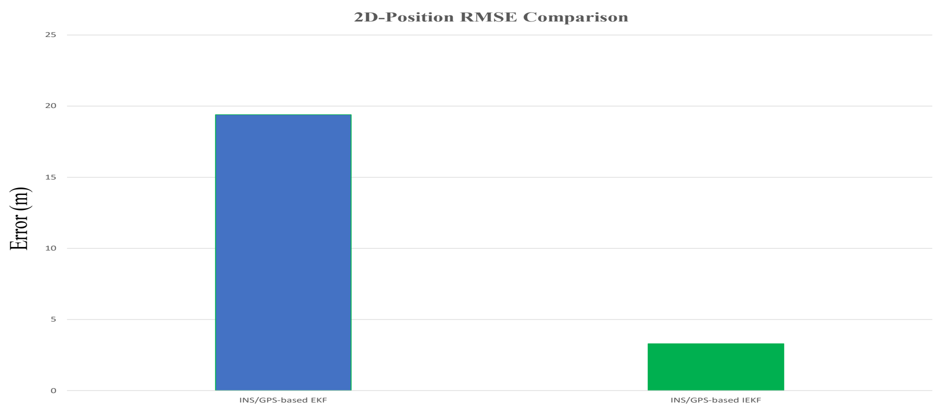

Figure 6. Bar graph for 2D-position RMSE Comparison

EKF. Moreover, the proposed system provides a better position solution with a minimum drift compared to the INS/GPS-based EKF. Moreover, when the GPS readings became available, the

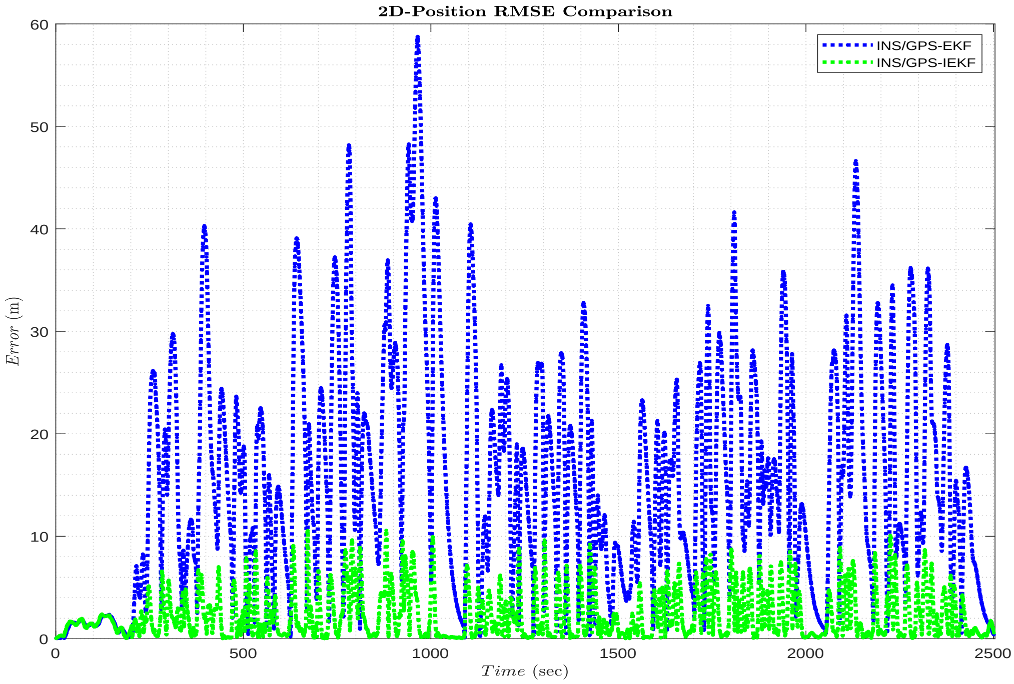

Figure 7. 2D-Position RMSE Comparison

proposed system converged directly, unlike the INS/GPS-based EKF system, which suffers from a convergence delay because of the drift error estimation.

Furthermore, from the previous figures (3, 4), the performance of the proposed INS/GPS-based LIEKF and the INS/GPS-based EKF has a close position estimation during straight driving. On the other hand, during the dynamics, the proposed integrated system provides a way more accurate position compared to the traditional system. This significant improvement is due to the capability of the proposed INS/GPS-based LIEKF in estimating the IMU associating nonlinear error components compared to the linearized EKF.

Moreover, the estimated Euler angles of the proposed INS/GPS-based LIEKF and the INS/GPS-based EKF over the whole trajectory compared to its correspondence reference are shown in figure 5. Moreover, The results in figure 5 show a small difference between the proposed INS/GPS-based LIEKF, the INS/GPS-based EKF, and the correspondence reference.

Table 2 shows the position RMS errors. Moreover, the tabulated results show a significant enhancement in the 2D position when using the proposed system, unlike the traditional system. Furthermore, the 2D-position RMS error reduced from $19.4m$ to $3.3m$ with 82.98% improvement. A further illustration of these results is shown in the bar graph in figure 6.

Finally, the results in figure 7 show 2D-Position RMSE Comparison between the proposed INS/GPS-based LIEKF and the INS/GPS-based EKF over the whole trajectory. Moreover, the overall trajectory takes about 2500 seconds with a stationary 180 seconds in the beginning, then after this period, the performance of the proposed system during the remaining trajectory produced a significant reduction in the 2D-position RMS error, which provides a great enhancement of the position information provided to the vehicle.

Table 2. 2D-Position RMSE Comparison

	INS/GPS-EKF	INS/GPS-LIEKF
E	$12.71m$	$2.21m$
N	$14.72m$	$2.45m$
2D-Position	$19.43m$	$3.32m$

5. Conclusions

LIEKF filter is applied to a loosely coupled INS/GPS integration for estimating land vehicle navigation solutions. Moreover, the proposed LIEKF showed a significant improvement in the positioning information provided to the vehicle compared to the traditional EKF, and the proposed LIEKF showed better performance during several dynamics, such as straight driving, turns, and consecutive turns at various speeds. Moreover, the results show that when using the LIEKF, the 2D-position RMS error reduced from $19.4m$ to $3.3m$ with 83% improvement. In subsequent studies, it is intended to assess the proposed approach during GPS outages and expand the update to include the velocity and the IMU sensors' measurements.

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