Journal of Physics: Conference Series

PAPER • OPEN ACCESS

Effective reduction of sidelobes in pulse compression radars using NLFM signal processing approaches

To cite this article: M G Selim *et al* 2023 *J. Phys.: Conf. Ser.* **2616** 012034

Effective reduction of sidelobes in pulse compression radars using NLFM signal processing approaches

M G Selim[1], G G Mabrouk[1], A K Elsherif[1] and A Youssef[2]

[1] Engineering Mathematics Department, Military Technical College, Cairo, Egypt.
[2] Radar Department, Military Technical College, Cairo, Egypt.

E-mail: mohamed.gamal@mtc.edu.eg

Abstract. Pulse compression techniques are widely used in modern radar systems to enhance range resolution and detection capability. As signal design is one of the basic factors of an efficient radar system, the problem of designing a radar signal with good characteristics using pulse compression is addressed in this paper. A linear frequency modulated (LFM) waveform has been widely used for conventional radars. However, it has a higher sidelobe level which makes the detection of weak targets very difficult in presence of strong targets returns and as a result the problem of masking occurs. In order to get suppressed sidelobes of radar matched filter output as well as preserve the main lobe resolution and level to overcome the problem of masking, nonlinear frequency modulated (NLFM) signals are used in modern radars. In this paper, we will introduce two different approaches to design an optimized NLFM signal which is characterized by optimized sidelobe level (SLL). The first is the exponential piecewise linear function (EPWL) which is the modification of piecewise linear (PWL) functions relying on an exponential predistortioning function. The second is the odd-term polynomial approximation (OTPA) in which the generation of NLFM signal depends only on odd-powered terms of the polynomial function. Furthermore, the simulation results of autocorrelation functions (ACF) of the proposed signals show its superiority over the traditional LFM signals and significantly enhancement compared to the recent background work. The Doppler sensitivity of the designed signals has been evaluated, revealing that the first approach offers Doppler tolerance and is suitable for surveillance radar systems, while the second approach with a lower sidelobe level is used in applications such as Synthetic Aperture Radar (SAR). Finally, the ambiguity function of the designed signals has been measured to illustrate the effect of the Doppler effect relative to different velocities.

Keywords: pulse compression, nonlinear frequency modulation, sidelobe level, exponential piecewise linear function, odd term polynomial approximation, Doppler tolerance, ambiguity function.

1. Introduction

A simple, unmodulated pulse demonstrates a relationship between energy and range resolution. Range resolution suffers when the pulse is lengthened to transmit more energy, and it also suffers when the pulse width is reduced to obtain better range resolution. Radar detection range is improved by increasing the power transmitted, however the traditional approach had issues with reliability and safety and required a huge power supply. By reducing the received signal pulse width, high resolution radar could be produced. However, as the pulse width decreases, the energy in the pulse also decreases, which reduces the radar range detection [1].

Pulse compression technique combines the benefits of high energy of long pulse and high resolution of short pulse as it transmits long-duration pulses to preserve the energy of the pulse to achieve the maximum detection range while compression is carried out at the receiver to produce a much narrower pulse width to achieve high range resolution [2].

Although linear frequency modulated signals have high sidelobe level, they achieve good ambiguity functions. The matched-filtered response of this signal has a peak sidelobe level (PSL) around -13 dB from the main lobe level. This sidelobe will make targets less detectable on radar, particularly those with weak target echoes. A solution to address this problem, is NLFM signals. Without any additional processing, such as frequency windowing or weighting, NLFM is regarded as a high-performing approach because it has a lower sidelobe level than that of LFM one[3]. NLFM signals have lately been used effectively due to their low side-lobe level, high adaptability, and variety of modulation techniques. Nevertheless, the development of NLFM is still going on today to achieve the best possible pulse compression with maintaining of PSL and Doppler shifts [4].

The idea of pulse compression using NLFM waveforms was first presented at the beginning of 1960s, after that it gained popularity in the 1990s. De Witte and Griff created Piecewise and continuous nonlinear FM waveform design which is the most widely used technique in 2004 [5]. Since the frequency deviation function in NLFM is variable, the relationship between the signal frequency and time is no longer linear. In NLFM, the changing rate of the phase of LFM waveform is effectively altered such that less amount of time is spent on the bandwidth edges.

Most authors began their approaches to pulse compression problem with the traditional rectangular linear frequency modulated (LFM) pulse. The nonlinear frequency modulated pulses are created using stationary phase principle in both the earlier work by Fowle as well as more contemporary ones [6,7]. A specific NLFM waveform shape with an instantaneous frequency which is symmetric piecewise linear (PWL) is proposed in [5]. The leading and following edges of the pulse, which are concentrated with the fastest frequency changes, where the instantaneous frequency in both articles [6,7] departs from linearity. Several works have acknowledged this since the initial work by Cook and Paolillo [8].

In this article, a technique to suppress the sidelobe level of PWL functions by using temporal predistortion FM signals is proposed. Also, we introduce a criterion to design the instantaneous frequency of NLFM signals based on polynomial approximation. Depending on this criterion using only the terms with odd powers, will give the same signal characteristics with reduced implementation complexity.

The remainder of this paper is organized as follows. In Section 2, we discuss the generation of NLFM waveform. In Section 3, the exponential piecewise linear (EPWL) function is used to generate the signal. In Section 4, an optimized polynomial is used to enhance the signal characteristics. in Section 5, the tolerance to Doppler shifts is also evaluated. in Section 6, our discussion is introduced. Finally, our conclusion and future work are given in Section 7.

2. Nonlinear frequency modulated (NLFM) waveforms design

LFM signals are popular in modern radar systems due to their ease of implementation and ability to maintain a high signal to noise ratio (SNR) through its autocorrelation function (ACF), but have a drawback of a high sidelobe level (SLL). Sidelobes are one of the main problems that affect the radar performance. Various sidelobe suppression procedures could be used to reduce SLL of the ACF obtained by different LFM waveforms [2]. Unlike the LFM waveforms, NLFM waveforms have lower sidelobe levels. In addition, if it is optimized, it will result a waveform with very low SLL and narrower mainlobe.

Numerous studies have been conducted in the theory of radar systems to provide the best NLFM signals, and all of these studies can generally be divided into two categories. One method involves designing a signal with a predefined power spectral density (PSD) function using various techniques, such as the principle of stationary phase (POSP) in which window functions are used to shape the PSD in order to get lower sidelobe level by letting the instantaneous frequency function spending less time at the edges or making a rapid change at the edges of the signal [9]. The other method is based on designing an NLFM signal using LFM signals that introduce predistortion at short intervals into a temporal domain or spectral domain [10]. According to the principle of stationary phase, the transmitted radar signal could be written as:

$$w(t) = b(t)e^{j\varphi(t)} \qquad -\frac{1}{2}T \leq t \leq \frac{1}{2}T \tag{1}$$

where $b(t)$ and $\varphi(t)$ are magnitude and phase of $w(t)$ respectively, while T is the pulse duration. The relation describing the instantaneous frequency of the pulse is described by:

$$f_I(t) = \frac{1}{2\pi}\frac{d\varphi(t)}{dt} \tag{2}$$

We assume that $f_I(t) \in (\frac{-B}{2}, \frac{B}{2})$, where B is the pulse bandwidth.

Specifically, an instantaneous frequency function of NLFM signals is regarded as a number of linear stages that are chosen and concatenated carefully such that it preserves the principle of continuity between different stages in order that it could generate nonlinear frequency function having a signal characterized by lower SLL and narrower main lobe. Such functions are called generalized PWL functions which are widely used to describe the complicated nonlinear functions [11]. There are mainly two approaches to design PWL functions, the first is the symmetric PWL functions in which the first half of the instantaneous frequency from stage 1 to stage $n/2$ is designed and the second half is directly the odd mirror of the first one as shown in Figure 1. This approach decreases the complexity of signal design as we have to design only the first half of the signal [5].

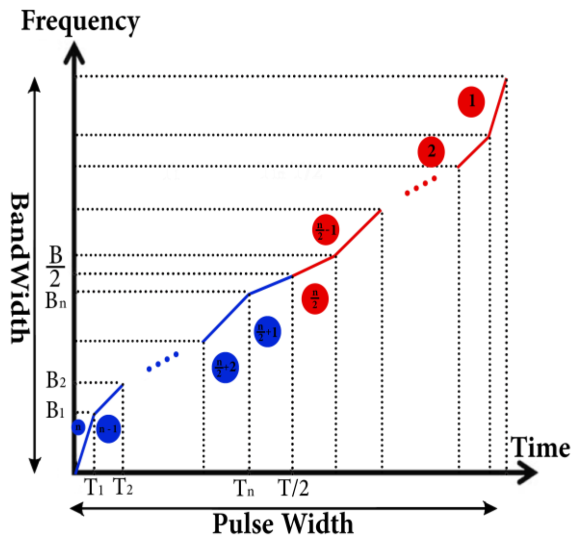


Figure 1. FM signal for n stages of PWL functions [12].

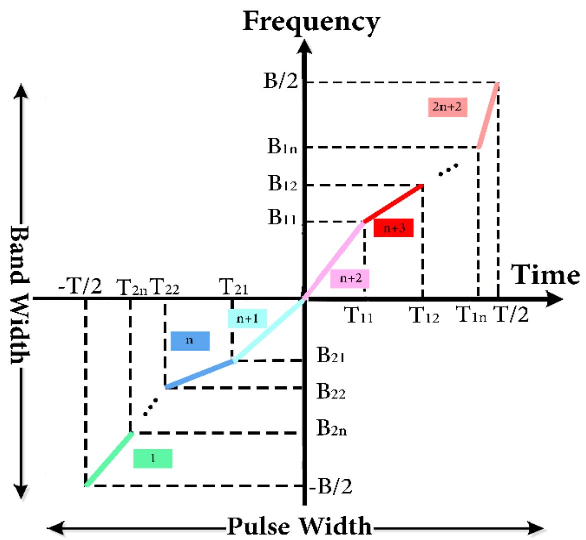


Figure 2. 2n + 2 stages for PWL functions [11].

Unlike the first method, Asymmetric PWL functions are also applied and results in very good features. Using this approach, the authors have presented a sidelobe cancelation idea by combining auto and cross correlation functions. As shown in Figure 2, the instantaneous frequency function is built using 2n + 2 stages (2n denotes the number of frequency control points) [11].

Our work will be concerned about the Tri-Stages PWL-NLFM chirp (TS-PWL-NLFM). The PWL-NLFM chirp, involves simpler mathematical calculations since it may be thought of as a concatenation of various linear frequency modulated stages with varied chirp speeds. The ideal chirp rate for each stage that when combined, can produce effective sidelobe suppression is still up for debate.

As shown in Figure 3, The traditional PWL functions have three stages of linear functions. The interval of changing of the instantaneous frequency $(-f_{max}, f_{max})$ determines the SLL at the output of ACF. The linear part of the frequency time function characteristic which is described by the frequency change interval $(-f_L, f_L)$ defines the mainlobe width [13]. The NLFM waveform designed by the instantaneous frequency of traditional PWL functions has an ACF characterized by -19 dB SLL. This high SLL could be reduced by our proposed predistortioning function as explained next.

Changing the first and the last stage from being linear to be nonlinear one using temporally pre-distorting LFM techniques is applied. The previous works stated that the predistortioning function could be either arcsine or a polynomial of degree n (t^n) and the SLL of the ACFs of the signals designed by these functions is a round -23 dB. The frequency modulation law of a temporal predistortioned FM signal can be generally written as follows:

$$f(t) = \begin{cases} f_d(t) & t \in (0, t_d] \\ f_L(t) = -\frac{F}{2} + \frac{F}{T}t & t \in (t_d, T - t_d] \\ -f_d(T - t) & t \in (T - t_d, T] \end{cases} \tag{3}$$

It should be noted that the predistortioning function is carefully selected to ensure slope continuity of the instantaneous frequency [14]. Furthermore, by increasing frequency modulation at the two edges, the nonlinearity of the frequency function curve increases by decreasing the time spent by the function in the first and last stages. This rapid change of the function as it goes up makes it able to generate enhanced NLFM signal.

In this paper, we will use another predistortion function instead of arcsine or t^n polynomial in the first and the last stages of the frequency function in order to increase the nonlinearity in the curve to reach the required S-shaped curve. This modification degrades the SLL of the designed NLFM waveform and it is explained briefly in the next section.

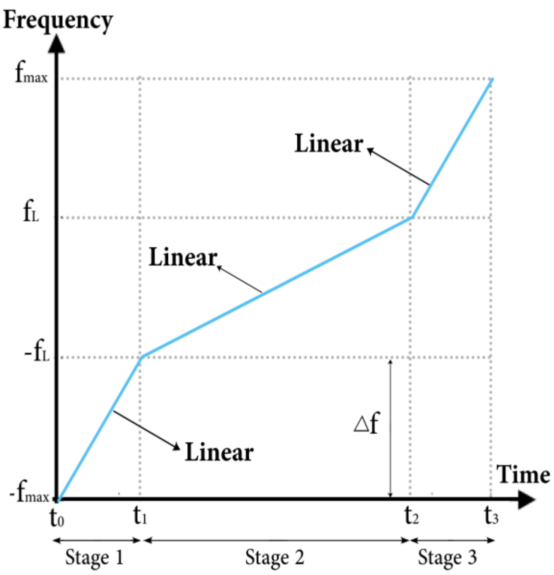


Figure 3. Instantaneous frequency of the TS-PWL-NLFM.

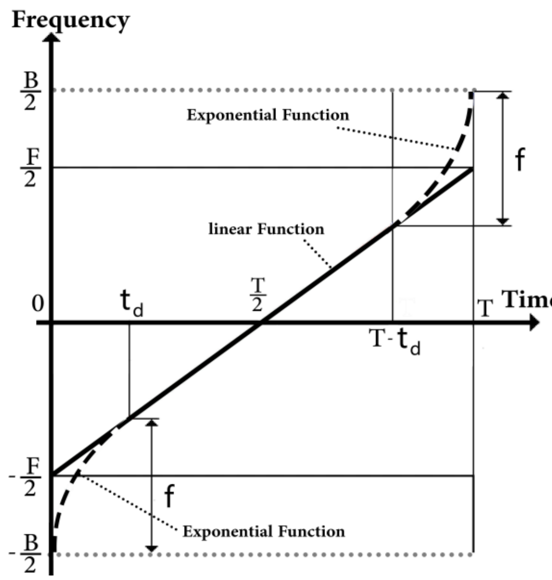


Figure 4. The LFM law's temporal predistortioning method.

Note that, we will rely on signal parameters shown in Table 1 in our simulation.

Table 1. Signal Parameters.

Name	Symbol	Value	Unit
Pulse width	T	40	µs
Bandwidth	B	16	MHz
Doppler value	f_d	5	kHz
Sampling frequency	f_s	60	MHz
Pulse repetition time	T_r	100	µs

3. Generation of the signal using exponential piecewise linear (EPWL) function

By replacing the linear parts at the first and last stages of the frequency function characteristic by our exponential function, there will be an enhancement in SLL, mainlobe width and Doppler tolerance. The explanation for this enhancement based on the fact that, the addition of small portions of higher frequency modulated (FM) rate signal at the beginning and end of the pulse is able to reduce the sidelobe level and improve radar system performance [15] as shown in Figure 4. The new frequency function results from our predistortion technique could be expressed as follows:

$$f(t) = \begin{cases} \frac{B_1}{T_1}(e^{\alpha_1 * t_1}) & t_1 \in (\frac{-T}{2}, T_1] \\ B_1 + \frac{B_2}{T_2} t_2 & t_2 \in (T_1, T_2] \\ B_1 + B_2 + \frac{B_3}{\mathrm{T}/2}(e^{\alpha_2 * t_3}) & t_3 \in (T_2, \frac{T}{2}] \end{cases} \tag{4}$$

Where t_1, t_2, and t_3 are the durations of the first, second and third stages respectively, B_1, B_2, and B_3 are the bandwidths of the first, second and third stages, respectively. Also, α_1 and α_2 are the predistortioning parameters. We set α_1= α_2= 11.4*10^4.

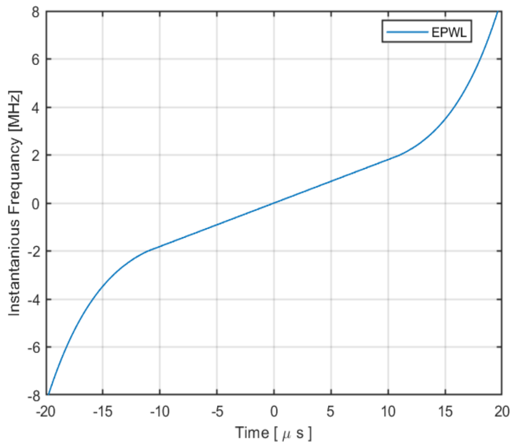


Figure 5. Instantaneous frequency using EPWL function for T = 40µs and B = 16MHz.

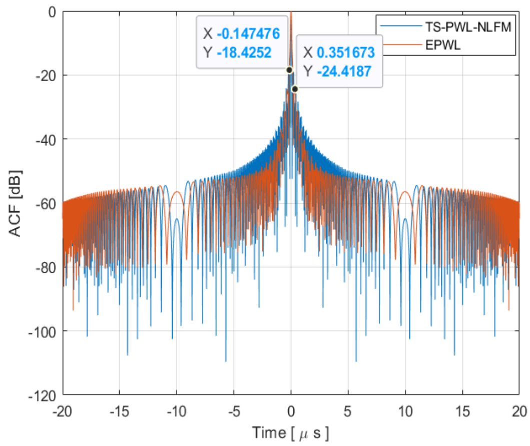


Figure 6. ACFs obtained by Instantaneous frequency using EPWL and TS-PWL-NLFM function.

The inverse of the first stage is concatenated with the other two stages to result in the instantaneous frequency function shown in Figure 5. Consequently, there will be an enhancement in SLL by -6 dB over the traditional PWL functions as shown in the ACFs of both signals in Figure 6. Besides the reduction of SLL, this signal preserves SNR, mainlobe width and it is Doppler tolerant as we will show next. The values of B_1, B_2 and B_3 should be chosen carefully to achieve continuity between different stages of the instantaneous frequency function. From simulation we can notice that, the bandwidth of the linear part (B_2) is almost twice the bandwidth of each exponential function either (B_1) or (B_3). The phase of the proposed NLFM signal is calculated by equation (2) using the frequency function obtained in this section.

Clearly, this approach has a lot of advantages including the simplicity in design, easy to be implemented, has promising SLL and mainlobe width, and has the property of being Doppler tolerant as explained in section 5.

4. Odd term polynomial approximation (OTPA) of NLFM signal

Another approach to design the proposed NLFM signal using polynomial approximation and curve fitting is introduced. Several attempts have been used to design the frequency function either by Legendre polynomials or by Taylor series approximation. The peak sidelobe ratio (PSLR) of the Legendre polynomials design-based signal and optimized by firefly algorithm (FA) was −34.45 dB but it leads to a 1.12 dB SNR loss [16]. While PSLR of Taylor series approximation for some nonlinear functions is around −39 dB with some losses in SNR [17]. Our proposed signal will depend on a polynomial approximation function to design the S-shaped instantaneous frequency that leads to a remarkable decrease in SLL. The ACF of this signal has PSLR of - 40.3 dB without losses in SNR and the other remained sidelobes converge asymptotically to -60 dB.

The instantaneous frequency function of this technique has the following form:

$$f(t) = c_0 + c_1 t + c_2 t^2 + \cdots + c_N t^N \tag{5}$$

where N is the polynomial order, and c_i where $i = 1,2, \dots, N$ represent the polynomial coefficients. In our case, the approximated polynomial will be of ninth order and it will have the form:

$$f(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4 + c_5 t^5 + c_6 t^6 + c_7 t^7 + c_8 t^8 + c_9 t^9 \tag{6}$$

We have found experimentally that if we only rely on terms with odd powers, we will have the same response. i.e., the same curve and the same SLL but with less complexity in both design time and hardware implementation. The time-frequency function of the new signal will have the form:

$$f(t) = c_1 t + c_3 t^3 + c_5 t^5 + c_7 t^7 \tag{7}$$

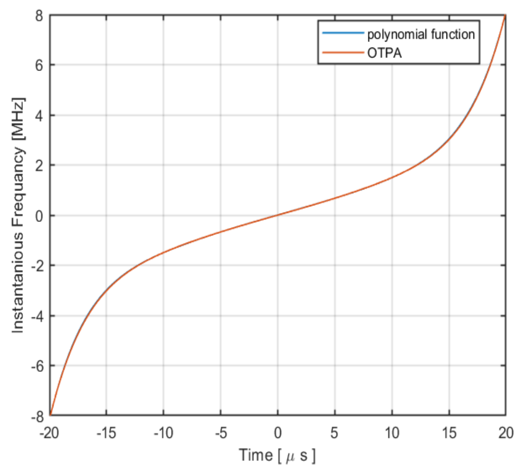


Figure 7. Instantaneous frequency of polynomial function and odd term polynomial approximation (OTPA) function.

The instantaneous frequency function in both cases, (polynomial function and odd term polynomial approximation (OTPA), are shown in Figure 7. The figure shows that the two curves are identical and confirms our assumption.

Substituting the instantaneous frequency function in equation (7) into equation (2) then integrating the result, the phase $\varphi(t)$ could be written as:

$$\varphi(t) = 2 * \pi * \left(\frac{c_1}{2}t^2 + \frac{c_3}{4}t^4 + \frac{c_5}{6}t^6 + \frac{c_7}{8}t^8\right) \tag{8}$$

By painstaking selection of the appropriate coefficients of the polynomial function in equation (7) and by using coefficients tabulated in Table 2, a very low SLL is obtained and it will be - 40.5 dB in this case. I.e., there is an enhancement of - 21dB in SLL as a result of using the odd term polynomial approximation (OTPA) function to design the NLFM signal compared to the SLL of the signal designed by traditional PWL functions. The coefficients of the proposed polynomial (polynomial I) are tabulated in Table 2.

Some important issues while generating NLFM signals using this approach that should be discussed are the hardware and time complexities. As shown in Table 2, the coefficients of the designed polynomials could hold floating numbers. Thereby, an implementation of such technique is challenging as it adds hardware complexity, although we decrease the complexity by selecting only the odd terms. Also, choosing appropriate and optimized coefficients adds time complexity as it could take a lot of time to choose the best coefficients that lead to the lowest SLL and narrowest mainlobe or to compromise both SLL and mainlobe width. Another designed signal (Polynomial II) is proposed. Such signal is characterized by ACF with SLL around -31dB but it has narrower mainlobe than that of polynomial I. This signal is used in radar system applications in which the mainlobe width is of great priority. Also, it could be used when a slight increase in SLL is accepted. The system operator could choose the best designing technique depending on the radar system application i.e., choosing such technique is a tradeoff between system application and required signal characteristics.

The coefficients of the designed polynomials, SLL, and mainlobe width of the ACF in both cases are tabulated as follows:

Table 2. characteristics of two designed polynomials.

Designed function	coefficients	SLL	Mainlobe width (μs)
Polynomial I	c_1= 12.9891*10^10 c_3= 19.0112*10^19 c_5= -3.4255*10^29 c_7= 3.9982*10^39	- 40.2585 dB	0.054545*
Polynomial II	c_1= 20.9875*10^10 c_3= 19.012*10^19 c_5= -4.9035*10^29 c_7= 2.1017*10^39	-30.5195 dB	0.044457*

* Compared to the mainlobe width of LFM signals and traditional PWL functions which are 0.02899μs and 0.0681μs, respectively.

The ACFs of the NLFM signals, designed by OTPA relying on Table 1 and Table 2 are shown in Figure 8.

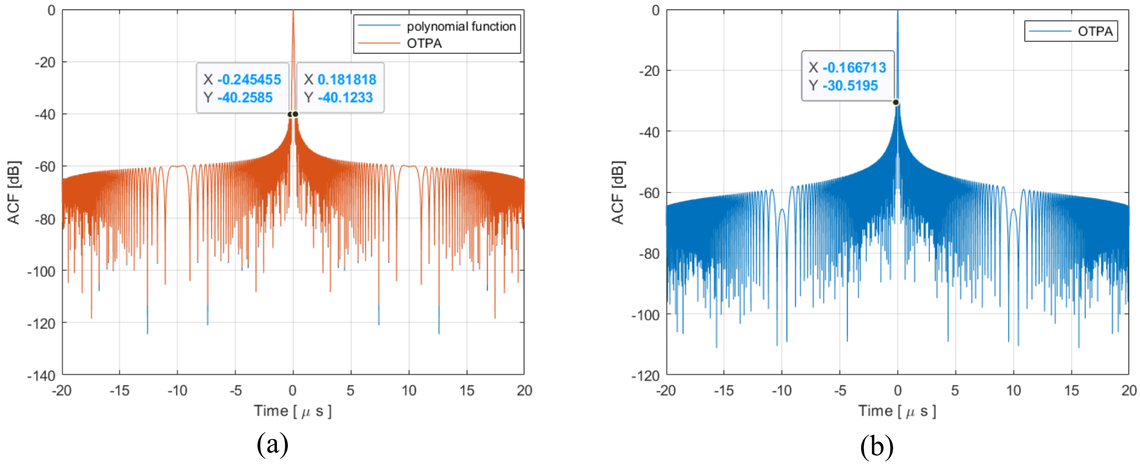


Figure 8. ACFs of the designed signals using OTPA function. (a) with priority of SLL (Polynomial I) and it is designed by the two approaches, polynomial function and OTPA. (b) with priority of mainlobe width (Polynomial II).

The measurements of the impulse response width (IRW) and integrated sidelobe Ratio (ISLR) for the designed waveforms compared to that of the LFM waveform are tabulated as follows:

Table 3. measurements of IRW and ISLR of the designed waveforms.

Waveform	IRW	ISLR
LFM	8.697 m	-9.6948 dB
EPWL	11.43 m	-18.4543 dB
OTPA	16.3635 m	-30.4091dB

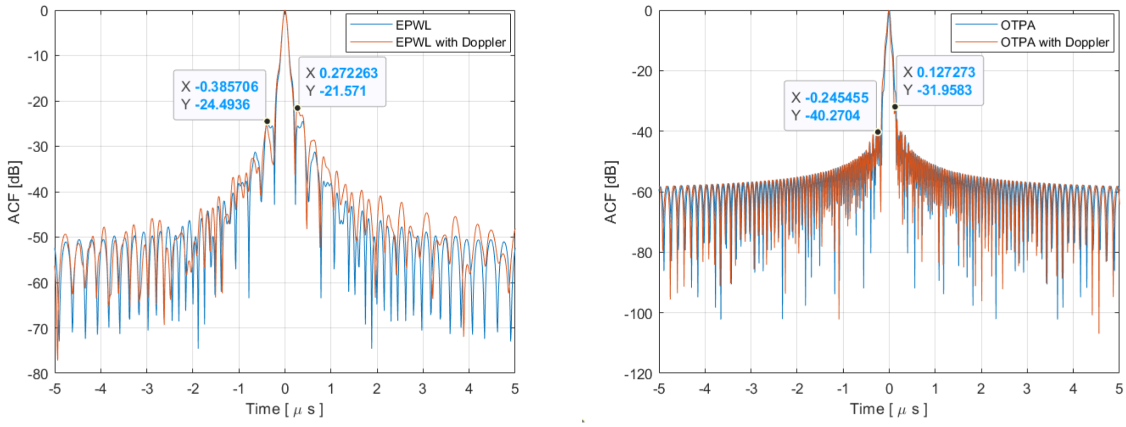
5. Effect of Doppler Shift

Consider the case in which there is a Doppler shift affecting the received signal due to the relative target-radar motion. Figure 9 shows the ACFs of NLFM signals designed by the two approaches discussed in this paper in presence of Doppler shift. It is clearly that, the SLL of the ACF of the signal designed by the EPWL function increased by 3 dB under the effect of Doppler shift. Also, some small notches in the ACF appear next to the mainlobe as shown in Figure 9 (a). However, the mainlobe width is not broadened so this signal could be regarded as Doppler tolerant.

Figure 9 (b) shows that the SLL of the ACF of the signal designed by the OTPA function increased by around 9 dB in presence of Doppler shift so this signal degraded under Doppler effect more than the waveform designed by EPWL function. In addition to the increase of SLL, is the appearance of a small notch and a local maximum on one of the two descending sides of the mainlobe with preserving the mainlobe width.

Further illustrations of the quality of the proposed signals are added by measuring the ambiguity function (AF). The AF, $X(\tau, f_d)$, for a waveform f(t) is defined as the mutual correlation between the waveform and its delayed and Doppler-shifted version and it could be expressed mathematically as:

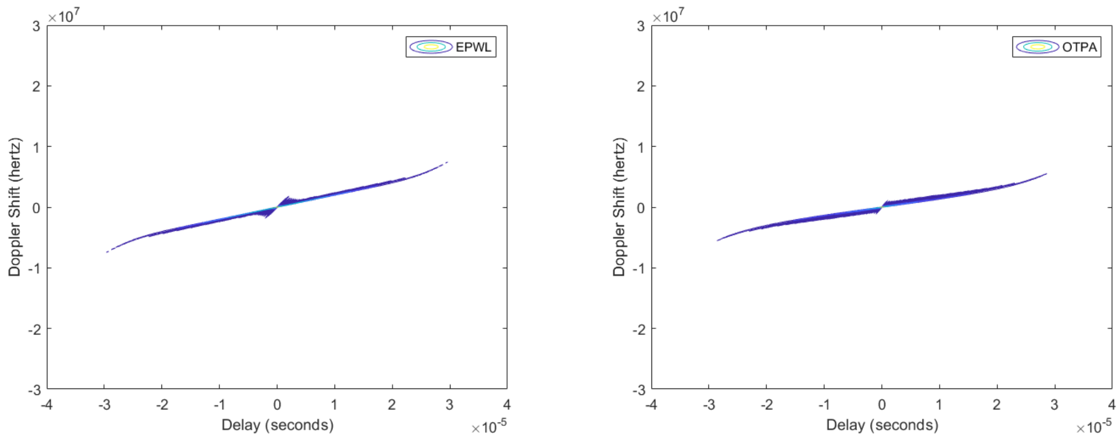
$$X(\tau, f_d) = \int_{-\infty}^{\infty} f(t) f^*(t+\tau) \exp(j2\pi f_d t) \mathrm{d}t \tag{9}$$



(a) (b)

Figure 9. ACFs of the designed signals in presence of Doppler shift. (a) EPWL function. (b) OTPA function.

Where τ is the delay. Radar designers typically utilize the radar AF to analyze and evaluate various waveforms. It can shed light on how various radar waveforms may be suitable for various radar applications [14]. For a certain radar waveform, it is also used to calculate the range and Doppler resolutions. The top view AF (contour) of the two designed signals is shown in Figure 10.



(a) (b)

Figure 10. Top view AF of the designed signals. (a) EPWL function. (b) OTPA function.

As shown in Figure 10, the two designed signals are in the shape of an oblique blade and it confirms our results. Figure 10 (a) shows the signal that designed by EPWL function has good Doppler coupling over wide range of Doppler frequencies with small SLL degradation at the two edges of the AF and hence it demonstrates good Doppler tolerance. However, although the signal generated by OTPA also offers good Doppler coupling over the same range of Doppler frequencies, its SLL degradation is higher compared to that of EPWL, as shown in Figure 10 (b).

6. Discussion

EPWL function approach results in an ACF characterized by almost -25 dB which is lower than that of the traditional PWL functions. Also, this approach has the advantage of easy implementation and less complexity. Furthermore, it is doppler tolerant because it preserves its features in presence of Doppler shift. The SLL of the ACFs of these signals raised by 3 dB under the effect of Doppler shifts.

The second approach is the OTPA which has an ACF with very low SLL almost - 40.5 dB but it asserts some complexities in hardware implementation and time design. This signal is affected by the Doppler shift more than that of the first approach as its ACF has 9 dB increase in SLL. However, it is still having lower SLL than that of the EPWL function technique. Both signals have the advantage of generating the proposed waveform with no losses in SNR.

7. Conclusion and Future Work

In this paper, two approaches for designing NLFM signals have been discussed. The first approach is the predistortioning technique which is called EPWL function and it is a modification of the traditional method of generating NLFM signals using PWL functions by replacing the first and the last stages in PWL functions which is linear by an exponential function. By doing so, an increase in frequency modulation at the beginning and the ending edges of the frequency-time function is achieved and this leads to an enhancement in SLL by almost -6 dB. The generated waveform is regarded to be Doppler tolerant as its SLL increases by only 3 dB. This technique has the advantage of simple and easy implementation. The second approach which is relying on the OTPA function is also introduced. This approach could suppress the SLL of the designed signal to be lower than -40 dB. There are some challenges while generating signals using this technique such as setting the polynomial coefficients, hardware implementation, time design and Doppler tolerance. The possible further improvements that could be achieved by applying some optimization techniques to the generated signals will be matter for future research.

References

[1] Kurdzo J M, Cheong B L, Palmer R D and Zhang G 2014 Optimized NLFM pulse compression waveforms for high-sensitivity radar observations *2014 International Radar Conference* pp 1-6

[2] Vizitiu, I C, Enache F and Popescu F 2014 Sidelobe reduction in pulse-compression radar using the stationary phase technique: An extended comparative study. *2014 International Conference on Optimization of Electrical and Electronic Equipment (OPTIM)* pp 898-901.

[3] Yichun P, Shirui P, Kefeng Y and Wenfeng D 2005 Optimization design of NLFM signal and its pulse compression simulation *IEEE International Radar Conference* pp 383-6

[4] Rohman B P, Indrawijaya R, Kurniawan D, Heriana O and Wael C B A 2016 Sidelobe suppression on pulse compression using curve-shaped nonlinear frequency modulation *1st International Conference on Information Technology, Information Systems and Electrical Engineering (ICITISEE)* pp 49-53

[5] Griffiths H D and Vinagre L 1994 Design of low-sidelobe pulse compression waveforms *Electronics Letters* **30** pp 1004-1005

[6] Zhiqiang G, Peikang H and Weining L 2008 Matched NLFM pulse compression method with ultra-low sidelobes *the 5th European Radar Conference* pp 92-95

[7] Zhang Y, Wang W, Wang R, Deng Y, Jin G and Long Y 2020 A Novel NLFM Waveform with Low Sidelobes Based on Modified Chebyshev Window *IEEE Geoscience and Remote Sensing Letters* **17** pp 814-8

[8] Argenti F and Facheris L 2021 Radar Pulse Compression Methods Based on Nonlinear and Quadratic Optimization *IEEE Transactions on Geoscience and Remote Sensing* **59** pp 3904-16

[9] Ghavamirad R, Babashah H and Sebt M A 2017 Nonlinear FM Waveform Design to Reduction of sidelobe level in Autocorrelation Function *Iranian Conference on Electrical Engineering (ICEE)* pp 1973-1977

[10] Adithya V N and Elizabath R D 2018 Modified PWNLFM Signal for Side-Lobe Reduction *International Journal of Engineering & Technology*

[11] Jin G, Deng Y, Wang R, Wang W, Wang P, Long Y, Zhang Z M and Zhang Y 2019 An Advanced Nonlinear Frequency Modulation Waveform for Radar Imaging with Low Sidelobe *IEEE Transactions on Geoscience and Remote Sensing* **57** pp 6155–68
[12] Saeedi J and Faez K 2016 Synthetic aperture radar imaging using nonlinear frequency modulation signal *IEEE Transactions on Aerospace and Electronic Systems* **52** pp 99-110
[13] Septanto H, Sudjana O and Suprijanto D 2022 A Novel Rule for Designing Tri-Stages Piecewise Linear NLFM Chirp *International Conference on Radar, Antenna, Microwave, Electronics, and Telecommunications (ICRAMET)* pp 62-7
[14] Vizitiu I C 2014 Some Aspects of Sidelobe Reduction in Pulse Compression Radars Using NLFM Signal Processing *Progress in Electromagnetics Research* **47** pp 119-129.
[15] De Witte E and Griffiths H D 2004 Improved ultra-low range sidelobe pulse compression waveform design *Electronics Letters* **40** pp 1448–50
[16] Xu Z, Wang X and Wang Y 2022 Nonlinear Frequency-Modulated Waveforms Modeling and Optimization for Radar Applications *Mathematics* **10** p 3939
[17] Swiercz E, Janczak D and Konopko K 2022 Estimation and Classification of NLFM Signals Based on the Time–Chirp Representation *Sensors* **22** p 8104