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Performance Analysis of OFDM System over LEO Satellite Channel

By

A. El-Mahdy

A. Mahran

and M. Karem

Abstract

In this paper an exact closed-form expression for the performance of the orthogonal frequency-division multiplexing (OFDM) system over LEO satellite channel is derived where each subcarrier is modulated by M-QAM. The effect of two channel parameters, namely the mean delay spread and maximum Doppler shift, on the system performance is investigated for a realistic wireless channel environment model.

1. Introduction

Satellite mobile communication has gained enormous attentions in the wake of third-generation (3G) and fourth-generation (4G) wireless communications systems. The central features of the future fourth-generation mobile communication systems are the provisioning of high-speed data transmissions (up to 1 Gb/s) and interactive multimedia services. Because of its resistivity to intersymbol interference (ISI), the OFDM system is a strong candidate for high-data-rate. OFDM system is used in many recent standards, such as IEEE 802.11, IEEE 802.16, and DVB-T. Moreover, the OFDM system is used in satellite mobile communication as a modulation scheme.

Satellite communication systems, especially non-geostationary satellite systems are the best candidates for providing communication services globally in a cost effective manner which will be used in 4G. Non-geostationary satellite systems will form a mobile telephony and data transmission network that would work without the need for complex ground-based infrastructures. Therefore, the cost of the installation is fixed with no relationship between cost and distance. For example, linking every home to internet with fiber links costs 300 billion dollars while via satellite it costs only 9 billion dollars [1]By using Low Earth Orbital (LEO) Satellite Systems the power requirements

in either on-board and on the ground, can be reduced. This results in lightweight low power radio telephones with small low profile antennas. Nonetheless, low altitude means minimized transmission delay to nearly equals that of the land-based networks. A more detailed description of Mobile Satellite Communication Systems can be found in [2].

A careful literature survey reveals the papers [3], [4], [5] and [6] which studied the performance of OFDM over LEO satellite channel. In [3], a comparison studies of the uplink performance of wideband code division multiple access (W-CDMA) and OFDM systems is presented for mobile multimedia communications via LEO satellites. In [4], the characteristics of interchannel interference (ICI) distortions are studied under fast fading LEO satellite channel as well as the effect of Doppler frequency shift on ICI. A theoretical analysis for a 4-QAM modulation based CDMA system in the presence of a nonlinear distortion over satellite mobile channel is studied in [5]. An average error probability for M-QAM modulation over a Rayleigh fading LEO satellite communication system is evaluated in [6] using finite state Markov channel.

In this paper, the performance of the OFDM-QAM system in LEO Satellite channel is investigated. The performance measure is the average probability of error. The effect of the number of the subcarrier, the guard interval, and the Doppler frequency on the performance is also studied. The paper is organized as follows. In section 2, the system model is described. a closed form of the expression of the probability of error of the OFDM-QAM system is derived in section 3. In section 4, simulations are carried out to validate the theoretical expression. Conclusions will be figured out in section 5.

2. System Model

The baseband equivalent discrete-time complex-valued model of the considered system is depicted in Fig 1. The source generates a sequence of serial data which is modulated with 4-QAM complex symbols with duration T . These symbols are fed to a serial-to-parallel converter to be converted to blocks of data with length NT . The set of symbols in each block, $\{0 \leq a_i(n) \leq N - 1\}$, is modulated by N -point IFFT onto N symbols of the i -th block, where each point n in IFFT represents the index of the carrier of OFDM system, and N is the total number of carrier. The IFFT saves the time of calculation and hardware implementation size. The cyclic prefix block adds the guard interval to reduce the ISI between blocks. This can be done by copying the first G elements and add them in the rare of the i^{th} block. The final stage in transmitter is the parallel-to-serial converter before transmitting the data over a LEO satellite channel.

In the receiver, the serial data is converted to parallel again using the serial-to-parallel converter. Following this, the guard interval is removed and the symbols are fed to N -point FFT block. The output from the FFT block is demodulated using 4-QAM demodulator and the data is converted from parallel to serial.

The LEO satellite channel, is classified as a wide-sense-stationary uncorrelated scattering (WSSUS) frequency selective Rayleigh fading channel. It is modeled as a tapped delay line model with time-varying coefficients and fixed tap spacing T_s , where T_s , is the sampling time. One of the advantages of the OFDM system is that it treats the frequency selective channel as a flat fading. The details of this fact are explained in [3]. In flat fading, the received instantaneous signal power is attenuated by α^2 , where α is the fading amplitude.

3. Performance Analysis of the System

In this section, the performance of the OFDM over LEO satellite channel is evaluated in terms of the average probability of error as a function of the instantaneous SNR per bit (γ). First we start by deriving γ and then the average BEP.

A. Derivation of average SNR per bit (γ).

The output symbols from modulator (MQAM) are assumed to be independent and identically distributed (i.i.d.), which is modulated by IFFT. The transmitted signal of the system before adding the guard interval can be written as:

$$s_i(k) = \frac{1}{\sqrt{N}} \sum_n^{N-1} a_i(n) e^{j \frac{2\pi nk}{N}}, \quad 0 \leq k \leq N-1 \quad (1)$$

where k represents the index of the discrete time, after a guard interval is appended to reduce the ISI between blocks, the transmitted sequence of the i^{th} block can be written as

$$s_i^g(k) = \frac{1}{\sqrt{N}} \sum_n^{N-1} a_i(n) e^{j \frac{2\pi nk}{N}}, \text{ for } -G \leq k \leq N-1 \quad (2)$$

If $s_i^g(k)$ is assumed to be zero for $k \geq N$, then:

$$s(k) = \frac{1}{\sqrt{N}} \sum_{i=-\infty}^{\infty} s_i^g(k - i(N + G)) \quad (3)$$

The received signal is distorted by the AWGN and the fading channel. Then, after removing the guarded interval, it can be written as

$$r(k) = \frac{1}{\sqrt{N}} \sum_{i=-\infty}^{\infty} \sum_{m=0}^M h_m(k) s_i^g(k - m - i(N + G)) + n(k) \quad (4)$$

where $h_m(k)$ is the tap coefficient of the channel, m represents the delay index caused by the imperial channel, and M is the maximum index of the delay spread time caused by the channel. In the following, we assume the worst case where the delay is longer than the guard interval, that is $MT_s \geq GT_s$ and also we assume that there exist ISI between, i^{th} and $(i^{\text{th}} - 1)$ block, therefore equation (4) can be rewritten as:

$$r_i(k) = \begin{cases} \sum_{m=0}^{k+G} h_{m,i}(k) s_i^g(k - m) \\ + \sum_{m=k+G-1}^M h_{m,i}(k) s_{i-1}^g(N + G - k + m) & \text{if } 0 \leq k \leq M - G - 1 \\ \sum_{m=0}^{M-1} h_{m,i}(k) s_i^g(k - m), & \text{if } M - G \leq k \leq N - 1 \end{cases} \quad (5)$$

after demodulation by FFT, $R_i(l) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} r_i(k) e^{j \frac{2\pi nl}{N}}$ or

$$\begin{aligned}
 R_i(l) = & \frac{1}{N} \sum_n^{N-1} a_i(n) \left[\sum_{k=0}^{M-G-1} \sum_{m=0}^{k+G} h_{m,i}(k) e^{-j\frac{2\pi nm}{N}} e^{j\frac{2\pi k(n-l)}{N}} \right. \\
 & + \left. \sum_{k=M-G}^{N-1} \sum_{m=0}^{M-1} h_{m,i}(k) e^{-j\frac{2\pi nm}{N}} e^{j\frac{2\pi k(n-l)}{N}} \right] \\
 & + \frac{1}{N} \sum_{n=0}^{N-1} a_{i-1}(n) \sum_{k=0}^{M-G-1} \sum_{m=k+G+1}^{M-1} h_{m,i}(k) e^{-j\frac{2\pi n(G-m)}{N}} e^{j\frac{2\pi k(n-l)}{N}} \\
 & + \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} n_i(k) e^{-j\frac{2\pi kl}{N}} \text{ for } 0 \leq l \leq N-1 \quad (7)
 \end{aligned}$$

The FFT output (7) can be separated into the signal component with multiplicative noise, interferences, and AWGN, as

$$R_i = H_i(l)a_i(l) + C_i(l) + S_i(l) + N_i(l) \text{ for } 0 \leq l \leq N-1 \quad (8)$$

where $N_i(l)$ is the AWGN of the l -th sub channel, $H_i(l)$ is the fading term over the l -th sub channel and it is given by

$$H_i(l) = \frac{1}{N} \left[\sum_{k=0}^{M-G-1} \sum_{m=0}^{k+G} h_{m,i}(k) e^{-j\frac{2\pi lm}{N}} + \sum_{k=M-G}^{N-1} \sum_{m=0}^{M-1} h_{m,i}(k) e^{-j\frac{2\pi lm}{N}} \right] \quad (9)$$

This fading term has a zero-mean complex Gaussian random variable, i.e., each subchannel is a Rayleigh fading channel. $S_i(l)$ is the interference from the adjacent blocks (ISI), and it is given by

$$S_i(l) = \frac{1}{N} \sum_n^{N-1} a_{i-1}(n) \sum_{k=0}^{M-G-1} \sum_{m=k+G+1}^{M-1} h_{m,i}(k) e^{-j\frac{2\pi n(G-m)}{N}} e^{j\frac{2\pi k(n-l)}{N}} \quad (10)$$

The ICI term, $C_i(l)$, can be written as

$$C_i(l) = \frac{1}{N} \sum_{n \neq l} a_i(n) \left[\sum_{k=0}^{M-G-1} \sum_{m=0}^{k+G} h_{m,i}(k) e^{-j \frac{2\pi l m}{N}} e^{j \frac{2\pi k(n-l)}{N}} + \sum_{k=M-G}^{N-1} \sum_{m=0}^{M-1} h_{m,i}(k) e^{-j \frac{2\pi l m}{N}} e^{j \frac{2\pi k(n-l)}{N}} \right] \quad (11)$$

The terms $C_i(l)$ and $S_i(l)$ are modeled as Gaussian random variables.

Under the isotropic scattering assumption, the autocorrelation of the tap coefficients is $E\{h_{m,i}(k)h_{m,i}^*(u)\} = \sigma_H^2 J_0(2\pi f_D T_s(k-u))$ [6] where f_D is maximum Doppler frequency which related to the velocity of the user v and the carrier frequency f_c , the variance of $C_i(l)$ can then be derived as $\sigma_C^2 = E\{C_i(k)C_i^*(u)\}$, The variances of $H_i(l)$ and $S_i(l)$ can similarly be calculated, then the average signal-to-noise ratio (SNR) is

$$\bar{\gamma} = \frac{E_S \sigma_H^2}{\sigma_C^2 + \sigma_S^2 + N_0} = \frac{\sigma_H^2}{\sigma_C^2 + \sigma_S^2 + \frac{N_0}{E_S} (1 + \frac{G}{N})} \quad (12)$$

where N_0 , is the one-sided power spectral density of the AWGN, σ_S^2 is the variance of ICI, and σ_H^2 is the variance of fading term $H_i(l)$.

B. Derivation of the average probability of Error

Now, the average probability of error of the M-QAM OFDM system is derived. First let us define some expressions: The Gaussian function Q can be written in the form

$$Q(x) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(\frac{-x^2}{2\sin^2(\theta)}\right) d\theta \quad [\text{Eq}(4.2),7]$$

$$\text{and } Q^2(x) = \frac{1}{\pi} \int_0^{\pi/4} \exp\left(\frac{-x^2}{2\sin^2(\theta)}\right) d\theta \quad [\text{Eq}(4.9),7],$$

The power density function (pdf) of the instantaneous SNR per bit for Rayleigh fading channel is given by

$$P_\gamma(\gamma) = \frac{1}{\bar{\gamma}} \exp\left(\frac{-\gamma}{\bar{\gamma}}\right) \quad \gamma \geq 0 \quad (13)$$

To compute the average error probability it should evaluate an integral whose integrand consist of the product of Q-function and fading pdf

$$I = \int_0^\infty Q(a\sqrt{\gamma}) P_\gamma(\gamma) d\gamma \quad (14)$$

Since the moment generating function (MGF) of γ is the Laplace transform of $P_\gamma(\gamma)$ with the exponent reversed in sign [8] i.e,

$$M_\gamma(s) = \int_0^{\pi/2} e^{s\gamma} P_\gamma(\gamma) d\gamma \quad (15)$$

So (14) could be written as

$$I = \frac{1}{\pi} \int_0^{\pi/2} M_\gamma\left(\frac{-a}{2\sin^2(\theta)}\right) d\theta \quad (16)$$

The Laplace transfer of the Rayleigh PDF can be evaluated in closed form as

$$M_\gamma(-s) = \frac{1}{1 + s\bar{\gamma}} \quad s > 0 \quad (17)$$

By substitution of (17) into (16) we have

$$\begin{aligned} I(a, \bar{\gamma}) &= \frac{1}{\pi} \int_0^{\pi/2} \left(1 + \frac{a^2\bar{\gamma}}{2\sin^2(\theta)}\right)^{-1} d\theta \\ &= \frac{1}{2} \left(1 - \sqrt{\frac{\frac{a^2\bar{\gamma}}{2}}{1 + \frac{a^2\bar{\gamma}}{2}}}\right) \end{aligned} \quad (18)$$

Similarly when $I = \int_0^\infty Q^2(a\sqrt{\gamma}) P_\gamma(\gamma) d\gamma$, we have

$$I = \frac{1}{4} \left[1 - \sqrt{\frac{\frac{a^2\bar{\gamma}}{2}}{1 + \frac{a^2\bar{\gamma}}{2}}} \left(\frac{4}{\pi} \tan^{-1} \sqrt{\frac{1 + \frac{a^2\bar{\gamma}}{2}}{\frac{a^2\bar{\gamma}}{2}}} \right) \right] \quad (19)$$

For M-QAM the symbol error probability (SEP) over the AWGN is the SEP for \sqrt{M} -AM, since a QAM modulation is composed of the quadrature combination of two \sqrt{M} -AM modulation, each with half the total power. So the probability of correct symbol decision for QAM can be expressed

$$P_s(c)|_{E_s^{M-QAM}} = \left[P_s(c)|_{\frac{E_s}{2}^{\sqrt{M}-AM}} \right]^2 \quad (20)$$

and the probability of error is

$$P(E) = 1 - P(c) \quad (21)$$

$$P_s(E) = 1 - 1 + 2P_s|_{\frac{E_s}{2}^{\sqrt{M}-AM}}(E) - P_s^2|_{\frac{E_s}{2}^{\sqrt{M}-AM}}(E) \quad (22)$$

Then, the SEP for symmetrical M-AM is given by

$$P_s(E) = 2 \left(\frac{M-1}{M} \right) Q \left(\sqrt{\frac{6E_s}{N_0(M^2-1)}} \right) \quad (23)$$

By substitution of (23) into (22) we have

$$P_s(E) = 4 \left(\frac{\sqrt{M}-1}{\sqrt{M}} \right) Q \left(\sqrt{\frac{3E_s}{N_0(M-1)}} \right) - 4 \left(\frac{\sqrt{M}-1}{\sqrt{M}} \right)^2 Q^2 \left(\sqrt{\frac{3E_s}{N_0(M-1)}} \right) \quad (24)$$

By replacing $\frac{E_s}{N_0} = \gamma \log_2 M$, we have

$$P_s(E; \gamma) = 4 \left(\frac{\sqrt{M}-1}{\sqrt{M}} \right) Q \left(\sqrt{\frac{3 \log_2 M}{N_0(M-1)}} \sqrt{\gamma} \right) - 4 \left(\frac{\sqrt{M}-1}{\sqrt{M}} \right)^2 Q^2 \left(\sqrt{\frac{3 \log_2 M}{N_0(M-1)}} \sqrt{\gamma} \right) \quad (25)$$

$$P_s(E) = \int_0^{\infty} P_s(E; \gamma) P_\gamma(\gamma) d\gamma \quad (26)$$

Finally, using (18), (19), (25) and (26), the average probability of error is given by

$$P_s(E) = 2 \left(\frac{\sqrt{M}-1}{\sqrt{M}} \right) \left(1 - \sqrt{\frac{1.5\bar{\gamma}_s}{M-1+1.5\bar{\gamma}_s}} \right) - \left(\frac{\sqrt{M}-1}{\sqrt{M}} \right)^2 \left[1 - \sqrt{\frac{1.5\bar{\gamma}_s}{M-1+1.5\bar{\gamma}_s}} \left(\frac{4}{\pi} \tan^{-1} \sqrt{\frac{M-1+1.5\bar{\gamma}_s}{1.5\bar{\gamma}_s}} \right) \right] \quad (27)$$

Where $\gamma_s = \gamma_s \log_2 M$

4. Numerical Results

In this section the performance of the system is evaluated for a 4-QAM OFDM system. The parameters which are used in the numerical evaluation of the system are: $T_s = 5 \mu\text{sec}$, $T_m = 5T_s$ and the mean delay τ_0 is T_s . The average signal to noise ratio $\bar{\gamma}$ as a function of the Doppler frequency (f_D) is shown in Fig.2 for various values of T_G . This figure shows that, when $T_G=T_m$ the ISI from the adjacent block is eliminated completely and only the ICI remains. In such a case, the average SNR decreases rapidly with f_D .

The BER for the 4-QAM OFDM system is shown in Fig, 3 and Fig. 4 for various values of N . Fig. 3 is plotted for $\text{SNR}=\infty$ and Fig. 4 is plotted for $\text{SNR}=15 \text{ dB}$. The results of these figures are obtained for a carrier frequency of 910 MHz, resulting in maximum Doppler frequencies of 5 Hz and 90 Hz for a person walking at 6 km/h and a vehicle moving at 100 km/h, respectively. In these figures, two extreme cases are considered. The first one when no guard interval is added ($T_G = 0$) and the second one when the guard interval is used. The figures show that the guard interval eliminates the ISI perfectly ($T_G = T_m$).

5. Conclusion

In this paper, an exact method for calculating the BER of an M-QAM OFDM system over LEO satellite channels has been developed which led to a closed-form BER expression. The exact BER expression was used to analyze the system performance under several wireless channel configurations. The simulation results of the derived closed-form expression are consistent with the published results in [8] As expected, the results show that the Doppler frequency causes the significant performance degradation.

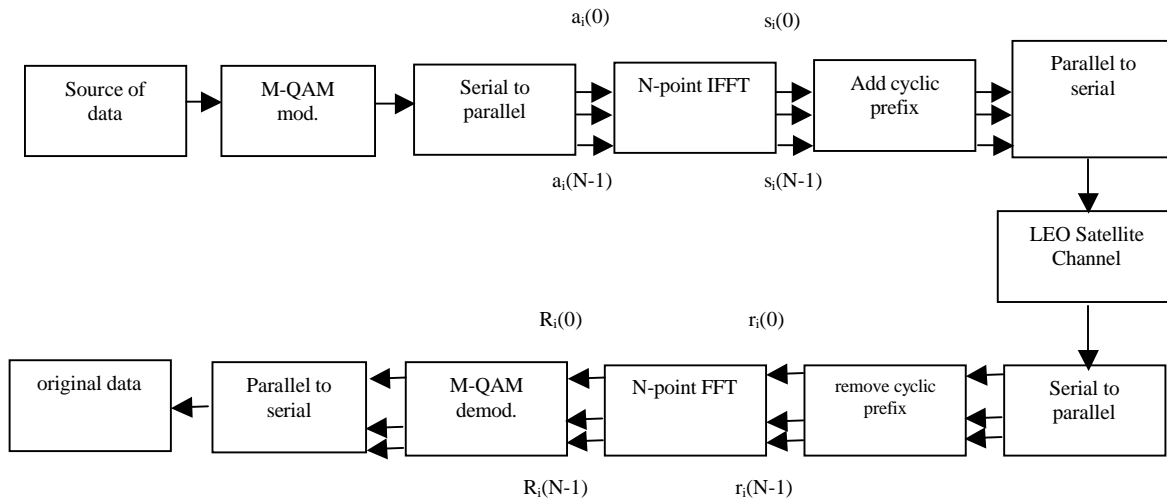


Fig.1. the baseband equivalent discrete-time system model

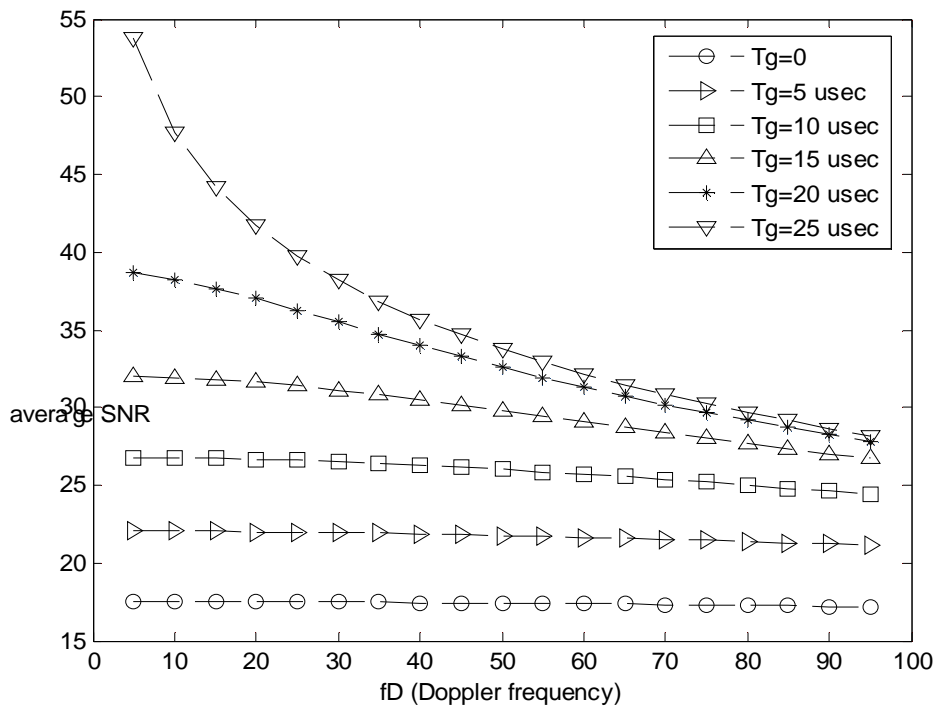


Fig. 2. Average SNR versus f_D when $N=64$ with different values of guard interval T_g

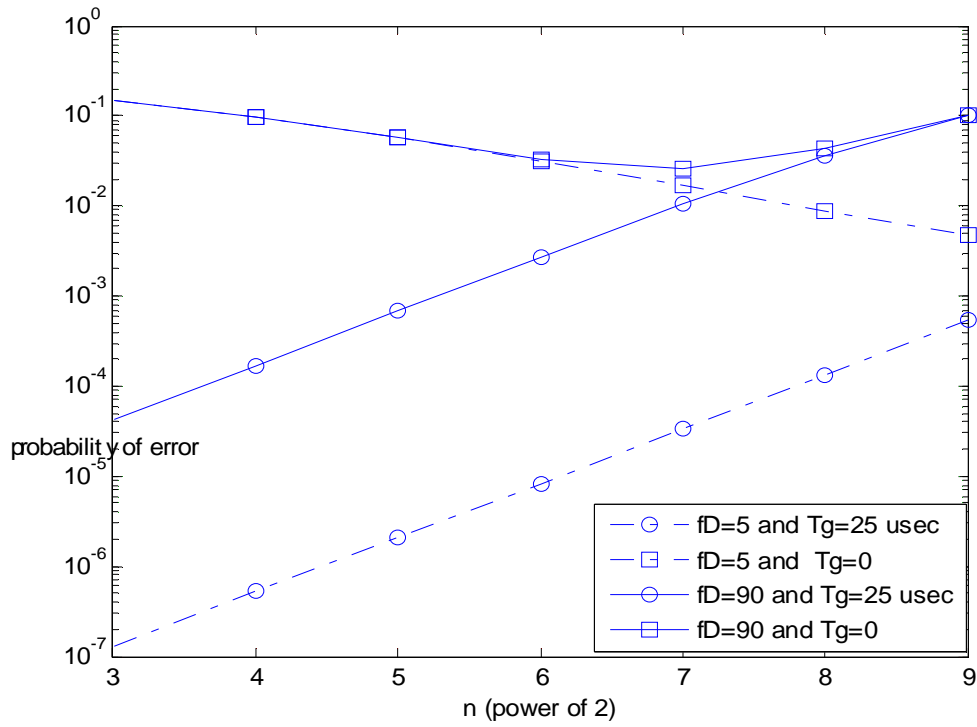


Fig. 3. BER versus N when $E_b/N_0 = \infty$

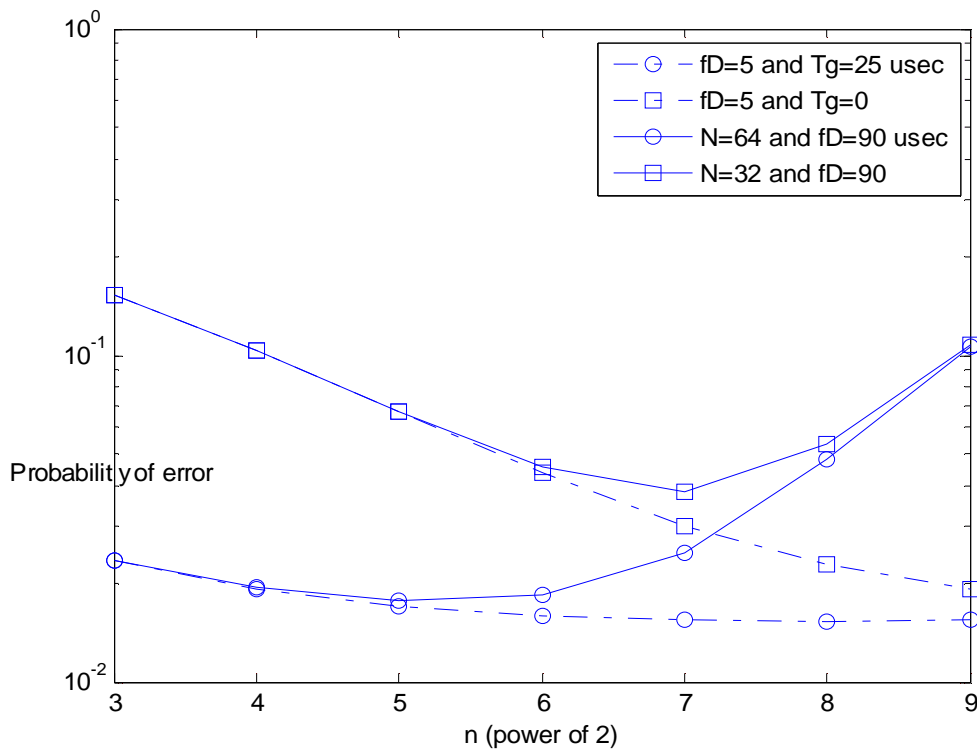


Fig. 4. BER versus N when $E_b/N_0 = 15$

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