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Interference Suppression with Total Least Square (TLS) Algorithm and Constant Modulus Algorithm (CMA)

By

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Abstract:

This paper proposes a beamforming scheme for suppressing interferences with a linear uniform antenna array interference cancellation is performed in two successive step steps, first estimation Direction Of Arrival's DOA of incident signals using high resolution spatial spectral estimation techniques. In this paper we concerned with the Multiple Signal Classification MUSIC techniques and it is modified version, Spatial Smooth MUSIC for DOA estimation in both case of uncorrelated and correlated incident signal to the array. The second step is to adjust the linear array weight vector to insert nulls in the direction of interference signals. A proposal for adaptive algorithms to adjust weight vector based on Least Square (LS) algorithm is presented, the LS algorithm could reduces the array pattern in the direction of interference signals, spatially when their Interference to Noise Ratio (INR) is high. However interference source with low INR is difficult to be reduced using LS algorithm, therefore another proposal for interference cancellation based on Total Least Square (TLS) algorithm is proposed. Both the LS and TLS algorithms require reference signal which reduce system throughput anther proposal for adaptive algorithms to adjust weight vector based on Constant Modulus Algorithm (CMA) algorithm, the CMA algorithm could reduces the array pattern in the direction of interference signals without using reference signal. The behaviours of the proposed algorithms are presented in graphs along with comments and discussions.

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1. Introduction:

A Linear antenna array consists of a set of antenna elements that are spatially distributed at known locations with reference to a common fixed point. By changing the phase and amplitude of the exciting currents in each of the antenna elements, it is possible to electronically scan the main beam and/or place nulls in any direction. The antenna elements can be arranged in various geometries, with the most common configurations along a line (linear array).
Adaptive antennas array refer to a group of antenna technologies that increase the system capacity by reducing the interference signal and increase the quality by reducing the fading effects.
Linear adaptive antenna array system that consists of M identical elements can steer a directional beam to maximize the signal from desired direction while nullify the signals from all other directions. An antenna array can nullify up to M-1 interference signals [1]. There are many adaptive techniques to insert nulls in the direction of interference sources.
In this paper we concerned with the Multiple Signal Classification MUSIC technique and it is a modified version, which is Spatial Smoothing MUSIC technique for DOA estimation.
In this paper also both the TLS and CMA algorithms are introduced for interference cancellation. The CMA has the advantage of working blind without need for reference signals, while the TLS provides deep nulls in the direction of interference signal regardless their INR. This is contradict to the original LS algorithm while provides deeper nulls for interference sources with high INR.
Both two proposed algorithms showed ability to reduce interference. The results are presented graphically with comments. The rest of this paper is organized as follows; section three describes the DOA estimation algorithms along with simulation to evaluate their behavior, section four provides the derivation of both TLS and CMA algorithms for interference cancellation in addition to comparison between them.

2. Signal and Noise Models:

Consider an M-element uniformly spaced linear array as illustrated in Figure 1. The array elements are equally spaced by a distance d, and a plane waves arrive at the array from a direction Φ of the array broadside. The angle $\Phi_i \quad 0 \le i \le D-1$ is called the DOA or angle-of-arrival (AOA) of the received signal

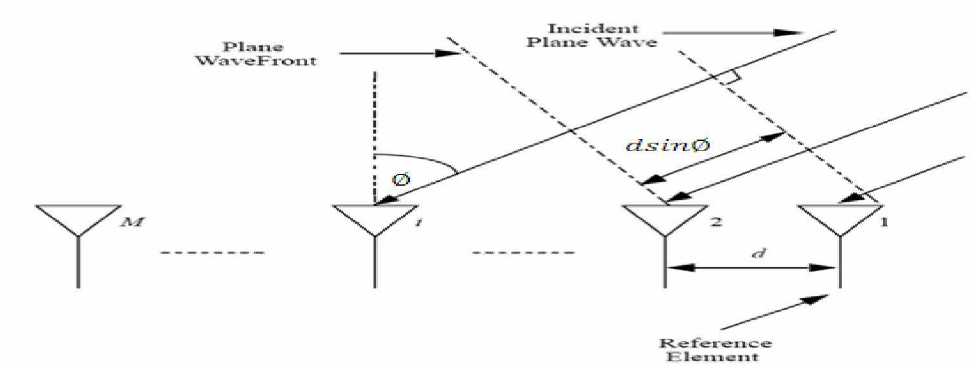


Figure (9): *Geometry of Linear Array*

The input signal to the liner antenna array is described by

$$x(t) = A(\emptyset)s(t) + n(t) \tag{1}$$

where

$s^T(t) = [s_0(t)\ s_1(t)\ \cdots\ s_{D-1}(t)]$ is the input signals vector.

$n^T(t) = [n_0(t)\, n_1(t)\ \cdots\ n_{M-1}(t)]$ is the noise vector at the array elements.

$A(\phi) = [a(\phi_0)\, a(\phi_1)\ \cdots\ a(\phi_{D-1})]$ is the steering vector of input of the antenna array with D signals.

and $a(\emptyset_i) = [1 \quad e^{j\pi \sin(\emptyset_i)} \quad e^{j2\pi \sin(\emptyset_i)} \quad \ldots \quad e^{j(M-1)\pi \sin(\emptyset_i)}]$

The equation (1) represents the model of the input data and noise that is most commonly used in the next sections.

3. Direction Of Arrival (DOA) Estimation

Direction of Arrival estimation (DOA) is one of the most demanding problems which one has to solve for localizing and tracking multiple rapidly moving targets as in radar, mobile communication and in other areas. This section presents the techniques that used for DOA estimation of uncorrelated and correlated signals.

3.1 MUSIC Algorithm

The word MUSIC stands for Multiple Signal Classification [2]. The basic idea of the MUSIC algorithm is to separate signal from noise by the orthogonality property of their spaces through eigen decomposition of the correlation matrix of the received signal.
The MUSIC algorithm can be described as follows; first, the correlation matrix R_{xx} is calculated From the input signal vector *x(0),x(1),…..x(M-1)* as follows:

$$R_{xx} = E[x(t)x^H(t)] = A(\phi)E[s(t)s^H(t)]A^H(\phi) + E[n(t)n^H(t)]$$

$$R_{xx} = A(\phi)R_s A^H(\phi) + \sigma_n^{\,2} I \tag{2}$$

where $\sigma_n^2 = E[n(t)n^H(t)]$ and $R_s = E[s(t)s^H(t)]$, second, use the eigen decomposition of the matrix R_{xx} to obtain the eigenvectors.

If the number of input signals D, is known, the large eigenvalues will correspond to signals and the remaining (M-D) small eigenvalues will correspond to noise. The eigenvectors of the matrix is given by:

$$E = [e_0 \; e_1 \; \ldots \ldots e_{M-1}] = [E_s \;\; E_n]$$

where E_S and E_n are the eigenvectors that correspond to signals and noise respectively. Both E_S and E_n are defined as:

$$E_s = [e_0 \; e_2 \; \ldots \ldots e_{D-1}] \text{ and } E_n = [e_D \; e_{D+1} \; \ldots \ldots e_{M-D}]$$

The spatial spectral function of the MUSIC algorithm is defined as

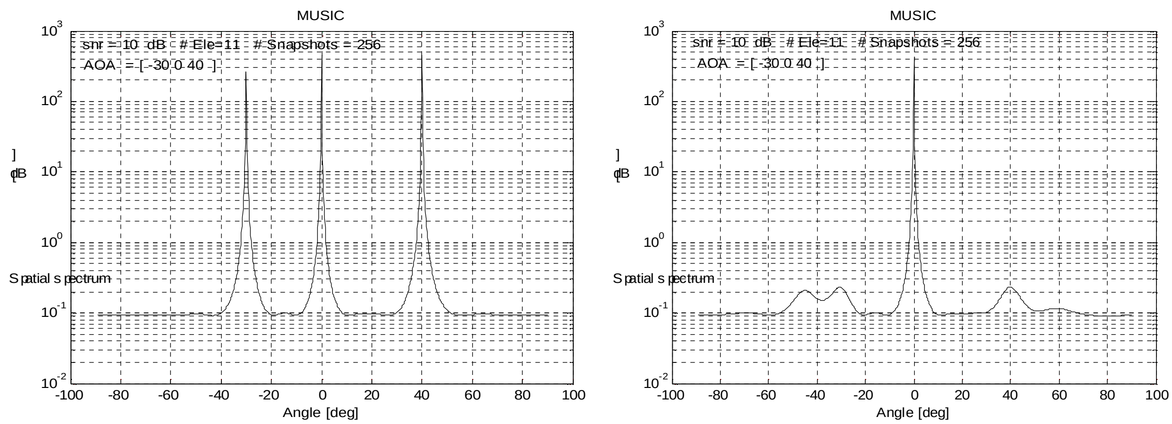
$$P_{music}(\phi) = \frac{1}{a^H(\phi) E_n E_n^H a(\phi)} \tag{3}$$

Finally, due to the orthogonality property [3], the denominator of $P_{music}(\phi)$ will be minimum, therefore the largest peaks of $P_{music}(\phi)$ will correspond to the direction of the incident signals.

Computer program using MATLAB is constructed to evaluate the performance of the MUSIC algorithm. It is considered to linear antenna array consists of 11 elements separated by half the wavelength of the transmitted signal of interest.

The environment consists of three incident signals at directions -30^o, 40^o, 0^o from the array broadside. The noise is zero mean white Gaussian noise and the SNR is assumed to be 10 dB for each signal. The number of snapshots that are used to construct the correlation matrix are 256 snapshots.

The performance of the MUSIC algorithm is evaluated in two different cases. In first case, the incident signals are assumed to be uncorrelated. The output of the MUSIC algorithm is presented in Figure 2(a). It is clear that the MUSIC algorithm has successfully determined the correction DOA of the incident signals. The second case, assumes that the signal at -30^o is multipath version of the signal at 40^o. Figure 2(b) shows the spatial spectral function obtained by the MUSIC algorithm for the case of correlated signals. One can see that the direction of both the correlated signals at -30^o, 40^o could not be estimated clearly, i.e. it cannot determine the individual DOA of each signal. This motivates us to apply the spatial smooth MUSIC algorithm as presented in the next subsection.



(b)

Figure 2: *the spatial spectrum of the MUSIC Estimator (a) Uncorrelated Signals (b) Correlated Signals*

3.2 Spatial Smoothing MUSIC Technique

The MUSIC algorithm failed to estimate the DOA for correlated signals. To overcome this problem, the spatial Smooth MUSIC is used for enhance MUSIC algorithm in the case of correlated signals. By dividing the M elements into (Q=M-P+1) overlapped sub arrays each of them contains P element (P<M), as Figure 3.

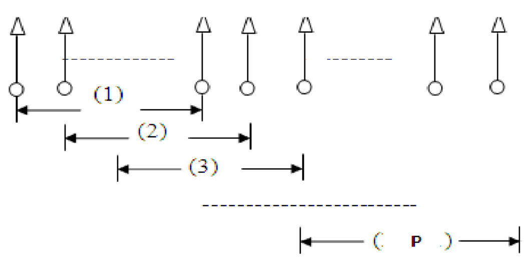


Figure 3: *Subarrays for Smooth MUSIC*

The received signal by the K^{th} subarrays is therefore given by:

$$x_k(t) = A(\phi)B^{k-1}s(t) + n_k(t) \quad (4)$$

where B is a (D x D) diagonal matrix defined as $B = diag[e^{-j\Phi}, \ldots, e^{-jD\Phi}]$. The K^{th} correlation matrix of the K^{th} subarrays is given by:

$$R_k = A(\phi)B^{k-1}R_s(B^{k-1})^H A^H(\phi) + \sigma^2 I \quad (5)$$

Then the average of these correlation matrices $1 \le K \le Q$ will results in non-singular correlation matrix regardless of the correlation of the incident signals. Then the spatial smoothing correlation matrix has the same form as the correlation matrix for

uncorrelation case if we apply MUSIC algorithm.
The spatial smooth MUSIC algorithm is applied to the same signals presented in the previous subsection and it overcomes the problem that happened when one of the incident signals is a multipath version of one anther.
From Figure 4, it is clear that the spatial smooth MUSIC algorithm can determine the DOA of the incident signals whether they are correlated or not.

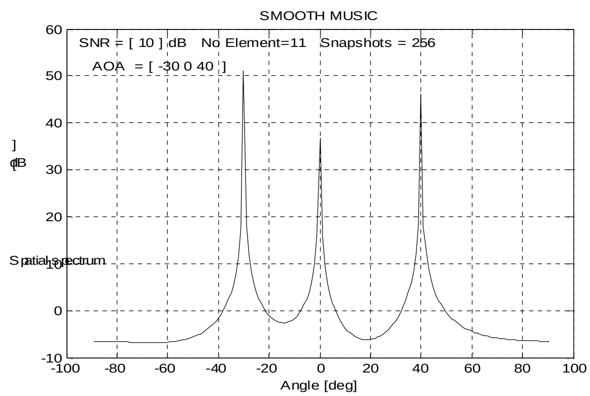


Figure 4: *The Spatial Spectrum of The MUSIC Estimator Correlated Signals*

4. Adaptive Beamforming Techniques

The next task of the adaptive array is nullify the direction of the unwanted signals. This could be obtained using LS, TLS and CMA. In adaptive antenna array, as shown in Figure 5, the weight vector w_k is adjusted or adapted, to maximize the quality of the signals at the output of the array.

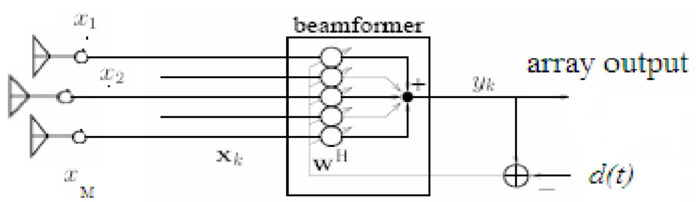


Figure 5: *An Adaptive Array Antenna*

Two of the most popular techniques which have been applied extensively in communication system are the LS and TLS criteria. In both of these two techniques, the square of the difference between the array antenna output, $y(t) = w_k{}^H x_i(t)$, and a desired output $d_k(t)$, or locally generated estimate of the desired signal, is minimized by finding an appropriate weight vector w_k[4]. In the LS approach, the cost function to be minimized is:

$$J(w_k) = E\left[\left|w_k{}^H x_i(t) - d_k(t)\right|^2\right] \quad (6)$$

The cost function is mean value of the square error between the antenna array output for the k^{th} signal and the desired version of that signal. So the cost function can rewrite as:

$$J(w_k) = {w_k}^H E[x_i(t){x_i}^H(t)]w_k - E[d_k(t){x_i}^H(t)]w_k - {w_k}^H E[[x_i(t){d_k}^H(t)] + E[d_k(t){d_k}^H(t)] \quad (7)$$

In general, we minimize a vector function by determining a location where the gradient of the function goes to zero. Therefore we can write

$$\nabla J(w_k) = 2E[x_i(t){x_i}^H(t)]w_k - 2E[[x_i(t){d_k}^H(t)] = 2R_{xx}w_k - 2r_{xd} \quad (8)$$

where R_{xx} is the correlation matrix of the input signal vector

$$R_{xx} = E[x_i(t){x_i}^H(t)] \quad (9)$$

And r_{xd} is the cross correlation vector between the input signal vector and desired signal,

$$r_{xd} = E[[x_i(t){d_k}^H(t)] \quad (10)$$

Setting the gradient of the cost function equal to zero, we find that the solution w_k which minimize $J(w_k)$ is

$$w_k = R_{xx}^{-1} r_{xd} \quad (11)$$

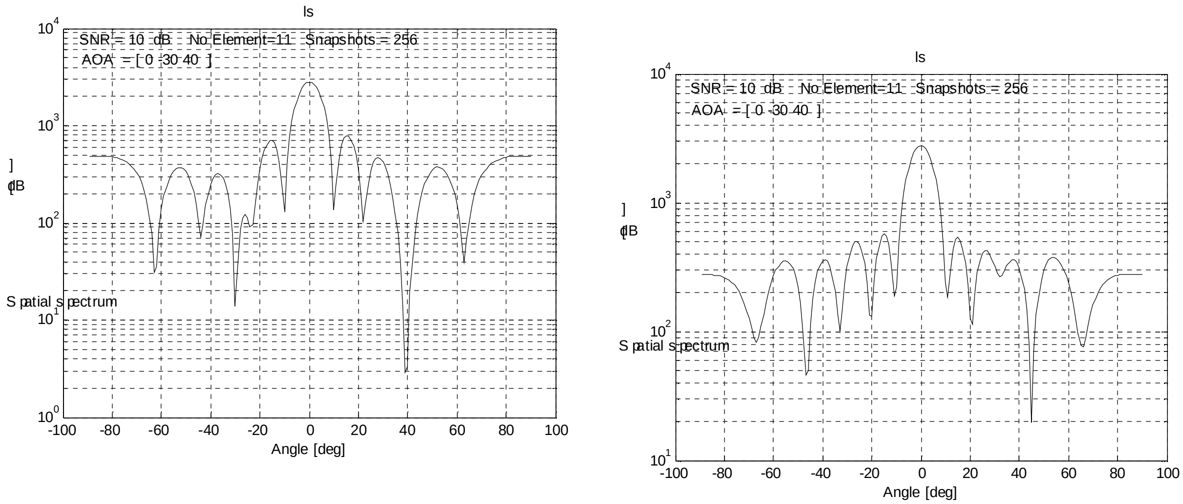
This is the optimal antenna array weight vector in the Minimum Mean Square Error (MMSE) sense.

We defined the Lest Squares cost function shown in equation (6) and after applying gradient operation, the solution of Lest Square gradient function tends to zero. This solution is defined as:

$$w_{LS} = \left(\left([x_i(t){x_i}^H(t)\right)^{-1} . x_i(t). d_k(t)]\right) \quad (12)$$

Computer simulation was carried out to evaluate the performance of LS beamformer. The test of the LS beamformer is performed in two different cases. In first one, the incident unwanted signals are assumed to be uncorrelated. The output of the LS beamformer is presented in Figure 6(a). It is clear that from this figure that the LS beamformer has successfully null the unwanted incident signals at directions -30^o, 40^o.

The second case, assumes that the unwanted signal at -30^o is multipath version of the signal at 40^o. It is clear from Figure 6(b) that the LS beamformer fails to null the unwanted incident signals.



(a) (b)

Figure 6*: The Spatial Spectrum for LS beamformer (a) Uncorrelated Signals (b) Correlated Signals.*

That the LS beamformer fails nullify unwanted signals if the array output y(t) is subject to error due to the DOA estimation error or when the INR is low, this motivates us to apply the to TLS beamformer as present in the next subsection.

4.1 The Total Least Square (TLS) algorithm

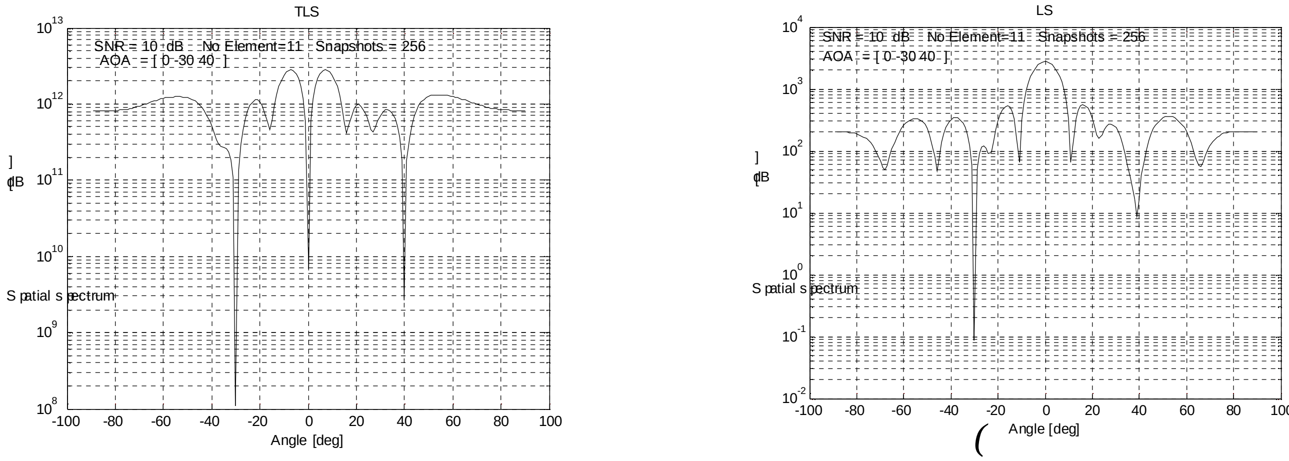
The TLS algorithm overcomes the problem of the LS algorithm by modifying the weight vector update equation to the form [5]

$$w_{TLS} = \left(\left([x_i(t)x_i^H(t) - \sigma^2 I\right)^{-1}.x_i(t).d_k(t)]\right) \quad (13)$$

Where σ^2_{n+1} is smallest non-zero singular value.

A comparison between the TLS and LS algorithms solution may be shows that the only different is the factor of $\sigma^2_{n+1}I$ that does not appear in the LS algorithm.

A computer program is constructed to evaluate the TLS algorithm when the input signals to the array are assumed to be two interference signals at DOA_1= 40^o ,INR_1= 0 dB and for and DOA_1= -30^o , INR_1=40 dB in addition to the desired signal. Figure 7(a,b) shows the performance of the TLS and LS algorithms respectively. One can see that the TLS algorithm inserted a deeper null in the direction of the week interference signal (DOA's) as well the stronger interference.



(b)

Figure 7: *The Spatial Spectrum of TLS and LS algorithms*

The LS and the TLS algorithms require reference signal which may not be available in some applications. This motivates us to apply the Constant Modules Algorithm (CMA) that will be presented in the next subsection.

4.2 Constant Modulus Algorithm (CMA)

A constant modulus algorithm CMA, can recover one signal from multiple incident signals without using reference signal. A priori knowledge of the CMA is that the envelope of the desired signal is constant. The cost function of the CMA is defined as

$$J(w_k) = [||y(k)|^p - |\alpha|^p|^q] \quad (14)$$

where $y(k) = w_k \times x_k$ (15)

where α is the desired signal amplitude at the array output.

The desired source is unknown but it has a constant modulus equals α=1. Then the cost function can be written as

$$J(w_k) = [||y(k)|^p - 1|^q] \quad (16)$$

An adaptive array which uses the CM cost-function will attempt to derive the signal at the array output to have a constant envelope with the specified amplitude, α. The exponent's p and q each are equal to either 1 or 2. The weight update equation is given by [6].

$$w_{k+1} = w_k - \mu \nabla J(w_k) \quad (17)$$

Where $\mu > 0$ is the step size.

We are interested to the CMA(1,2) case of both p and q have values of 1 and 2 respectively the corresponding Geometry of this CMA(1,2) is shown in Figure 8.

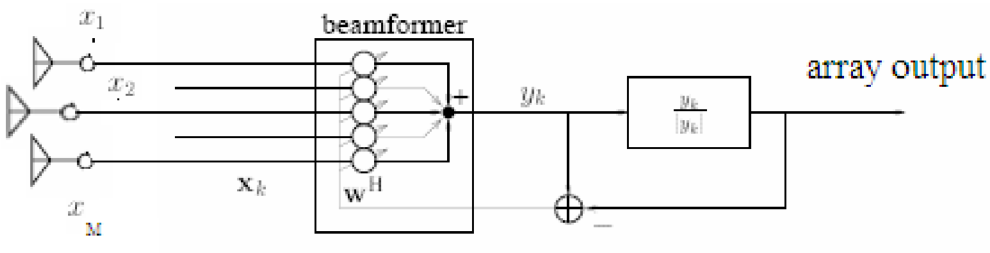


Figure 8: *Geometry of CMA(1,2)*

$$J(w_k) = E(|y_k| - 1)^2 = E(|w^H x_k| - 1)^2 \tag{18}$$

The gradient vector is given by

$$\nabla\big(j(w_k)\big) = 2\frac{\partial J(w_k)}{\partial w_k}$$
$$= \left[(y_k - \frac{y_k}{|y_k|})x_k\right] \tag{19}$$

Therefore the weight update equation can be written as

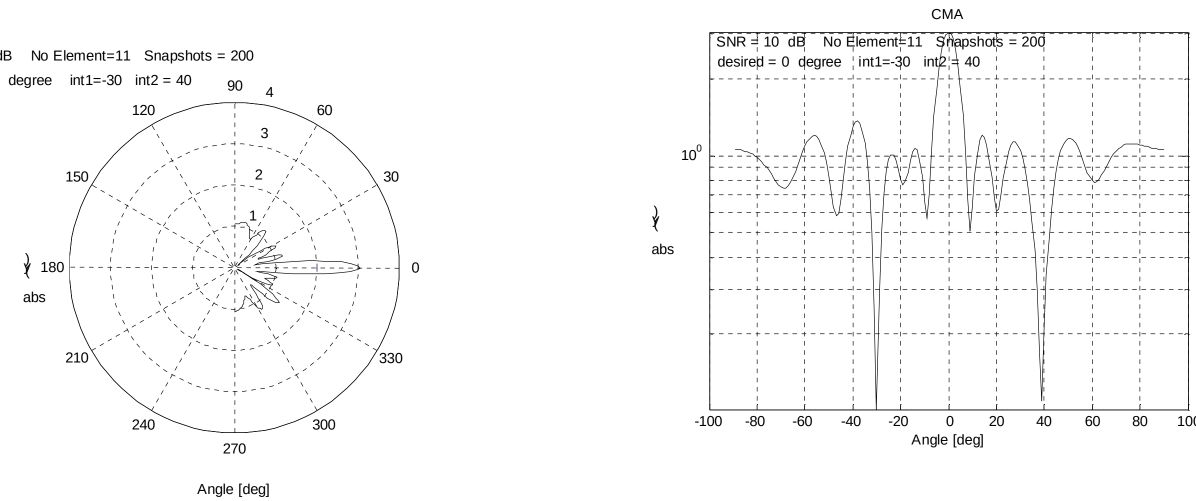
$$CMA(1,2): \quad \begin{cases} w_{k+1} = w_k - \mu x_k(\bar{y}_k - \frac{\bar{y}_k}{|y_k|}) \\ \qquad y_k = w_k^H x_k \end{cases} \tag{20}$$

Similar to Least Mean Square (LMS), but with update error $\bar{y}_k - \frac{\bar{y}_k}{|y_k|}$.

The desired signal is estimated by $s_k = \frac{\bar{y}_k}{|y_k|}$

Computer simulation were carried out to evaluate the performance of CMA (1,2) beamformer. The test of the CMA (1,2) beamformer is performed in two different cases.

In first case, the incident unwanted signals are assumed to be uncorrelated. The output of the CMA beamformer is presented in Figure 9(a). It is clear that the CMA beamformer has successfully null the unwanted incident signals at directions -30^{o,} 40^o, The second case, assumes that the unwanted signal that is at -30^o is multipath version of the signal that is at 40^o. It is clear from Figure 9(b) successfully null the unwanted incident signals.


SNR = 10 dB No Element=11 Snapshots = 200
desired = 0 degree int1=-30 int2 = 40
abs
Angle [deg]
CMA
SNR = 10 dB No Element=11 Snapshots = 200
desired = 0 degree int1=-30 int2 = 40
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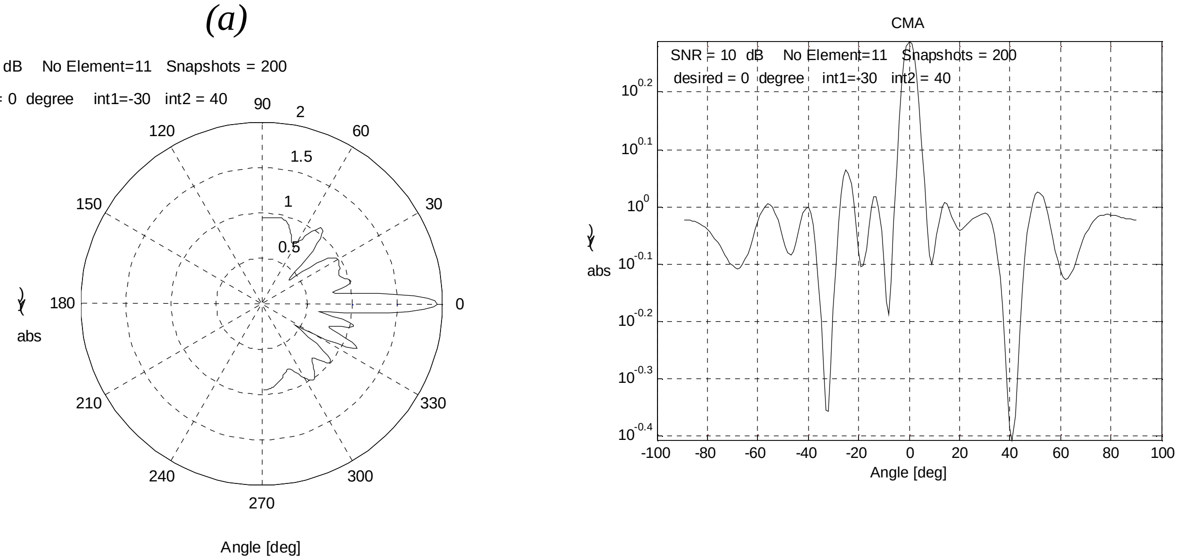
(a)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SNR = 10 dB No Element=11 Snapshots = 200
desired = 0 degree int1=-30 int2 = 40
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(b)

Figure 9: *Gain Pattern of The CMA(1,2) Beamformer (a) Uncorrelated Signals (b) Correlated Signals.*

5. Conclusions:

An adaptive antenna array for DOA estimation and nulling interference signals is proposed.

The proposed system estimates the DOA's of the incident signals using both MUSIC and Spatial Smooth MUSIC algorithms. Then it nullifies the signals that are comes from undesired directions using different adaptive beamforming algorithm such as LS,TLS and CMA.

The results show that the MUSIC algorithm can be applied properly to uncorrelated signals, while spatial smooth MUSIC is applied to signals that are multipath version of each other. The TLS algorithm provides better performance than the LS specially, when the INR is too low up to 0 dB. In the case when there is no reference signal, the CMA is applied and the result show that it better performance tan the other traditional adaptive algorithm such as LS, TLS algorithms.

References:

[1] B. Allen and M. Ghavami, *Adaptive Array Systems*, John Wiley and Sons, Ltd *2005*.

[2] Joseph C. Liberti, *Smart Antenna for Wireless Communications*, Prentice Hall, PP. 255-270, 1999.

[3] R. O. Schmidt, Multiple *estimator location and signal parameter estimation*, IEEE Trans. No Antenna and Propag,. Vol. AP-34, March 1986

[4] S. Haykin, *Adaptive Filter Theory*, Prentice Hall, nj 1991.

[5] A. Faeina. G. Golino and L.Timmoneri, *Comparison Between LS and TLS in Adaptive Processing for Radar Systems*, IEEE Proc.-Radar Sonar Navig, vol 150, NO 1, Feb 2003.

[6] M. G. Larimore and J. R. Treichler, *Convergence Behavior of the Constant Modulus Algorthim*, IEEE. Int.conf. Acoustic, Speech, and Signal Processing, PP.30.6.1-30.6.4,Apr.1985.