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OPTIMAL DESIGN OF DYNAMIC VIBRATION ABSORBER FOR ROLLING SYSTEMS

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ABSTRACT

This work deals with the reduction of vibration levels for rolling systems such as ropeway gondola, cranes and ships using a dynamic vibration absorber (DVA). The optimum values of the DVA damping factor and the spring constant are determined numerically through minimizing a maximum response of the main system. Single DVA (SDVA) and multi-masses DVA (MDVA) in parallel configurations are examined. A Mini-max criterion optimization method is utilized to calculate the optimum design parameters of the DVA for many numbers of DVA masses with a constant total mass. The results obtained that the rising of the mass ratio leads to decrease the vibration levels. Using the MDVA leads to enhanced robustness and vibration attenuation in the main system at the expense of growing the vibration in the absorber. MDVA compared with SDVA have some advantages such as lower frequency ratios and low fatigue.

KEYWORDS

Dynamic vibration absorber; Min-max technique; ropeway carrier; passive control; multiple tuned mass damper

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INTRODUCTION

Dynamic vibration absorbers (DVA) are effective control devices, which attached to a vibrating system for attenuating unwanted vibrations. The DVA's parameters are tuned with the primary structure, therefore a vibrating energy is transmitted to the DVA and then absorbed by a damper consequently the safety of the primary structure is extremely boosted [1-20]. For rolling systems in engineering applications such as, ships and ropeway gondola several control devices have been suggested and studied for improving their dynamic response under various types of excitation. Passive DVAs display more advantages over other because they have a simple design and low cost. Moreover, the efficiency of a DVA depends on appropriate selection of its parameters [1, 2]. Several optimal design techniques for the DVA have been suggested to reduce the unwanted vibration under various types of excitation. Asami et al. [3] presented analytical optimal solutions to H1 (Taylor series approximation) and H2 (algebraic solution) optimization of single mass DVA. Lin [4], proposed a new design of the DVA by connecting the damper of the DVA to the ground rather than the primary structure, and derived the new optimum tuning which is best over the conventional one. Elliott et al. [5] presented the optimal parameters for the kinetic energy of the primary structure is minimized or the energy absorption by the DVA is maximized. Tigli [6] presented the optimal design of the DVA connected to the damped system under random excitation.

Cheung and Wong [7] proposed a H2 optimization of the DVA. They used H_{∞} optimization to drive new optimal parameters to minimize the maximum system response [8]. Many other researchers studied the effectiveness of multiple absorbers under dynamic loads. Kareem and Kline [9] studied the multiple absorbers under random forcing. Zuo and Nayfeh [10, 11] proposed Multi-masses DVA (MDVA) with distributed masses. They obtained the design parameters by modeling the system as a decentralized control problem Love and Tait [12], proposed a numerical method to simulate the structure-MDVA systems. MDVAs, which distribute the tuning of several masses in a range close to the natural frequency, were more effective than traditional DVA systems. For a damped system, the optimal parameters of the DVA are determined numerically and an analytical optimization cannot be performed [13, 14]. Morga and Marano [15] investigated two different criteria about displacement and acceleration for DVAs on slender structures under wind excitations. Also, a tuning process has been developed by Salvi and Rizzi [16] considering the minimax optimization algorithm for the reduction of seismic vibrations with the pas with the passive DVAs.

Pennestrì [17] developed a Mini-max procedure to determine the optimum design parameters. Six constraint equations with seven design parameters were solved. Brown [18] developed a Mini-max procedure of a passive DVA for uncertainties frequency range. One may observe from the literature that numerical optimization method based on Minimax technique has been probably the most common computational techniques adopted to evaluate the optimal DVA parameters in the case of damped primary structures, when the main system is subjected to standard typical loadings, such as harmonic or Gaussian white noise force or base excitation [19,20,21].

In this study, a simple and effective procedure for vibration reduction of rolling systems under external excitation is presented through the optimal design of the

DVA. The response properties of a SDVA are analyzed. The performances of a MDVA applied to the rolling system are compared. A Mini-max approach is presented for the optimal design of the DVA's parameters for the uncertain frequency range. This method decreases the system amplitude to a value lower than the other available methods. The optimum design parameters of DVA such as the damping factor and frequency ratio are determined with a Mini-max numerical optimization. The results for the undamped main system are compared with those studied by Matshuhisa [22, 23]. The effect of the structure damping, mass ratio, relative position and number of DVA masses on the design parameters and the maximum system response were studied.

EQUATIONS OF MOTION (EOMs)

Single Mass DVA (SDVA)

Referring to Fig. 1, the rolling system is modeled as a pendulum with a mass m , length l and damping constant C . the main system has an angular variation θ . The DVA is mounted at a distance l_1 from the point o with a displacement u . where, m_1 , k_1 and C_1 are the mass, stiffness and damping constant of the DVA. f_o and ω are the amplitude and frequency of the external force. The locations of the main system mass (x, y) and the DVA mass (x_1, y_1) can be expressed as:

$$x = l \sin \theta \quad , \quad y = l \cos \theta \quad (1)$$

$$x_1 = l_1 \sin \theta + u \cos \theta \quad , \quad y_1 = l_1 \cos \theta - u \sin \theta$$

The kinetic energy K.E. is given by:

$$K.E. = \frac{1}{2} m l^2 \dot{\theta}^2 + \frac{1}{2} m_1 (\dot{\theta}^2 (l_1^2 + u^2) + 2 l_1 \dot{u} \dot{\theta} + \dot{u}^2) \quad (2)$$

The potential energy P.E. is given by:

$$P.E. = m g l (1 - \cos \theta) + m_1 g (l_1 (1 - \cos \theta) + u \sin \theta) + \frac{1}{2} K u^2 \quad (3)$$

The dissipation function D.E. is given by:

$$D.E. = \frac{1}{2} C_1 \dot{u}^2 + \frac{1}{2} C \dot{\theta}^2 \quad (4)$$

The EOMs of the system can be obtained using Lagrange equations as follows:

$$\ddot{\theta} \left(m l^2 + m_1 (l_1^2 + u^2) \right) + m_1 l_1 \ddot{u} + 2 m_1 u \dot{\theta} \dot{u} + g (m l + m_1 l_1) \sin \theta + m_1 g u \cos \theta + C \dot{\theta} = f_o l e^{i \omega t} \quad (5)$$

$$m_1 \ddot{u} + m_1 l_1 \ddot{\theta} + C_1 \dot{u} + m_1 g \sin \theta + (K - m_1 \dot{\theta}^2) u = 0 \quad (6)$$

Considering that θ and u are small values, the EOMs can be linearized by neglecting higher order terms;

$$\begin{bmatrix} ml^2 + m_1 l_1^2 & m_1 l_1 \\ m_1 l_1 & m_1 \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{u} \end{bmatrix} + \begin{bmatrix} C & 0 \\ 0 & C_1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{u} \end{bmatrix} + \begin{bmatrix} g(ml + m_1 l_1) & m_1 g \\ m_1 g & Ku \end{bmatrix} \begin{bmatrix} \theta \\ u \end{bmatrix} = \begin{bmatrix} f_0 l e^{i\omega t} \\ 0 \end{bmatrix} \quad (7)$$

Introducing the following dimensionless terms:

$$\mu = \frac{m_1}{m}, \gamma = \frac{l_1}{l}, r = \frac{\omega}{\omega_n}, r_a = \frac{\omega_a}{\omega_n}, \omega_a = \sqrt{\frac{K_1}{m_1}}, \omega_n = \sqrt{\frac{g}{l}}, \zeta_a = \frac{C_1}{2m_1\omega_a},$$

$$\zeta_s = \frac{C}{2ml^2\omega_n}, U_n = \frac{U}{f_0 l / mg}, \theta_n = \frac{\theta}{f_0 / mg}$$

where, θ_n and U_n are the normalized vibration amplitudes of the main system and the DVA respectively, μ is the mass ratio, γ is the relative position, r_a is the DVA frequency ratio, ω_a is the natural frequency of the DVA, ω_n is the natural frequency of the main system, ζ_a is the DVA damping factor and r is the excitation frequency ratio.

The steady state responses of the main system and the DVA are given by:

$$\theta = \Theta e^{i\omega t} \quad \text{and} \quad u = U e^{i\omega t} \quad (8)$$

Combining Eqs. 7 and 9 yields:

$$\begin{bmatrix} (1 + \mu\gamma) - (1 + \mu\gamma^2)r^2 + 2ir\zeta_s \\ \mu(1 - \gamma r^2) \\ \mu(1 - \gamma r^2) \\ \mu r_a^2 - \mu r^2 + 2i\mu r r_a \zeta_a \end{bmatrix} \begin{bmatrix} \theta_n \\ U_n \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (9)$$

where θ_n and U_n are the normalized vibration amplitudes of the main system and the DVA respectively.

Multi-Masses DVA (MDVA)

For the main system with MDVA, the EOM can be formulated in a matrix form as:

$$\begin{bmatrix} D_1 & A_1 A_2 \dots A_N \\ B_1 & 0 \dots 0 \\ & B_2 \dots 0 \\ & \dots 0 \\ \text{sym} & & & B_N \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{u}_1 \\ \ddot{u}_2 \\ \vdots \\ \ddot{u}_N \end{bmatrix} + \begin{bmatrix} D_2 & 0 & 0 \dots 0 \\ E_1 & 0 & 0 \\ & E_2 & 0 \\ & \dots & 0 \\ \text{sym} & & & E_N \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{u}_1 \\ \dot{u}_2 \\ \vdots \\ \dot{u}_N \end{bmatrix} + \begin{bmatrix} D_3 & gB_1 & gB_2 \dots gB_N \\ B_1 \omega_{a1}^2 & 0 & \dots 0 \\ & B_2 \omega_{a2}^2 & \dots 0 \\ & \dots & \dots 0 \\ \text{sym} & & & B_N \omega_{aN}^2 \end{bmatrix} \begin{bmatrix} \theta \\ u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix} = \begin{bmatrix} \frac{f_0 g}{m} e^{i\omega t} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (10)$$

where;

$$D_1 = (1 + \sum_{i=1}^N \mu_i \gamma^2) g l, \quad A_i = g \mu_i \gamma, \quad B_i = \mu_i \omega_n^2$$

$$D_2 = \frac{2\zeta_s g^2}{\omega_n}, E_i = 2\mu_i \omega_{ai} \omega_n^2 \zeta_{ai}, D_3 = (1 + \sum_{i=1}^N \mu_i \gamma) g^2$$

And the displacement of i^{th} mass of DVA is u_i ($i = 1, 2, \dots, N$). The mass ratio is expressed as;

$$\mu = \frac{1}{m} \sum_{i=1}^N m_i$$

For i^{th} mass the design parameters are the mass ratio μ_i , the damping factor ζ_{ai} , and the frequency ratio ω_{ai} . where;

$$\zeta_{ai} = C_i / 2 m_i \omega_{ai}, \omega_{ai} = \sqrt{k_i / m_i}, \mu_i = \frac{\mu}{N}$$

The normalized vibration amplitudes of the main system and of the MDVA are:

$$\begin{bmatrix} d_1 & a_1 & a_2 & \cdot & a_N \\ & b_1 & 0 & 0 & 0 \\ & & b_2 & 0 & 0 \\ \text{sym} & & & \cdot & \cdot \\ & & & & b_N \end{bmatrix} \begin{bmatrix} \theta_n \\ U_{1n} \\ U_{2n} \\ \cdot \\ U_{Nn} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \cdot \\ 0 \end{bmatrix} \quad (11)$$

where;

$$d_1 = 1 + \mu\gamma - (1 + \mu\gamma^2)r^2 + 2ir\zeta_s, \\ a_i = (1 - \gamma r^2)\mu_i, b_i = \mu_i r_{ai}^2 - \mu_i r^2 + 2ir\mu_i r_{ai} \zeta_{ai}$$

NUMERICAL OPTIMIZATION OF THE DESIGN PARAMETERS

The minimax optimization offers the advantage to find the values of design variables which minimize the maximum objective function value over a given range of a set of uncertain variables. In the current optimization case, the uncertain variable is the frequency of the excitation function and the objective function is the normalized vibration amplitude of the main system $\theta_{n\text{max}}$. The design variables are the damping factor ζ_a and frequency ratio r_a of the vibration absorber subsystem. The main system damping factor ζ_s and the mass ratio μ are assumed to be given parameters which can be decided firstly by the designer according to space restriction for added mass on the main system. Therefore, for SDVA the parameters to be searched are the SDVA optimum damping factor ζ_{ao} and optimum frequency ratio r_{ao} . While the MDVA requires a search over combinations of N of optimum frequency ratios $r_{ao})_i$ and N optimum damping factors $\zeta_{ao})_i$ where $i = 1$ to N .

Therefore, the optimization problem is defined as:

$$\text{Find } r_{ao} \text{ and } \zeta_{ao} \quad (12)$$

$$\text{To minimax } \theta_n \text{ for } r_{\min} \leq r \leq r_{\max}$$

$$\text{With } lb_1 \leq r_a \leq ub_1, lb_2 \leq \zeta_a \leq ub_2$$

where: r_{min} and r_{max} are the upper and lower limits for the excitation frequency ratio respectively. The MATLAB code is written to simulate the minimax problem of the main system with SDVA and MDVA. To start the optimization process, it is necessary to initialize the values of the two variable parameters r_a and ζ_a . For engineering applications, it is suitable to assume $r_a=1$ and $\zeta_a=0.1$ as starting values. The tests of the numerical procedure have displayed that such presumption does not influence significantly the effectiveness of the optimization process. The process is unconstrained except that the lower and upper bound vectors of the two parameters r_a and ζ_a are also taken into account. To optimize the DVA, every researcher considers a range of these bounds [23, 24]. In this manuscript, the range for frequency ratio and damping factor of DVA (in MATLAB vector notation) has been chosen as $lb = [0.5; 0.001]$ and $ub = [1.5; 1]$, which provide quite wide intervals for the optimization process.

RESULTS AND DISCUSSION

The suggested numerical process is first tested on the pendulum system without the damped element. The results for the undamped main system with SDVA are emulated with those presented by Matshuhisa [21, 22]. As shown in TABLE I the two results are very close each other.

Table 1. Optimum frequency ratio for undamped sdof system.

μ	γ	r_{ao}	r_{ao} [21]	DIV %
0.05	0.25	1.0097	1.00926	0.042
	0.5	1.0126	1.01204	0.054
	0.75	1.0091	1.00845	0.063
0.1	0.25	1.0175	1.018330	0.081
	0.5	1.0231	1.023228	0.012
	0.75	1.0176	1.015271	0.227

Effect of the Main System Damping

Figure 2 shows that the increases in the mass ratio increase the optimum damping factor ζ_{ao} for any value of the damping factor of the main system ζ_s . The increasing of the damping factor for the main system ($\zeta_s > 0.1$) leads to increase ζ_{ao} for a given μ . Referring to Fig. 3, increasing the damping factor ζ_s leads to decrease the optimum frequency ratio r_{ao} . Furthermore, the optimum frequency ratio r_a is close to unity. The maximum normalized amplitude θ_{nmax} is plotted in Fig. 4 for many values of ζ_s by employing the calculated optimum values ζ_{ao} and r_{ao} . From this figure, it can be noticed that the rise of the damping factor ζ_s leads to a reduction in the normalized amplitude. Moreover, when ζ_s is low, θ_{nmax} is mainly dependent on μ , but for the high value of ζ_s it becomes less sensitive to μ . Actually when $\zeta_s \geq 0.2$, the curves of θ_{nmax} vary little whatever the value of μ is. It may seem from these results that the DVA is suitable for the systems with a high damping factor, but in fact, the efficiency of the

DVA does not calculate directly from the value of θ_{nmax} at a given μ . The efficiency of the DNA can be expressed as following;

$$\text{DVA efficiency \%} = \frac{\theta_{nmax})_b - \theta_{nmax}}{\theta_{nmax})_b} \times 100$$

where; $\theta_{nmax})_b$ is the maximum normalized amplitude of the main system without the DVA.

Figure 5 illustrates the efficiency of the DVA for various values of ζ_s , it can be observed that the rising the damping in the main system leads to reduce the DVA efficiency. The DVA is ineffective when the damping of the main system becomes high.

Effect of the Relative Position

The effect of mass ratio on the optimum damping factor and optimum frequency ratio respectively is illustrated in Figs. 6 and 7 for three relative positions $\gamma = 0.25, 0.5$ and 0.75 . From these figures, it can be observed that the rising of μ leads to an increase in both optimum frequency ratio and optimum damping factor for any relative position. Furthermore, Fig. 6 displays that the optimum damping factor grows considerably as the DVA moves to the center of the swing. For example it is increasing from $\zeta_{ao} = 0.06$ to $\zeta_{ao} = 0.29$ when the DVA is moved from $\gamma = 0.75$ to $\gamma = 0.25$. Fig. 7 clearly shows that the influence of rising the relative position on the optimum frequency ratio is small particularly for low mass ratios. Referring to Fig. 8, the variation of the damping factor of the main system with SDVA in free vibration is obtained for different relative positions. For $\mu=0.1$ and $\zeta_s = 0.02$, the total damping factor (the damping factor of structure- DVA system) is increasing from $\zeta = 0.027$ to $\zeta = 0.058$ when the DVA is moved from $\gamma = 0.75$ to $\gamma = 0.25$, the total damping factor is increased by about 114%. Fig. 9 shows the maximum normalized amplitude of the main system under external excitation function with an optimally tuned SDVA and $\zeta_s = 2\%$. It can be noticed that the maximum normalized amplitude increases with the relative position. In the case of using SDVA with $\mu = 0.05$, the maximum normalize amplitude ($\theta_{nmax} = 13.4$) for $\gamma = 0.75$ is greater than the maximum normalized amplitude ($\theta_{nmax} = 6.5$) for $\gamma = 0.25$. It is clear that the DVA is effective and that it is dependent on the relative position.

Effect of the MDVA

Figures 10 to 14 show the effect of the MDVA on a dynamic performance of the rolling system. The total absorber mass is equally distributed among N masses (N = 2, 3, 4). Fig. 10 shows the main system normalized amplitude in the frequency domain for various values of N. The maximum response of the main system decreases by raising the number of MDVA masses with a constant total mass. In addition, the frequency response includes N troughs at the MDVA natural frequencies and N+1 crests. Moreover, the flatness of the frequency response curve increases with raising the number of DVA masses.

Figure 11 shows the influence of the number of DVA masses on the maximum normalized amplitude. It can be noticed that the increasing the number of DVA masses leads to a steady decrease in the maximum normalized amplitude of the main

system. For example at $\gamma = 0.25$ the maximum normalized amplitude is decreasing from $\theta_{nmax} = 5.65$ to $\theta_{nmax} = 4$ for $N=1$ to 4, nearly 30% improvement. As shown in Fig. 12, The effectiveness of the MDVA can be improved by increasing its number of masses, for example for SDVA with mass ratio 20%, the efficiency is 92%, which can be achieved by a MDVA with $N = 4$ and mass ratio 10%.

Figure 13 shows the effect of the number of DVA masses on the optimum frequency ratios. It can be noticed that the optimum frequency ratio is nearly equal to unity. The lower frequency ratio decreases by decreasing the relative position and raising the number of DVA masses. Referring to Fig. 14, optimum damping factor increases by increasing the relative position of the MDVA and it decreases by raising the number of masses.

CONCLUSIONS

From the results discussed in this paper, it may be concluded that growing the damping of the main system leads to decrease the efficiency of the DVA and increases its optimal mass. Moreover, they will be totally inactive when the system damping factor is greater than 0.1. As the mass ratio is increased, the vibration level decreases. Generally, high mass ratios are not recommended due to functional considerations of the whole system. The effect of growing the relative position on the optimum frequency ratio is small, particularly for low mass ratios. Raising the number of DVA masses without changing the total DVA mass leads to reduce the system vibration and improves the DVA efficiency. The MDVA has several features over the SDVA such as a lower ratio of tuning frequencies, lower requirement of anti-fatigue materials and lower mass ratio. The presented optimization approach shows improvement compared with the findings given in the previous studies by profiting knowing of the excitation frequency range for a damped rolling system.

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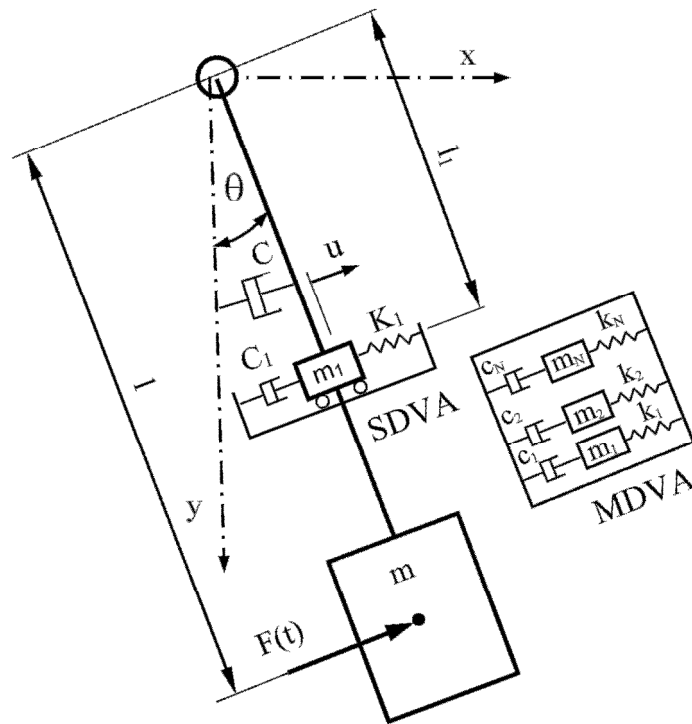


Fig. 1. Dynamic model of a rolling system with a dynamic absorber.

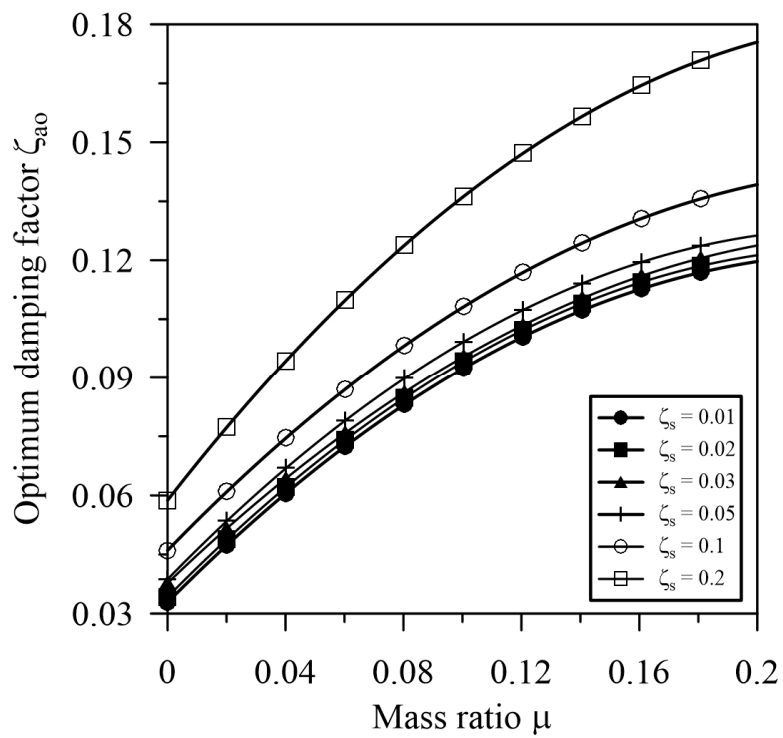


Fig. 2. Optimum damping factor variations with mass ratio for different main system damping factors.

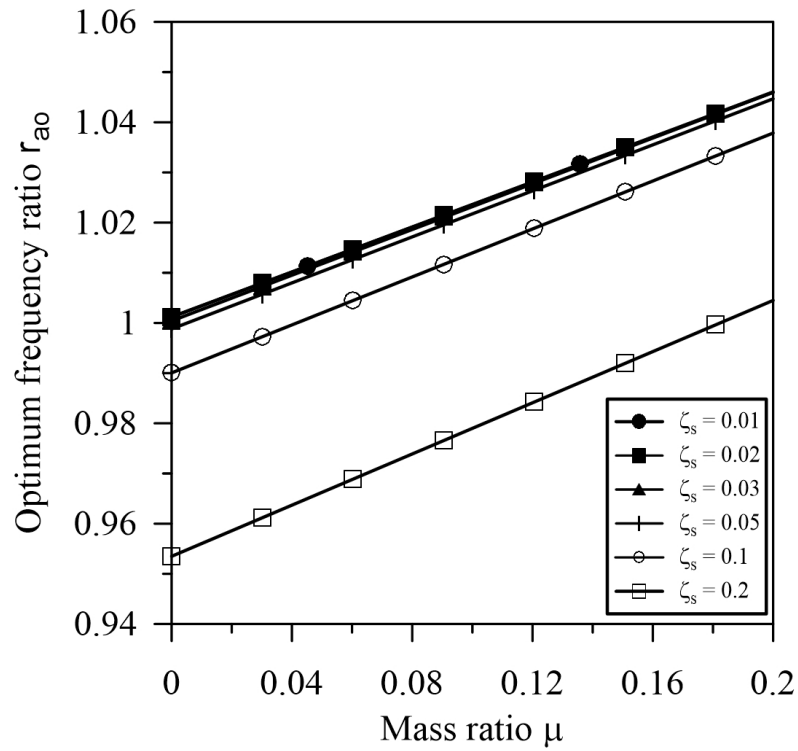


Fig. 3. Optimum frequency ratio variations with mass ratio for different main system damping factors.

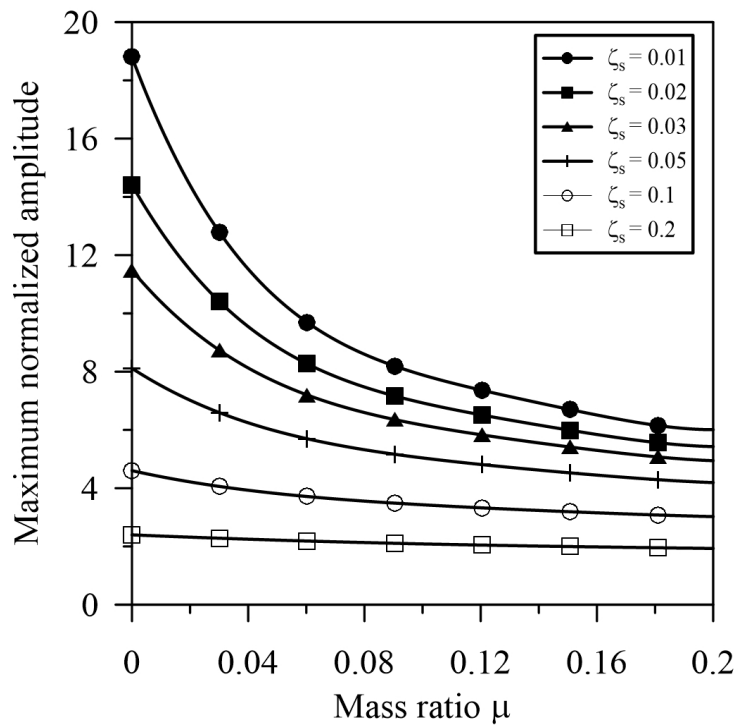


Fig. 4. Maximum normalized amplitude for different values of damping factor at $\gamma = 0.5$.

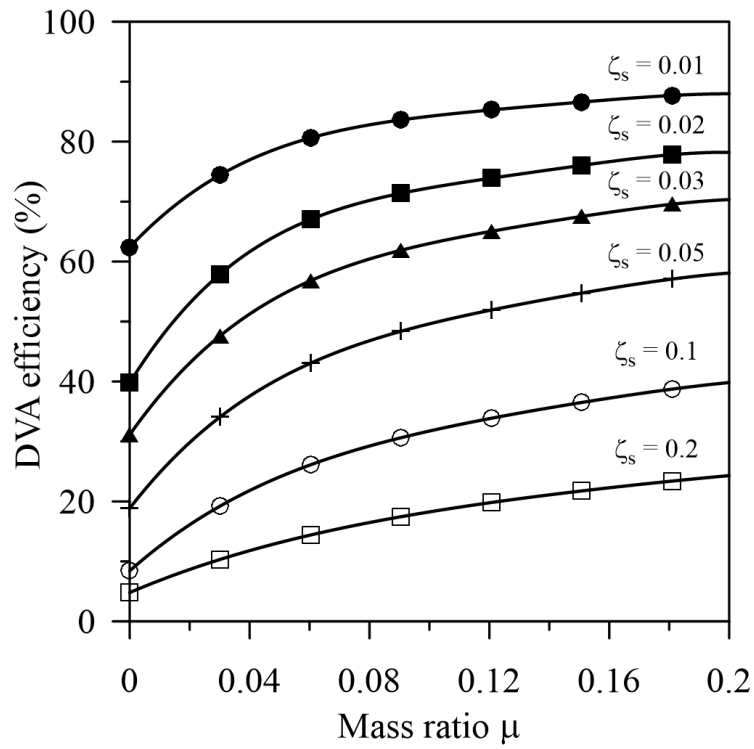


Fig. 5. Variation of DVA efficiency versus mass ratio for different main system damping factors.

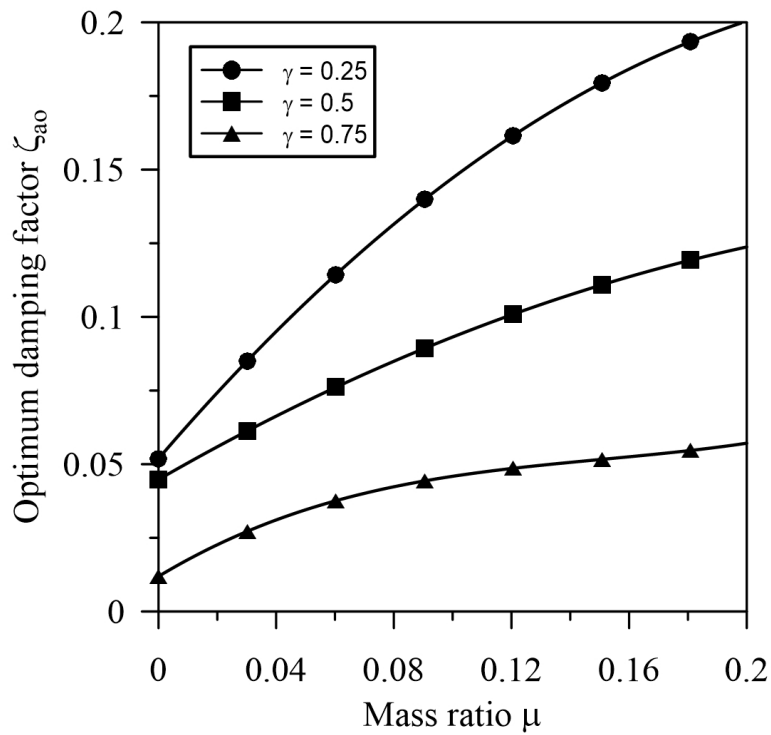


Fig. 6. Optimum damping factor variations with mass ratio for different relative positions at $\zeta_s = 0.02$.

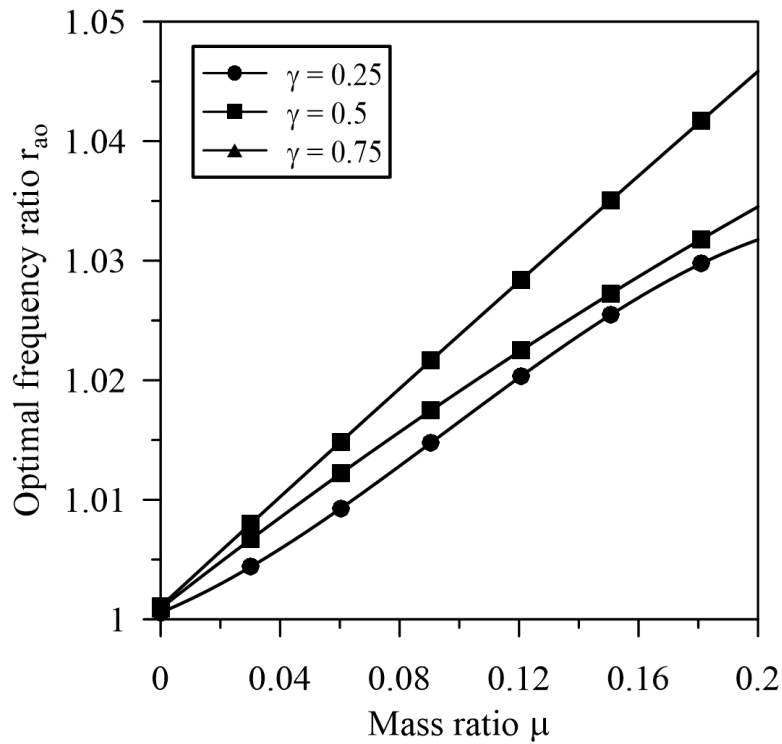


Fig. 7. Optimum frequency ratio variations with mass ratio for different relative positions at $\zeta_s = 0.02$.

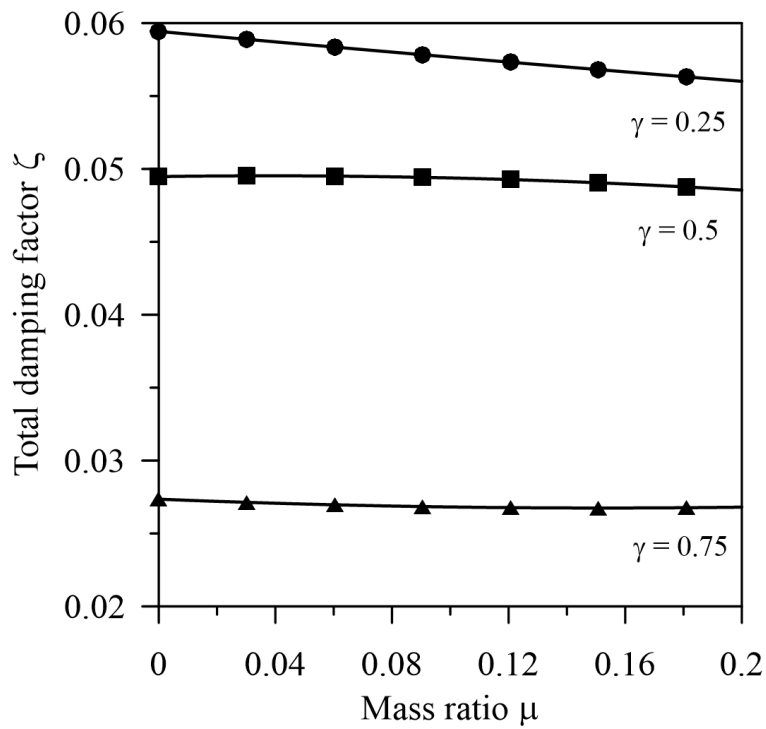


Fig. 8. Total damping factor variations with mass ratio for different relative positions at $\zeta_s = 0.02$, ζ_{ao} and r_{ao} .

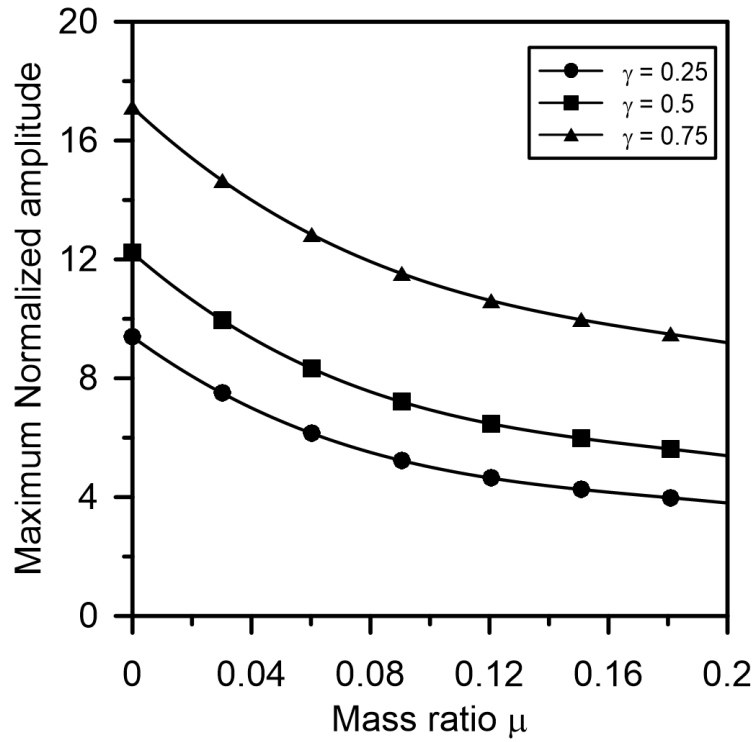


Fig. 9. Variation of maximum normalized amplitude with mass ratio for different relative positions at $\zeta_s = 0.02, \zeta_{a0}$ and r_{a0} .

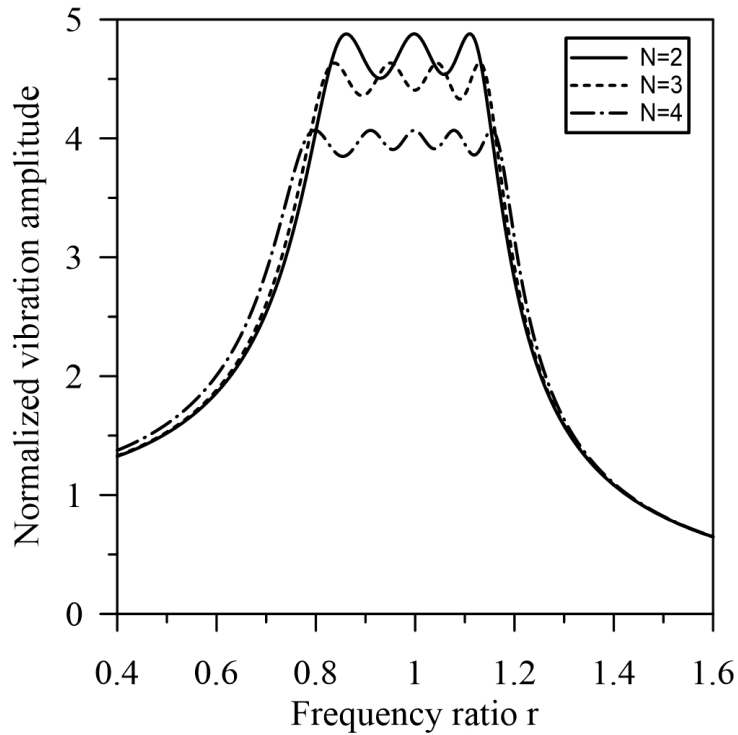


Fig. 10. Effect of number of DVA masses for $\zeta_s = 0.01, \gamma = 0.25$ and $\mu = 0.1$.

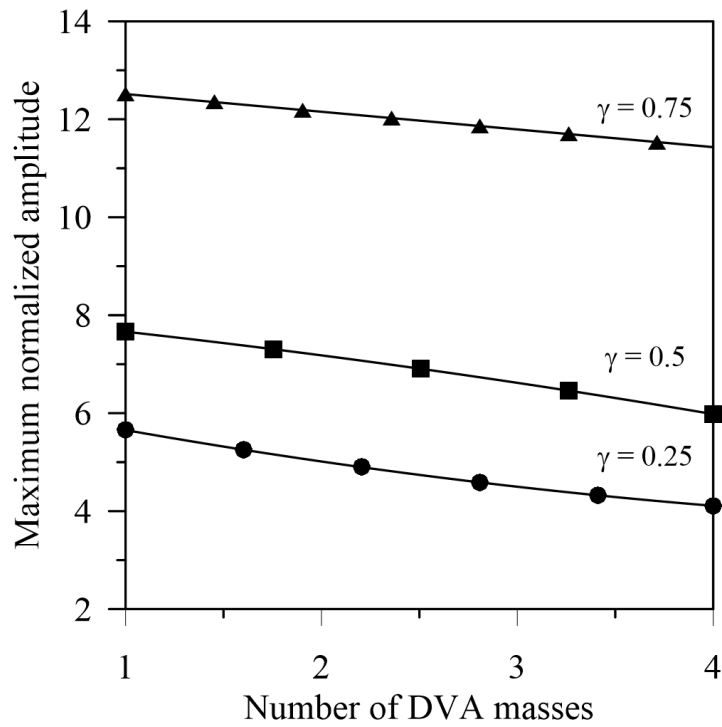


Fig. 11. Maximum normalized amplitude vs number of DVA masses for different relative positions at $\zeta_s = 0.01$ and $\mu = 0.1$.

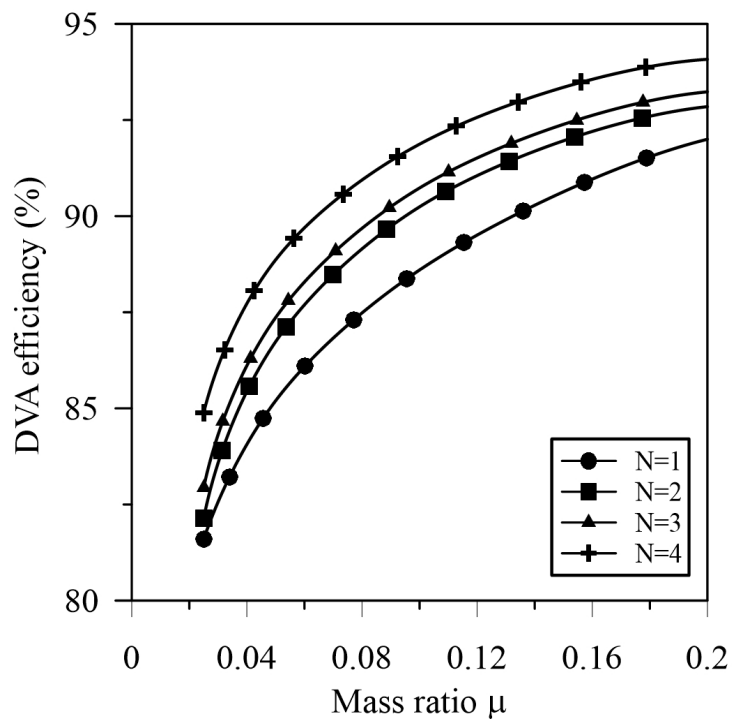


Fig. 12. Variation of DVA efficiency with mass ratio for different number of DVA masses when $\zeta_s = 0.01$ and $\gamma = 0.25$.

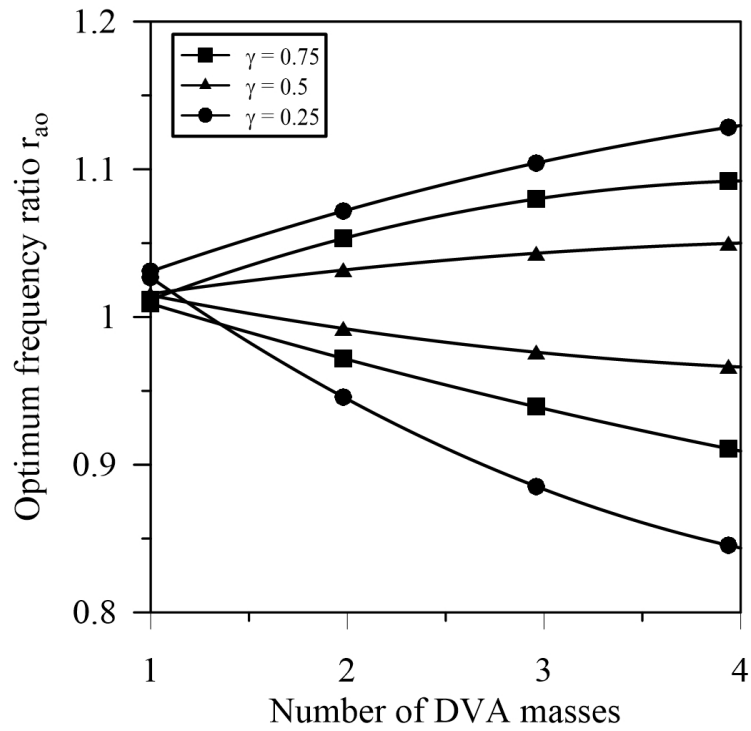


Fig. 13. Optimum DVA frequency ratio vs number of DVA masses for different relative positions when $\zeta_s = 0.01$ and $\mu = 0.1$.

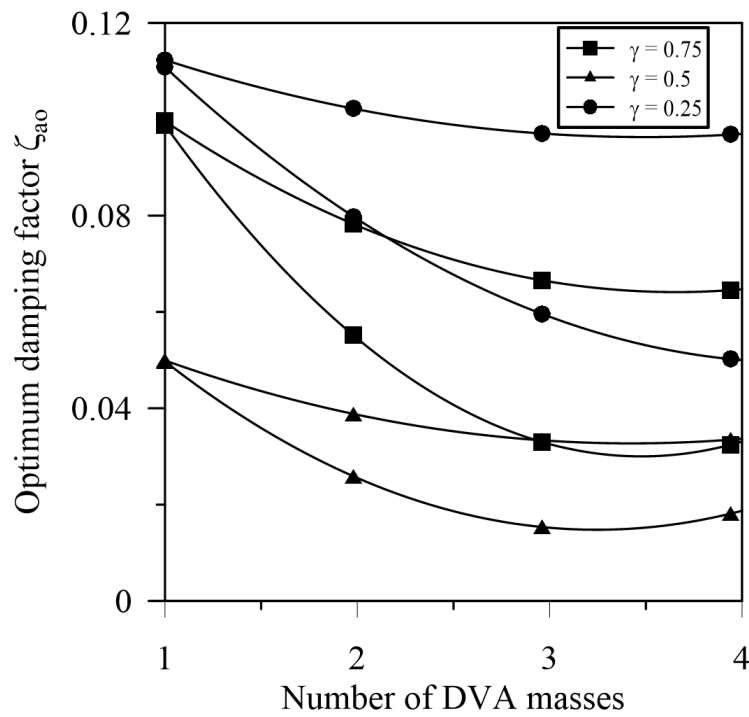


Fig. 14. Optimum DVA damping factor vs number of DVA masses for different relative positions when $\zeta_s = 0.01$ and $\mu = 0.1$.