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Research Article

MATHEMATICS

The Beta-Truncated Lomax Distribution with Communications Data

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ABSTRACT

The beta-truncated Lomax distribution is investigated (BTLD). The goal of this paper is to investigate an extension to Lomax distribution (LD) which is my new distribution BTLD. Also, the LD is a special case of the BTLD. Our distribution can be used as an alternative to the LD. For the BTLD, we obtain the probability density function (PDF), cumulative distribution function (CDF), moments, distribution of order statistics, and a single moment of order statistics. We derive PDF of BTLD minimum order statistics, the PDF of the biggest order statistics. The BTLD's random sum is discussed. An application of the model BTLD to communications real data set is presented to show the superiority of this new distribution by comparing the fitness with LD and exponential distribution. The survival function (SF), hazard function (HF), odd function (OF), reserved hazard function (RHF), moment generating function (MGF), and several moments are all deduced. We get some special cases.

Introduction

A new three-parameter lifetime distribution is proposed by **Al-Zahrani, and Sagor, 2014.**

Javid, and Abdullah, 2015 introduced the distribution of order statistics (OS) for the two parametric Lomax distribution (LD). The exponential LD is obtained by **El-Bassiouny et al., 2015**. In **Wake et al. 2003**, they derived the form of the probability density function (PDF) from the evaluation in time of a previously truncated frequency distribution of animal live weights. Plenty of problems can be solved by reference to the random sum (RS). **Yates and Goodman, 1999** introduced the RS of independent random variables. Several studies are in the RS **(Teamah and Abd El-Bar, 2009; El-Damrawy and Teamah, 2007; El-Damrawy and El-Shiekh, 2014; El-Damrawy and Abou-Gabal, 2015).**

The weighted exponential Gomperetz distribution WE-G distribution was introduced by **Abd El-Bar and Ragab, 2019**. The generalized weighted exponential- G family, was obtained by **Teamah et al., 2020**. Recently, many studies have generated distributions **(El-Damrawy and El-Shiekh, 2021; El-Damrawy et al., 2022; El-Damrawy et al., 2024).**

The goal of this paper is to investigate an extension to truncated Lomax distribution which is BTLD. The contribution of this paper as follows: The two parametric truncated Lomax distribution (TPTLD) is considered, we derive the PDF and the CDF of BTLD in Section 1. In addition, we exhibit the PDF of the BTLD for various parameter values. In Section 2, an application of communications real data set is presented. The SF, HF, OF, RHF, MGF, and several moments are all deduced in Section 3. The BTLD's RS is obtained in Section 4. Section 5 includes some distributions and moments order statistics. In Sections 6, we obtain some special cases.

1. The Beta-Truncated Lomax Distribution

The PDF of the two parametric Lomax distributions (TPLD) is

$$f(x) = \frac{\alpha}{\lambda}\left(1+\frac{x}{\lambda}\right)^{-(\alpha+1)}, \; x>0, \alpha, \lambda>0,$$

where α and λ are the parameters for shape and scale, respectively **(Javid and Abdullah, 2015).**

If X has the TPTLD, then we will derive PDF as follows:

$$f_{LT}(x) = \frac{\alpha(1+a/\lambda)^{\alpha}}{\lambda}\left(1+\frac{x}{\lambda}\right)^{-(\alpha+1)},$$

$$x>a, \alpha, \lambda>0. \qquad (1)$$

The CDF for equation (1) is calculated as:

$$F_{LT}(x)=\int_a^x f_{LT}(x)dx=1-(1+a/\lambda)^{\alpha}$$

$$\times\left(1+\frac{x}{\lambda}\right)^{-\alpha}\quad, x>a,\alpha,\lambda>0. \quad (2)$$

The following three curves:

$\alpha=0.25,\ 0.5, 0.9$ and $\lambda=10$, $a=2$

are drawn from the PDF for TPTLD.

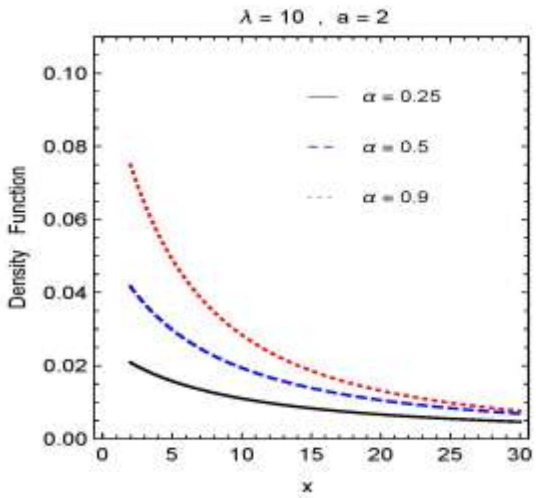

Fig. (1): The relationship between α and the PDF of TPTLD

From Fig. (1), when the value α increases, the value of the PDF of TPTLD increases.

The PDF of the beta-generated distributions is:

$$f_d(x)=\frac{g(x)}{\beta(c,b)}\left(G(x)\right)^{c-1}$$

$$\times\left(1-G(x)\right)^{b-1} \quad (3)$$

where $G(x)=F_{LT}(x), g(x)=f_{LT}(x)$ are the CDF and PDF, respectively **(El-Damrawy et al., 2024)**.

When c=1, the BTLD's PDF and CDF are determined using the relation (3).

$$f_d(x)=\ \alpha b(\lambda+a)^{\alpha b}(\lambda+x)^{-(\alpha b+1)} \quad ,$$

$$x>a,\alpha,b,\lambda>0 \quad (4)$$

and

$$F_d(x)=1-(\lambda+a)^{\alpha b}\left(\lambda+x\right)^{-\alpha b},$$

$$x>a,\alpha,b,\lambda>0. \quad (5)$$

We draw the following three curves for PDF of the BTLD when $\alpha=0.25,\ 0.5,\ 0.9$, $b=2,\ 3, 4$, $\lambda=10$ and $a=2$

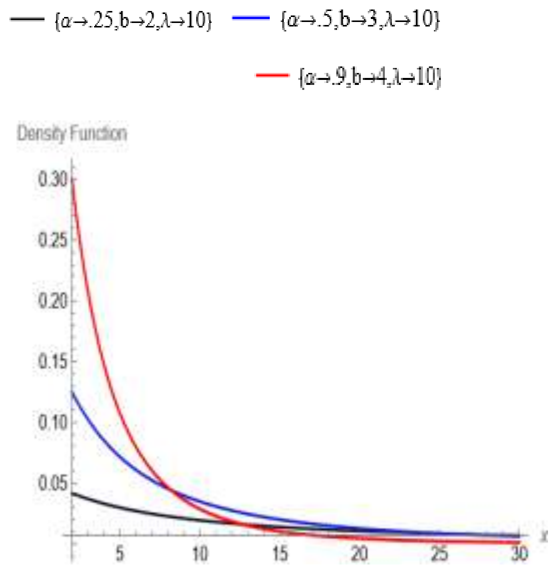

Fig. (2): The relationship between α and the PDF of BTLD

From Fig. (2), when the parameter values α, b increase, the value of PDF increases in the BTLD. The maximum likelihood estimation **(**MLE) for the unknown parameters of BTLD is derived. If $X_1, X_2, ..., X_n$ is a random sample in this section using the BTLD (α, b, λ) model, then it's log of maximum likelihood function equals $Q(x_i, \alpha, b, \lambda)$:

$$Q=Q(x_i;\alpha,\mathrm{b},\lambda)=n\,L\,\mathrm{og}\left(\alpha b\right)$$

$$+n\,\alpha b\ L\,\mathrm{og}(\lambda+a)$$

$$-(\alpha b+1)\sum_{i-1}^{n} L\,\mathrm{og}(\lambda+x_i). \quad (6)$$

By partially differentiating both sides of equation (6) with respect to α, b *and* λ equating them to zero, we obtain

$$\frac{\partial Q}{\partial \alpha}=\frac{n}{\alpha}+n\,b\,Log(\lambda+a)-b\sum_{i=1}^{n} Log\left(\lambda+x_i\right)=0,$$

$$\frac{\partial Q}{\partial b}=\frac{n}{b}+n\,\alpha\,Log(\lambda+a)-\alpha\sum_{i=1}^{n} Log\left(\lambda+x_i\right)=0,$$

$$\frac{\partial Q}{\partial \lambda}=\frac{n\,\alpha b}{(\lambda+a)}-(\alpha b+1)\sum_{i=1}^{n}\frac{1}{\left(\lambda+x_i\right)}=0.$$

We cannot obtain the exact solutions for parameters, then the MLEs will be calculated by using Wolfram Mathematica.

2. Real Data Application

In this section we need to fit the BTLD model to a real data set. The following set of data presents the repair times (Hours) for an airborne communication transceiver **(Muhammadz et al., 2021)**. The values are: 0.50, 0.60, 0.60, 0.70, 0.70, 0.70, 0.80, 0.80, 1.00, 1.00, 1.00, 1.00, 1.10, 1.30, 1.50, 1.50, 1.50, 1.50, 2.00, 2.00, 2.20, 2.50, 2.70, 3.00, 3.00, 3.30, 4.00, 4.00, 4.50, 4.70, 5.00, 5.40, 5.40, 7.00, 7.50, 8.80, 9.00, 10.20, 22.00, 24.50.

In table (1) the MLEs for the parameters of BTLD (λ , α, b), LD (λ , α) and exponential distribution (ED) are obtained. The value of the negative log likelihood function (NLOG), Bayesian information criterion (BIC), Akaike information criterion (AIC), Hannan-Quinn information criterion (HQIC) and second order of Akaike information criterion (AICc) are evaluated in table (2).

Table (1): MLEs for the data set

Distribution	$\hat{\alpha}$	$\hat{\lambda}$	$\hat{b}$
BTLD	4.386	3.514	0.465
LD	4.678	14.758	-
ED	-	0.249	-

Table (2): The statistics -Log likelihood, AIC, BIC, AICc and HQIC for the data set

Distribution	**NLOG**	**AIC**	**BIC**	**AICc**	**HQIC**
BTLD	86.68	179.36	184.42	180.03	181.19
LD	94.50	193.01	196.38	193.33	194.23
ED	95.58	193.15	194.84	193.26	193.77

It showed from table (1) and table (2) that, the above application of BTLD shows the superiority of BTLD by comparing the fitness with LD and ED.

3. Beta-Truncated Lomax Distribution Properties

For BTLD, the PDF and CDF have the following functional relationship.

$$f_d(x)=\alpha b\left(\lambda+x\right)^{-1}-\alpha b\left(\lambda+x\right)^{-1}F_d(x). \qquad (7)$$

The SF is:

$$S_d(x)=(\lambda+a)^{\alpha b}\left(\lambda+x\right)^{-\alpha b}$$

The HF is:

$$h_d(x)=\frac{\alpha b}{(\lambda+x)} \quad .$$

The OF is:

$$o_d(x)=\frac{F_d(x)}{S_d(x)}=\frac{1-(\lambda+a)^{\alpha b}\left(\lambda+x\right)^{-\alpha b}}{(\lambda+a)^{\alpha b}\left(\lambda+x\right)^{-\alpha b}}$$

The RHF is:

$$r_d(x)=\frac{f_d(x)}{F_d(x)}=\frac{\alpha\,\mathrm{b}(\lambda+a)^{\alpha b}\left(\lambda+x\right)^{-(\alpha b-1)}}{1-(\lambda+a)^{\alpha b}\left(\lambda+x\right)^{-\alpha b}}$$

The MGF is:

$$M_d(t)=E[e^{tX}]=\sum_{m=0}^{\infty}\frac{(t\lambda)^m}{m!}\sum_{j=0}^{m}\binom{m}{j}$$
$$\times(-1)^{m-j}\left(1+a/\lambda\right)^j\left(\frac{\alpha b}{\alpha b-j}\right) \quad .$$

Theorem 1:

The k^{th} moments, $k=1,2,\ldots$

represented by, $\mu_d^{(k)}$ are given by

$$\mu_d^{(k)}=\lambda^k\sum_{j=0}^{k}\binom{k}{j}(-1)^{k-j}\left(1+a/\lambda\right)^j$$
$$\times\left(\frac{\alpha b}{\alpha b-j}\right), \qquad (8)$$

where the value α is chosen in such a way that $\alpha b-j\neq 0$.

Proof: We are well aware of this.

$$\mu_d^{(k)}=\int_a^{\infty}x^k f_d(x)dx \, .$$

Using relation (4) to obtain relation (8). The formula for $k=1$ is:

$$\mu_d=\lambda\left(\frac{\alpha b(1+a/\lambda)}{\alpha b-1}-1\right)=\frac{\alpha b a+\lambda}{\alpha b-1} \quad .$$

If $k=2$, $\alpha b\geq 2$, in relation (8), then

$$\mu_d^{(2)}=\lambda^2\left[1-\frac{2\alpha b(1+a/\lambda)}{\alpha b-1}\right.$$
$$\left.+\frac{\alpha b(1+a/\lambda)^2}{\alpha b-2}\right].$$

As a result, the connection can be used to calculate the variance.

$$V_d(X)=(\lambda+a)^2\left[\frac{\alpha b}{(\alpha b-2)}-\frac{(\alpha b)^2}{(\alpha b-1)^2}\right].$$

4. The random Sum for Beta-Truncated Lomax Distribution

Allow the BTLD's PDF to be used

$$f_d(x_i)=\alpha b\,(\lambda+a)^{\alpha b}\left(\lambda+x_i\right)^{-(\alpha b+1)} \quad ,$$

$$x_i>a,\alpha,b,\lambda>0\,.$$

The following is how the aforementioned PDF's Laplace transform may be written:

$$f_X^*(s)=L\left\{f_d(x)\right\}$$
$$=\frac{\alpha b}{(1-\alpha b)(a+\lambda)}\left(\frac{e^{-as}}{s}\right) \, . \quad (9)$$

Assume the RS $S_N=\sum_{i=1}^{N}X_i$, X_i follows BTLlD , N is a random variable with a probability mass function (PMF) that is non-negative and integer-valued $g(n)=pr(N=n)$.The following connection gives the Laplace transform of the PDF for the RS of the BTLD:

$$L\left\{f_{S_N}(t)\right\}=f_{S_N}^*(s)=P_N\{f_{X_i}^*(s)\}, \quad (10)$$

where $P_N(z)$ is the probability generating function (PGF) of N .

Substituting from relation (9) to relation (10), we get:

$$f^*_{S_N}(s)=\sum_{n=0}^{\infty} g(n)\left(\frac{\alpha b}{(1-\alpha b)(a+\lambda)}\right)^n$$

$$\times\left(\frac{e^{-ans}}{s^n}\right) \qquad (11)$$

Using the inverse Laplace transform of relation (11), we can now find the PDF of the RS:

$$f_{S_N}(t)=L^{-1}\left\{f^*_{S_N}(s)\right\}$$

$$=\sum_{n=0}^{\infty} g(n)\left(\frac{\alpha b}{(1-\alpha b)(a+\lambda)}\right)^n$$

$$\times\left(\frac{(an+t)^{n-1}UnitStep[an+t]}{\Gamma(n)}\right). \quad (12)$$

Unit step [t] denotes the unit step function, which has values of 0 if $t<0$ and 1 if $t\geq 0$.

5. Properties of Order Statistics

In this part we derive some functions and moments for OS.

Let $X_1, X_2, ..., X_n$ be a random sample from the BTLD of any size n, and let $X_{1:n}\leq X_{2:n}\leq ...\leq X_{n:n}$ be the associated OR. The PDF $X_{r:n}$, $1\leq r\leq n$ is then provided (**Javid and Abdullah, 2015**).

$$f_{r:n}(x)=C_{r:n}\left\{\left[F(x)\right]^{r-1}\left[1-F(x)\right]^{n-r}\right.$$

$$\left.\times f(x)\right\}, \qquad (13)$$

where

$$C_{r:n}=\frac{n!}{(r-1)!(n-r)!}.$$

The PDF of $X_{r:n}$ and $X_{s:n}$, $1\leq r<s\leq n$, this is provided by **Javid and Abdullah, 2015.**

$$f_{r,s:n}(x,y)=C_{r,s:n}\left\{\left[F(x)\right]^{r-1}\left[F(y)-F(x)\right]^{s-r-1}\right.$$

$$\left.\times\left[1-F(y)\right]^{n-s} f(x)f(y)\right\}$$

for $\quad C_{r,s:n}=\dfrac{n!}{(r-1)!(s-r-1)!(n-s)!}$

and $-\infty<x<y<\infty$. (14)

Using (4), (5), and (13), we get the following PDF of BTLD minimum order statistics:

$$f_{d\ 1:n}(x)=\alpha b\, n\,(\lambda+a)^{\alpha b n}\left(\lambda+x\right)^{-(\alpha b n+1)}.$$

Similarly, the PDF of the biggest order statistics for the BTLD is obtained using (4), (5), and (13):

$$f_{d\ n:n}(x)=\alpha b\, n\,(\lambda+a)^{\alpha b}\left(\lambda+x\right)^{-(\alpha b+1)}$$

$$\times\left[1-\left(\lambda+a\right)^{\alpha b}\left(\lambda+x\right)^{-\alpha b}\right]^{n-1}.$$

Theorem 2:

If CDF and PDF of the BTLD are given, then PDF of the OS is given.

$$f_{d\ r:n}(x)=\alpha b\, C_{r:n}\sum_{k=0}^{r-1}\binom{r-1}{k}(-1)^k$$

$$\times\left(\lambda+a\right)^{\alpha b[(n-r)+k+1]}$$

$$\times\left(\lambda+x\right)^{-\alpha b[(n-r)+k+1]-1}.$$

Proof: Replace (4) and (5) with (13) using the binomial expansion.

Theorem 3:

Assume that the BTLDs $X_{r:n}$ and $Y_{s:n}$, $1 \le r < s \le n$ are a single PDF, the joint PDF is

$$f_{d\ r,s:n}(x,y) = \frac{(b\,\alpha)^2 C_{r,s:n}}{\lambda^2} \sum_{i=0}^{s-r-1} \sum_{j=0}^{n-s} \binom{s-r-1}{i}$$

$$\times \binom{n-s}{j} (-1)^{i+j} \left(1-\left(1+a/\lambda\right)^{\alpha b}\left(1+\frac{x}{\lambda}\right)^{-\alpha b}\right)^{i+r-1}$$

$$\times \left(1-\left(1+a/\lambda\right)^{\alpha b}\left(1+\frac{y}{\lambda}\right)^{-\alpha b}\right)^{s-r-1-i+j}$$

$$\times \left(1+a/\lambda\right)^{2\alpha b}\left(1+\frac{x}{\lambda}\right)^{-(\alpha b+1`)}\left(1+\frac{y}{\lambda}\right)^{-(\alpha b+1)} .$$

Proof: Use the binomial expansion to replace (4) and (5) into (14).

We use the BTLD to create explicit expressions for OS moments**.**

Theorem 4:

Assume that there $X_1, X_2, \ldots, X_n$ is a sizeable random sample from the BTLD and that it $X_{1:n} \le X_{2:n} \le \ldots \le X_{n:n}$ is the associated OS. The k^{th} moments $k = 1, 2, \ldots$ of order statistics are denoted $\mu_{d\ r:n}^{(k)}$ and supplied by

$$\mu_{d\ r:n}^{(k)} = \lambda^k C_{r:n} \sum_{l=0}^{n-r} \sum_{j=0}^{l+r-1} \sum_{i=0}^{k} \binom{n-r}{l}\binom{l+r-1}{j}$$

$$\times \binom{k}{i} (-1)^{l+i+j} \left(1+a/\lambda\right)^{k-i}$$

$$\times \left(\frac{\alpha b}{\alpha b\, j + i - k + \alpha b}\right),$$

the value α is selected in a way that $\left(\frac{i-k}{\alpha\, b} + j\right) \ne -1$.

Proof: We are aware of this.

$$\mu_{d\ r:n}^{(k)} = E\left(X_{r:n}^{\,k}\right) = \int_a^\infty x^k f_{d\ r:n}(x)\,dx$$

$$= C_{r:n} \sum_{l=0}^{n-r} \binom{n-r}{l} (-1)^l$$

$$\times \int_a^\infty x^k \left(1-\left(1+a/\lambda\right)^{\alpha b}\left(1+\frac{x}{\lambda}\right)^{-\alpha b}\right)^{l+r-1}$$

$$\times \frac{\alpha b}{\lambda}\left(1+a/\lambda\right)^{\alpha b}\left(1+\frac{x}{\lambda}\right)^{-(\alpha b+1)} dx \,. \quad (15)$$

Permit me to elaborate.

$$w = \left(1+a/\lambda\right)^{\alpha}\left(1+\frac{x}{\lambda}\right)^{-\alpha}$$

$$\Rightarrow -dw = \frac{\alpha}{\lambda}\left(1+a/\lambda\right)^{\alpha}\left(1+\frac{x}{\lambda}\right)^{-\alpha-1} .$$

As a result, (6.3) becomes

$$\mu_{d\ r:n}^{(k)} = \lambda^k C_{r:n} \sum_{l=0}^{n-r} \sum_{j=0}^{l+r-1} \binom{n-r}{l}\binom{l+r-1}{j}$$

$$\times (-1)^{l+j} \int_0^1 w^{\,j} \left(w^{-\frac{1}{\alpha b}}\left(1+a/\lambda\right)-1\right)^k dw \,.$$

Alternatively,

$$\mu_{d\ r:n}^{(k)} = \lambda^k C_{r:n} \sum_{l=0}^{n-r} \sum_{j=0}^{l+r-1} \sum_{i=0}^{k} \binom{n-r}{l}\binom{l+r-1}{j}\binom{k}{i}$$

$$\times (-1)^{l+i+j} \left(1+a/\lambda\right)^{k-i}$$

$$\times \left(\frac{\alpha b}{\alpha b\, j + i - k + \alpha b}\right). \quad (16)$$

We'll offer some cases:

When this $k = 1$ is combined with relation (16), we get the following:

$$\mu_{d\ r:n} = E\left(X_{\ r:n}\right) = \int_a^\infty x\, f_{d\ r:n}(x)\,dx$$

$$= \lambda C_{r:n} \sum_{l=0}^{r-1} \binom{r-1}{l} (-1)^l$$

$$\times \left[\frac{\alpha b(1+a/\lambda)}{\alpha b(l+n-r+1)-1} - \frac{1}{l+n-r+1} \right] .$$

Now, if we put this $k = 2, \alpha \geq 2$ in relation (16), we may get the r^{th} OS's second-order moment as

$$\mu^{(2)}_{d\ r:n} = E\left(X^{\ 2}_{\ r:n}\right) = \int_a^\infty x^{\ 2} f_{d\ r:n}(x)\,dx$$

$$= \lambda^2 C_{r:n} \sum_{l=0}^{r-1} \binom{r-1}{l} (-1)^l$$

$$\times \left[\frac{\alpha b(1+a/\lambda)^2}{\alpha b(l+n-r+1)-2} \right.$$

$$\left. - \frac{2\alpha b(1+a/\lambda)}{\alpha b(l+n-r+1)-1} + \frac{1}{l+n-r+1} \right].$$

As a result, the r^{th} order statistic's variance can be calculated using the relationship.

$$V\,(X_{\ r:n}) = \mu^{(2)}_{d\ r:n} - \left(\mu_{d\ r:n}\right)^2 .$$

The extreme orders of statistics' mean, variance, and other statistical metrics are always of great interest. We can get the mean of the smallest order statistic by taking this r^{th} :

$$\mu_{d\ 1:n} = \frac{n\,\alpha\, b\lambda(1+a/\lambda)}{\alpha\, bn-1} - \lambda\ .$$

The smallest order statistic's second-order moment can also be calculated as:

$$\mu^{2}_{d\ 1:n} = \lambda^2 \left[\frac{\alpha\, b\, n\,(1+a/\lambda)^2}{\alpha\, b\, n-2} \right.$$

$$\left. - \frac{2\alpha\, b\, n(1+a/\lambda)}{\alpha\, b\, n-1} + 1 \right]$$

Therefore,

$$V_d\,(X_{\ 1:n}) = \mu^{(2)}_{d\ 1:n} - \left(\mu_{d\ 1:n}\right)^2$$

$$= \frac{\alpha\, b\, n\,\lambda^2(1+a/\lambda)^2}{\alpha\, b\, n-2}$$

$$- \left(\frac{\alpha\, b\, n\,\lambda\,(1+a/\lambda)}{\alpha\, b\, n-1} \right)^2$$

As a result, the greatest order statistic's mean, second-order moment, and variance can be calculated as

$$\mu_{d\ n:n} = \lambda\, n\alpha b(1+a/\lambda) \sum_{l=0}^{n-1} \binom{n-1}{l} (-1)^l$$

$$\times \frac{1}{\alpha b(l+1)-1} - \lambda\ .$$

$$\mu^{(2)}_{d\ n:n} = \lambda^2 n\alpha b(1+a/\lambda)^2 \sum_{l=0}^{n-1} \binom{n-1}{l} (-1)^l$$

$$\times \frac{1}{\alpha b(l+1)-2}\ - 2\alpha b\,\lambda^2(1+a/\lambda) \sum_{l=0}^{n-1} \binom{n-1}{l}$$

$$\times (-1)^l\ \frac{1}{\alpha b(l+1)-1} + \lambda^2,$$

$$V_d\,(X_{\ n:n}) = \lambda^2 n \sum_{l=0}^{n-1} \binom{n-1}{l} (-1)^l$$

$$\frac{\alpha b(1+a/\lambda)^2}{\alpha b(l+1)-2}\ - \left(\lambda\, n\alpha b(1+a/\lambda)\right)^2$$

$$\times \left(\sum_{l=0}^{n-1} \binom{n-1}{l} (-1)^l\ \frac{1}{\alpha b(l+1)-1} \right)^2 .$$

6. Special cases

Case (1):

If $a=0, b=1$ in relations (4) and (5), then the PDF and CDF of the LD will be

$$f(x)=\frac{\alpha}{\lambda}\left(1+\frac{x}{\lambda}\right)^{-(\alpha+1)},$$
$$F(x)=1-\left(1+\frac{x}{\lambda}\right)^{-\alpha} \quad x>0, \alpha, \lambda>0.$$

The PDF and CDF of the LD are the same as in (**Javid and Abdullah, 2015**).

Case (2):

The PDF for RS of the LD is obtained where $a=0, b=1$:

$$f_{S_N}(t)=\sum_{n=0}^{\infty} g(n)\left(\frac{\alpha}{\lambda(1-\alpha)}\right)^{n}\left(\frac{t^{n-1}}{\Gamma(n)}\right).$$

Case (3):

If this is the case $a=0, b=1$, the PDF of the LD's minimum OS is

$$f_{d\,1:n}(x)=\frac{\alpha n}{\lambda}\left(1+\frac{x}{\lambda}\right)^{-(\alpha n+1)}.$$

And the PDF for the LD's largest OS is

$$f_{d\,n:n}(x)=\frac{\alpha n}{\lambda}\left(1+\frac{x}{\lambda}\right)^{-(\alpha+1)} \times\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]^{n-1}.$$

These are the identical PDFs for the LD's minimum and maximum OS. **(Javid, and Abdullah, 2015)** provide these.

Conclusion

The BTLD is investigated and various mathematical properties are calculated. The BTLD's PDF and CDF were created. In addition, the BTLD's PDF and CDF were collected. The SF, HF, OF, RHF, MGF, and a few more moments are calculated. We obtained the BTLD's RS. For the BTLD, the PDF of the smallest OS and the PDF of the largest OS are introduced, and various theorems are proven. We developed expressions for OS moments from the BTLD. The real data set is presented to show the superiority of BTLD by comparing the fitness with LD and ED. The BTLD is used to obtain some distributions as in special cases.

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توزيع بيتا لوماكس المبتور مع بيانات الاتصالات

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الهدف من هذه البحث هو تحقيق تعميم لتوزيع لوماكس وهو توزيع بيتا لوماكس المبتور ، وتم الحصول على دالة كثافة الاحتمال ودالة التوزيع التراكمي ، والعزوم ، وتوزيع اصغر متغير فى العينة العشوائية وتوزيع أكبر متغير فى العينة العشوائية وتمت مناقشة المجموع العشوائي لمتغيرات تتبع توزيع بيتا لوماكس المبتور.

وتم تقديم تطبيق لنموذجنا المقترح على مجموعة من البيانات الحقيقية للاتصالات لإظهار تفوق هذا النموذج الجديد من خلال مقارنة درجة مرونته مع توزيع لوماكس والتوزيع الأسي. وتم استنتاج دالة البقاء وداله الخطر والدالة الفردية ودالة الخطر المحجوزة والدالة المولدة للعزوم. وقدمنا بعض الحالات الخاصة.