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MULTIVARIATE PROCESS CAPABILITY ASSESSMENT AND IMPROVEMENT: A CASE STUDY

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ABSTRACT

In today's competitive manufacturing environment, the challenge is to responsively produce products with minimum cost and high quality. Achieving and controlling the targeted quality level in manufacturing processes does not only increase customer satisfaction, but it can also result in significant cost and time savings. Further, measuring the process performance is a critical issue in process improvement initiatives. The common practice in several industries is using the Univariate Process Capability Indices (UPCIs) to measure the process performance, which are based on only a single quality characteristic. In most of the applications, it is not acceptable to judge the performance based on a single quality characteristic as it actually relies on more than one characteristic. In this paper, univariate and multivariate PCIs are used to measure the performance of the flare making process. This process is a critical step in the straight fluorescent light bulb production line. In addition, multivariate control charts such as the Hotelling T^2 as well as the Multivariate Exponentially Weighted Moving Average (MEWMA) are constructed for the collected data to verify that the process is in control before assessing its capability. Besides, Principal Component Analysis (PCA) and Joint Normal Distribution (JND) techniques are applied in the multivariate process capability assessment. In this paper, Multivariate Process Capability Indices (MPCIs) have been evaluated to compare the process performance before and after improvement efforts. In the considered case study, MPCIs provide the user with an overall assessment of process capability regardless of the fluctuations in the individual variables capabilities.

KEYWORDS

Process capability, Multivariate process capability, Principal component analysis, Multivariate control charts.

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INTRODUCTION

Most of the manufacturing organizations are struggling to survive in today's highly competitive marketplace through adopting continuous quality improvement programs. Quantitative assessment of the processes performance is a key step in the application of improvement methodologies such as the widely used Six Sigma DMAIC (Define- Measure-Analyze-Improve-Control) methodology [1]. Process Capability Indices (PCIs) are mainly utilized to compare the natural variation of a process with the specification limits. Process capability analysis is critical not only for evaluating the current status of a manufacturing process, but also for observing the effects of improvement efforts.

Process capability indices such as C_p or process performance such as P_p are commonly demonstrated by a histogram as well as calculations for predicting the number of parts out-of-specifications. Process capability analysis necessitates the process to be statistically in control. Thus, if any assignable cause exists, then corrective actions must be taken and the control chart must be revised by eliminating the assignable cause signal. Univariate Process Capability Indices (UPCIs) are applied to measure the process performance for individual quality characteristics. As a result of modern technology and advances in manufacturing processes, a single quality characteristic cannot reflect the overall quality of a product. Furthermore, the rapidly changed customer requirements may result in including more complicated features in the product design. In this context, multiple quality characteristics must be assessed simultaneously using Multivariate Process Capability Indices (MPCIs). To measure the capability of a single quality characteristic, only the inherent variation in its structure is required. However, multivariate capability assessment does not only involve the individual variances of each quality characteristic, but the correlations between the quality characteristics are also considered [2].

The relation between the process variability and the specified tolerance has been formalized by considering the standard deviation σ of the process. Producing within the specification limits necessitates that the distance between the Upper Specification Limit (USL) and Lower Specification Limit (LSL) to be equal to or greater than the process width. Several research work focused on the application of capability measures either to univariate or multivariate quality characteristics. UPCIs such as C_p and C_{pk} were applied to the solder bump processing and boring operation as illustrated in [3, 4]. Medles et. al.[5] applied the UPCIs on the electrostatic separation processes after using control charts to verify that the process is in statistical control. Liu [6] studied the performance of thermal process (hardening at continuous furnaces) to ensure producing components within the tolerance limits. Besides, univariate and multivariate PCIs have been employed to quantify the process performance in the manufacturing of locomotive wheels [7].

This paper demonstrates a case study devoted to the assessment of multivariate capability of the flare making process at Toshiba fluorescent light bulb factory of ElAraby Group, Egypt. The main purpose of this paper is measuring the multivariate capability of the process in phase *I* after insuring that the process is in control using multivariate control charts. If the process is found to be incapable or not achieving the desired goal, then corrective actions are recommended in order to improve the capability of the process. In such a case, multivariate capability must be reevaluated in phase *II* to quantify the achieved process improvements. The ultimate objective of

this research is to highlight the effectiveness of multivariate capability in assessing the performance of multivariate processes using principal component analysis and joint normal distribution techniques. Essentially, this can be considered as motivating application encouraging quality practitioners in industry to cease dependence on univariate capability assessment in situations necessitate multivariate capability assessment.

The paper is organized as follows: the subsequent section presents the general concepts of the multivariate control charts such as Hotelling T^2 and Multivariate Exponentially Weighted Moving Average (MEWMA); followed by a section that demonstrates in details the univariate process capability, provides a review of the multivariate process capability indices, explains how the multivariate process capability can be assessed using the Principal Component Analysis (PCA) technique; then a section devoted to the application on the considered industrial case study with the results and discussions; and conclusions are highlighted in last section.

MULTIVARIATE CONTROL CHARTS

The product quality mainly relies on the joint effect of several variables instead of their individual contributions. Unfortunately, univariate control charts can only monitor a single quality characteristic at a time. Despite the ease of signal interpretation in these charts, it does not count for the interactions between the other variables within the process. However, it is a fact that in practical applications, most of the processes are multivariate in nature. When these variables are correlated, multivariate control charts are more appropriate to monitor these variables simultaneously [8]. The most widely used multivariate control charts are Hotelling T^2 that is devoted to detect large shifts in the process and MEWMA that is faster and more sensitive in detecting small shifts.

Generally, the main issue in multivariate control charts is the interpretation the out-of-control signals, i.e., the identification of which variable or variables are responsible for the out-of-control signals. However, univariate control charts and decompose approach are helpful methods that can be used to overcome this problem and these have been used in this work for explaining the out-of-control signals resulted in the Hotelling T^2 and MEWMA charts. For more illustration, one may refer to the previous work of the authors of this paper [9].

Hotelling T^2 control chart is the most popular chart in the conventional multivariate process monitoring and control. It is applied for monitoring the mean vector of the process. It integrates multiple correlated variables together which are Independent and Identically Distributed (IID) and also correlated. In case of unknown parameters, assume the multivariate normal distribution with unknown mean vector μ and unknown covariance matrix Σ . The values of \bar{X} and S that are estimated from the preliminary data are substituted for μ and Σ . The test statistic of the Hotelling T^2 is shown in (1), as illustrated in [10].

$$T^2 = n(\bar{X} - \bar{\bar{X}})'S^{-1}(\bar{X} - \bar{\bar{X}}) \quad (1)$$

The mean of each subgroup is defined by \bar{X} , the mean of all subgroups is denoted by $\bar{\bar{X}}$, the variance-covariance matrix denoted by S , and n is the number of

observations in subgroup (size of subgroup). Hotelling T^2 control chart is normally implemented in two phases. The phase *I* analysis is sometime called a retrospective analysis and the control limits for this phase are illustrated in (2). Moreover, phase *II* control limits are constructed using (3). These equations were illustrated in [10]. Note that m is the number of preliminary subgroups, P is the number of variables, and α is the type *I* error.

$$UCL = \frac{P(m-1)(n-1)}{mn-m-p+1} F_{\alpha,P,mn-m-p+1} \quad (2)$$

$$LCL = 0$$

$$UCL = \frac{P(m+1)(n-1)}{mn-m-p+1} F_{\alpha,P,mn-m-p+1} \quad (3)$$

$$LCL = 0$$

The MEWMA control chart is more sensitive than the Hotelling T^2 for detecting small shifts in the process. This chart operates by determining the statistic T^2 , that illustrated in (4) and the parameters of this equation are calculated using (5) and (6) [10].

$$T_i^2 = Z_i' \Sigma_{z_i}^{-1} Z_i \quad (4)$$

$$Z_i = \lambda X_i + (1 - \lambda) Z_{i-1} \quad (5)$$

$$S_{z_i} = \frac{\lambda}{2-\lambda} [1 - (1 - \lambda)^{2i}] \Sigma \quad (6)$$

where, S_{z_i} is the covariance matrix, λ is the weight assigned to the current measurement ($0 < \lambda < 1$), and i is the sample number. Denote that the $Z_0 = zero$. Since the statistic T^2 is always non-negative, only the UCL is needed to work with this chart. As the covariance of Z_i (S_{z_i}) relies on the considered number of samples, the UCL will also depends on i . The upper control limit becomes narrow at the initial start of the MEWMA chart and increases with i . The upper control limit of this chart is shown in (7), which illustrated in [11].

$$UCL_i = L \left[\frac{\lambda(1-(1-\lambda)^{2i})}{n(2-\lambda)} \right]^{1/2} \quad (7)$$

After running the EWMA control chart for several times, the variances and upper control limit will approach a steady state value given as (8), and (9), respectively.

$$S_{z_i} = \Sigma \left[\frac{\lambda}{n(2-\lambda)} \right] \quad (8)$$

$$UCL = L \left[\frac{\lambda}{n(2-\lambda)} \right]^{1/2} \quad (9)$$

PROCESS CAPABILITY

Basically, a process capability may be specified by the range that bounds all the possible values of a particular quality characteristic resulted from the process under certain conditions [12]. Besides, Process capability indices (PCIs) are defined to compare the process natural variation with the specification limits of the product quality characteristic. This comparison can be realized by dividing the spread

between the process specifications over the spread of the process values as measured by six process standard deviation units for normally distributed processes. Practically, the use PCIs is more convenient in assessing the process performance as it helps in reducing the multifaceted information related to a process to a single measure [13].

For instance, process capability index (C_p) is developed to quantify the ratio between the overall process variation and the specified manufacturing tolerance. It just relates the process capability to the specification range, but it does not relate the location of the process with respect to the specifications. Hence, it is called the potential capability index. However, another capability index (C_{pk}) is called the actual process capability index [14]. The value of C_{pk} relative to C_p represents a direct measure of the process off-centering effect. As a general rule, if process distribution is centered between the specification limits, the value of C_{pk} will be similar to the C_p value. Furthermore, the actual process capability index C_{pk} is commonly applied as it measures how far is the average of a process from the closer specification limit in terms of 3σ distances. Besides, Table 1 demonstrates the equations that can be used to estimate the different UPCIs as well as their usage.

Many authors such as [15-19] developed and presented multivariate process capability indices for assessing the capability of the process. Wang and Du & Wang and Chen [17, 19] proposed multivariate extension for C_p , C_{pk} , C_{pm} , and C_{pmk} based on the principal component analysis, which transforms number of originally related measurement variables into a set of uncorrelated linear function. The principal component analysis technique can be used to assess the capability of any manufacturing process as long as it has multiple related variables. This technique was applied by many authors in many different situations such as Xekalaki and Perakis [20], Shinde and Khadse [21], and Perakis and Xekalaki [22]. Chan et. al. [15] introduced a version of the multivariate process capability index C_{pm} or multivariate normal case. This index takes into account both the proximity to the target and the variation observed in the process. In addition, it explains the shape of bivariate specification for independent, dependent uncorrelated, and correlated variable cases. Multivariate capability index defined as a ratio of two volumes R_1 and R_2 , where R_1 is a modified tolerance region and R_2 is a scaled 99.73% percent region or elliptical region under the normality assumption [18].

Chen [16] proposed a multivariate process capability index over a rectangular solid tolerance zone. Then, it proceeds to deal with a general type of tolerance zone, which includes rectangular solid as a special case. The multivariate capability vector [C_{pm} , PV , LI] consists of three components. The first one is a ratio of areas or volumes which is equivalent to the lengths ratio in the C_p . The second component represents the significance level PV of the observed value with the Hotelling's statistic T^2 . Whereas, the third component LI takes a value of 0 or 1 [18]. Furthermore, a recent review of multivariate and univariate process capability indices is provided in [23].

Wang and Du [17] proposed a process performance indicator for multivariate data using principal component analysis (PCA). This method is capable of reducing the number of p variables to a fairly fewer set of k -derived variables that preserves most of the information in the original p variables [24]. Beltran [25] explains the procedure

of PCA as it is the most effective technique that can be applied to transform a set of correlated variables into a set of liner combination of uncorrelated variables that account for decreasing proportions of the variation of the original observations. One of the advantages of the PCA technique is that it is readily available in most of the software packages and it is a well-established technique in statistical multivariate analysis, as well. In addition, the application of PCA does not necessitate the multivariate normal assumption. The i^{th} principal component defined as in (10). Note that, U is composed of the columns of $U_1, U_2 \dots U_m$, which are the eigenvector of covariance matrix S , and x is the vector of the observations on the original variables. The specification limits and target value of PC_i are defined in (11). The multivariate process capability index can be simply determined by (12), and (13). In addition, C_p ; PC_i can be replaced by C_{pk} ; PC_i or C_{pm} ; PC_i .

$$PC_i = U^T x, \quad i = 1, 2, \dots, m \quad (10)$$

$$\begin{cases} LSL_{PCi} = U_i^T LSL \\ USL_{PCi} = U_i^T USL \\ T_{PCi} = U_i^T T, \quad i = 1, 2, \dots, m \end{cases} \quad (11)$$

$$\widehat{MC}_p = (\prod_{i=1}^m \widehat{C}_p; PC_i)^{1/m} \quad (12)$$

$$\widehat{C}_p; PC_i = (USL_{PCi} - LSL_{PCi})/6S_{PCi}^2 \quad (13)$$

On the other hand, Joint normal distribution (JND) technique can be employed to assess the multivariate process capability, but it operates only under the assumption that the data is coming from normal distribution. Mainly, JND procedure determines the probability that the items characterized by two or more variables meet established specifications limit. Particularly, in case of correlated variables, it is critical to consider their joint behavior as illustrated in [26]. Because of looking at each variable individually may result in a misleading depiction of the overall process capability.

CASE STUDY

The case study presented in this paper has been accomplished through the investigation of the Toshiba straight fluorescent light bulb production line at El-Araby Group. The factory is located in Kofor El Ramel, Quesna Industrial City, El-Menoufia, Egypt. The manufacturing process has been analyzed and it has been found that the flare making process is a critical step in the manufacturing process as it has a significant effect on the defect rates. In this paper, the main target is to assess and improve the multivariate capability of the flare making process using multivariate statistical process control techniques and multivariate process capability analysis. This paper depends on the previous work conducted and published by the same authors of this paper as illustrated in [9], which presents monitoring the dynamic behavior of the flare making process using multivariate control charts. In addition, decompose approach was used to interpret the out-of-control signals, and Autoregressive Integrated Moving Average (ARIMA) models have been applied to reduce the effect of the dynamic behavior. In flare making process, the variables to be monitored are flare diameter, stem diameter, flare height, and temperature. However, the output variables which are considered in the process capability

assessment are flare diameter with specification limits 25.5 ± 0.5 mm and flare height with specification limits 22 ± 0.5 mm.

Multivariate Control charts (Phase I)

Multivariate Hotelling T^2 is constructed for residual data as illustrated in Fig. 1. Besides, Table 2 demonstrates the actual correlation among residual variables. Hotelling T^2 chart illustrated in Fig. 1 reveals that the process has failed out at samples 6 and 22. The flare diameter variable is responsible for the out-of-control signal in sample 6 and stem diameter variable is responsible for the signal in sample 22. By investigating the process, it has been found that these signals are due to stopping and restarting the process during data collection. Therefore, the causes of out-of-control signals have been identified as assignable causes in these samples and have been eliminated. Thus, Hotelling T^2 control chart is revised as illustrated in Fig. 2. Revised Hotelling T^2 control chart reveals that the process become in-control. These charts and the procedure to interpret the out of control signals were illustrated in reference [9].

Univariate and Multivariate Process Capability (Phase I)

The primary step in the process capability assessment is checking the normality assumption of the data in order to assure the accuracy of the calculations. In this paper, the Shapiro-Wilk test examines how closely the points fall along a straight line on a normal probability plot. If the smallest P-value for the Shapiro-Wilk test performed is less than 0.05, then the normality assumption is violated at 95% confidence limit. Figures 3 and 4 show the probability plot for flare diameter and flare height; respectively. Flare diameter and flare height have p-values 0.75198 and 0.2387; respectively. Therefore, the assumption that the flare diameter and flare height come from normal distribution cannot be rejected.

Univariate process capability was applied on the revised flare diameter and the revised flare height. For the flare diameter with specification 25.5 ± 0.5 mm, it has been found that all the values of process capability indices are less than one ($C_p=0.81$ and $C_{pk}=0.62$). Therefore, the process is incapable of producing flare diameter within specification. However, the process is capable of producing the flare height within the specification limits (22 ± 0.5 mm), as the results indicate that $C_p=1.42$, and $C_{pk}=1.34$.

Joint normal distribution technique is applied to determine the percentage of items outside a set of multivariate specification limits. In the considered case, the estimated frequency of non-conformities with respect to at least one of the two variables equals 33324.3 per million (DPM). Table 3 illustrates the result of the capability test. Besides, Fig. 5 reveals that the process is beyond the USL and LSL in the direction of flare diameter. However, the flare height is within the specification limits.

Principal component analysis (PCA) technique was employed for assessing the multivariate process capability using the variance-covariance matrix that is illustrated in Table 4 to estimate the eigenvalue and eigenvector for each component.

Equations 14 and 15 can be constructed from concluded eigenvector from the analysis of variance covariance matrix. The eigenvalue of the first principle

component equals to 0.044273 and it contributes by 78.5% of the overall variance of the data. In the meanwhile, the second one equals to 0.012103.

$$PC_1 = 0.975 \times \text{Flare diameter} + 0.224 \times \text{Flare height} \quad (14)$$

$$PC_2 = -0.224 \times \text{Flare diameter} + 0.975 \times \text{Flare height} \quad (15)$$

Analysis of multivariate process capability using PCA for phase I indicates that the overall potential process capability (MC_p) equal to 0.95 and the overall actual multivariate process capability (MC_{pk}) equal to 0.75. Furthermore, the overall defect part per millions equals to 33324. Analysis of multivariate process capability reveals that the overall process capability is less than one. Therefore, the process is incapable to produce flare diameter within specification limits. Consequently, corrective actions must be taken to improve the capability of the process. The main purposes of these corrective actions are to reduce the variability in the flare diameter and make sure that the center of the process is close to the target values. The corrective actions that have been carried out practically on the flare making process involve the following: cleaning all the burners regularly before feeding the stem to the machine in order to stabilize the temperature during the production, changing the pins and sleeves of the mechanism that produces flare diameter more frequently, and finally, adjusting the stroke of the cam mechanism in order to achieve the target of flare diameter (Target= 25.5 mm). After implementing the corrective actions, another data set has been collected from the process to evaluate the effect of process improvements.

Multivariate Control charts (Phase II)

After taking the corrective actions, multivariate control charts must be constructed to verify that the process is in-control before assessing the process capability. Hotelling T^2 and MEWMA control charts reveal that the process is in-control as displayed in Figs. 6 and 7. Where, MEWMA control chart has a parameter λ equals to 0.2.

Univariate and Multivariate Process Capability (Phase II)

Normality test for flare diameter and flare height reveals that the flare diameter data has P-value equals to 0.00291446 (i.e., flare diameter is non-normally distributed). Therefore, Johnson transformation technique is used to convert the data of the flare diameter to normal. Johnson transformation technique is used to produce normally distributed variables with approximate mean (μ) =0 and approximate standard deviation (σ) =1 in order to be more accurate in determining the capability indices. Fig. 8 shows the Johnson transformation for flare diameter. The P-value of flare diameter after transformation increased to 0.604951.

The univariate capability indices assessed during phase II are presented as follows: for the transformed flare diameter variable the values become $C_p=1.27$ and $C_{pk}=0.7$, while for the flare height variable they become $C_p=1.31$ and $C_{pk}=1.12$. Univariate capability analysis indicates improvements in the process as the performance of the flare diameter has been significantly improved than before.

In multivariate process capability techniques, JNDT illustrates that the process has been improved and the overall defect part-per-million has been decreased to 18419.2

as shown in Fig. 9. Analysis of multivariate process capability using PCA after corrective actions indicates that the overall potential process capability (MC_p) equals to 1.1 and the overall actual process capability (MC_{pk}) equals to 0.91. Tables 5 and 6 illustrate the eigenvalues and eigenvectors for each principal component; respectively.

CONCLUSION

This paper has been devoted to the assessment and improvement of multivariate capability of the flare making process at Toshiba fluorescent light bulb factory of ELARABY Group, Egypt. To do so, multivariate control charts have been constructed and these reveal that the process in phase *I* is out-of-control. However, process capability assessment requires the process to be in a state of statistical control. Therefore, the Hotelling T^2 control chart developed in phase *I* was revised after identifying and eliminating its out-of-control signals. The multivariate process capability indices estimated in phase *I* reveal that the process is incapable to produce parts within specification limits. After carrying out some corrective actions in phase *II*, it has been observed that there is no indication of out-of-control signals and the process capability has been significantly improved.

The presented application emphasized the effectiveness of the multivariate capability in assessing the performance of multivariate processes using principal component analysis and joint normal distribution techniques. Besides, this application may encourage the quality practitioners in different industries to start ceasing the dependence on univariate capability assessment for manufacturing process having multiple related variables. Instead, the results demonstrate that MPCIs can help in appraising the current status of the process as well as evaluating the improvement efforts through an overall assessment of the process performance regardless of the fluctuations in the individual capabilities of its variables. In the considered case study, improvement efforts have resulted in decreasing the defect rate from 33324 DPM to 18419 DPM. In addition, the potential multivariate process capability index (MC_p) has improved from 0.95 to 1.1, while the actual multivariate process capability index (MC_{pk}) has improved from 0.75 to 0.91. Analysis of the process capability reveals that the MC_p has improved by 15.8% and MC_{pk} has improved by 20%. It means that the corrective actions that have been taken reduced the variability in the process by 15.8% and the center of the process become closer to the target of the specifications by 20% than before.

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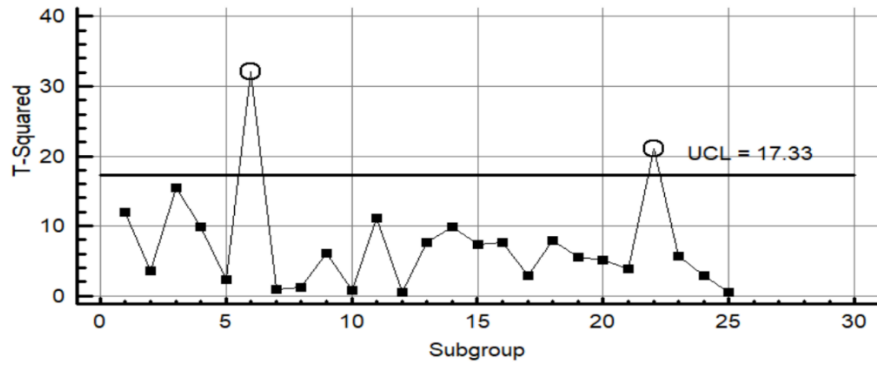


Fig. 1. Hotelling T^2 control chart for four residual variables.

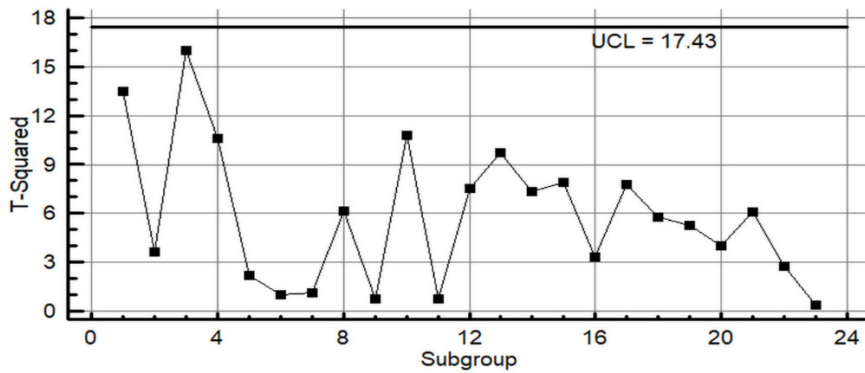


Fig. 2. Hotelling T^2 control chart for revised data.

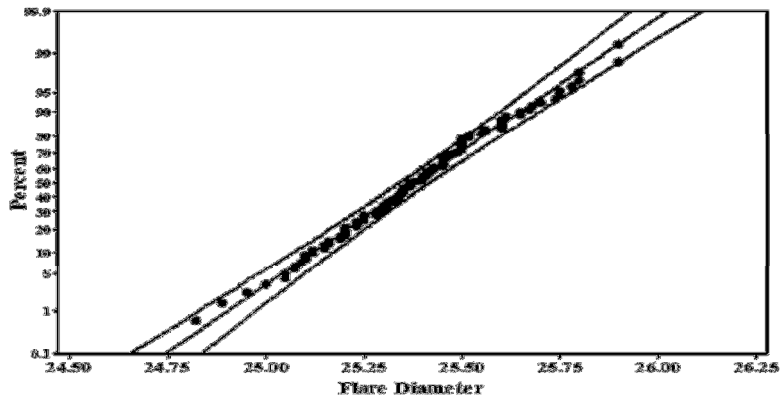


Fig. 3. Probability plot for flare diameter.

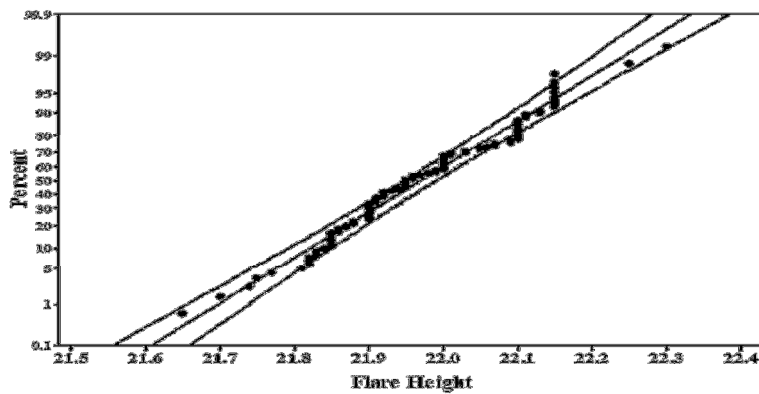


Fig. 4. Probability plot for flare height.

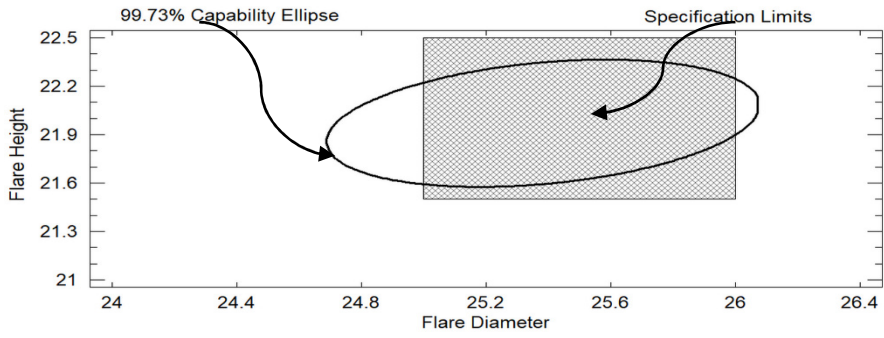


Fig. 5. Top view of the 3D effect for two variables in phase I.

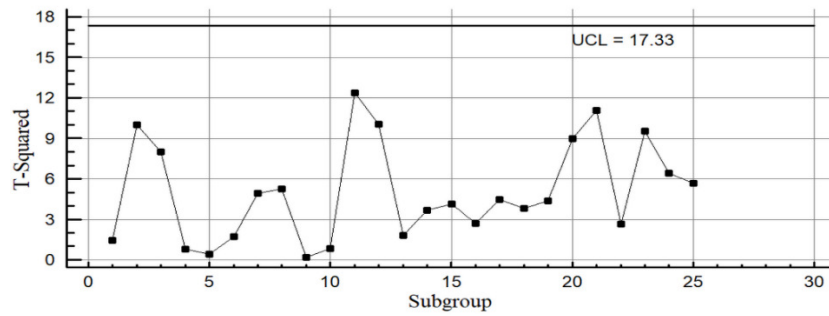


Fig. 6. Hotelling T^2 control chart for phase II.

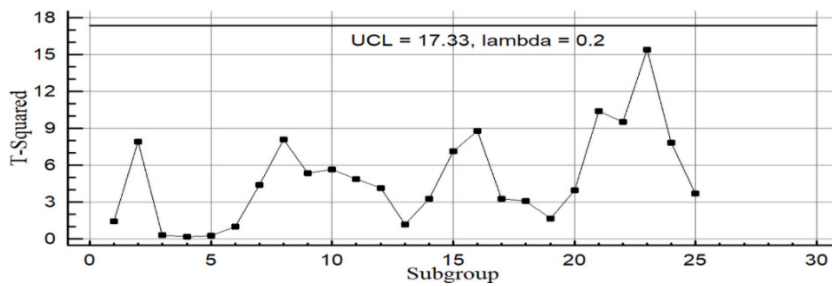


Fig. 7. MEWMA control chart for phase II.

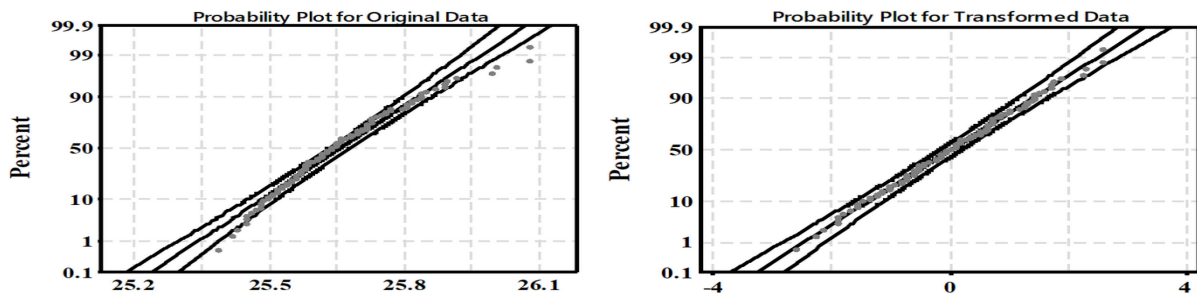


Fig. 8. Johnson transformation technique for flare diameter.

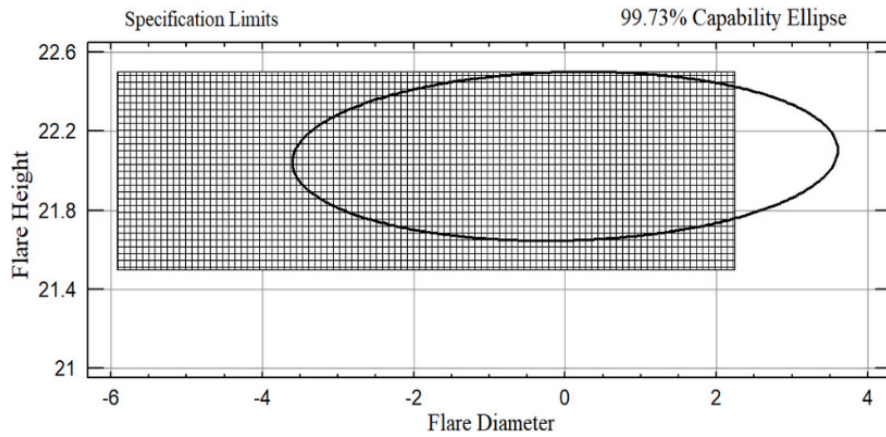


Fig. 9. Top view for the 3D effect for two variables in phase II.

Table 1. The equations of UPCIs.

Index	Estimation Equation	Usage
C_p	$C_p = \frac{USL - LSL}{6\sigma}$	It estimates what the process is capable of producing if the process mean is to be centered between the specification limits.
C_{pl}	$C_{pl} = \frac{\bar{X} - LSL}{3\sigma}$	It estimates process capability for specifications that consist of a lower limit only.
C_{pu}	$C_{pu} = \frac{USL - \bar{X}}{3\sigma}$	It estimates process capability for specifications that consist of an upper limit only.
C_{pk}	$C_{pk} = \text{Min} (C_{pl}, C_{pu})$	It measures how far the process mean is from the nearer specification limit.

Table 2. The correlation among residual variables.

Variables		Flare diameter	Stem diameter	Flare height
Stem diameter	Correlation	0.383		
	P-value	0.000		
Flare height	Correlation	0.071	-0.054	
	P-value	0.433	0.553	
Temperature	Correlation	-0.030	-0.110	0.153
	P-value	0.742	0.221	0.088

Table 3. Estimated frequency of non-conforming the specification.

Variable	Estimated Beyond Spec.	DPM
Flare diameter	3.32938 %	33293.8
Flare height	0.00325184 %	32.5184
Joint	3.33243 %	33324.3

Table 4. Variance covariance matrix.

Variables	Flare diameter	Flare height
Flare diameter	0.0426592	0.0070231
Flare height	0.0070231	0.0137173

Table 5. Eigenvalue for each principal component.

Component Number	Eigenvalue	Percent of variance	Cumulative percentage
1	0.018616	54.3%	54.3%
2	0.015674	45.7%	100%

Table 6. Eigenvector for each principal component.

Components Variables	PC_1	PC_2
Flare diameter	0.926	-0.377
Flare height	0.377	0.926