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Research Paper

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The Effect of Laser Pulses on a Porous Semiconductor Elastic **Medium through a Photothermal Process**

Kh. Lotfy¹, A. El-Bary², N. Anwer^{1, 3} and R. S. Tantawi¹

¹Department of Mathematics, Faculty of Science, P.O. Box 44519, Zagazig University, Zagazig, Egypt. ²Arab Academy for Science, Technology and Maritime Transport, P.O.Box 1029, Alexandria, Egypt. ³Department of Basic science. Zagazig Higher institude of engineering & Technology. Zagazig. Egypt. khlotfy 1@yahoo.com, aaelbary@aast.edu , nerminanwar456@gmail.com, ramadan tantawi1@yahoo.com

ABSTRACT: The primary objective of the ongoing research is to comprehend how waves generated in porous semiconductor elastic media behave during the photothermal process when impacted by laser pulses. In this process, the surface of the semiconductor material is illuminated with a laser pulse. Through the analysis of normal modes, the researchers aim to derive precise expressions for the main physical fields involved in photoexcitation processes. To achieve comprehensive solutions for these physical fields, mechanical, thermal, and recombination plasma loads are applied at the surface of the medium. The governing field equations are formulated in non-dimensional forms, incorporating coupled elasticity theory, plasma diffusion equations, void equation, and moving equations. The study focuses on the deformation in two-dimensional (2D) space. The graphical results obtained consider the influences of laser pulses and porosity, and these findings are compared to the behavior of silicon semiconductor material

KEYWORDS: Photo-thermoelasticity; Semiconductor; Laser pulses; Silicon; porosity; Normal mode method.

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I. INTRODUCTION

In recent times, there has been a rise in the variety of contemporary materials that can be categorized as conductive or insulating. These materials do not exhibit the extreme insulating properties of glass or the high conductivity of aluminum. They are known as semiconductors, with substances like silicon and carbon being prominent examples. These semiconductors are abundantly found in our surroundings. Recent studies have emphasized the significance of heat conduction in solid semiconductor materials, particularly in relation to changes in mass and heat transfer, often referred to as thermal diffusivity [1, 2]. Numerous researchers have employed the generalized thermoelasticity theory to explain how elastic and thermal waves behave in elastic materials, including semiconductors, particularly those with semi-insulating properties. Previously, semiconductors were primarily examined solely as elastic media. However, contemporary studies emphasize the significance of heat conduction in solid semiconductor materials, particularly concerning changes in mass and the transport of heat (thermal diffusivity). In today's renewable energy-focused technology, the semiconductor material industry, particularly in solar cells, is extensively utilized.

When the laser beam interacts with the surface of the semiconductor material, the electrons that are generated create both free charge and plasma waves. As a result, it is important to examine the interaction between thermal waves, plasma waves, and elastic materials. Currently, there is a growing interest in studying this interaction through both analytical and experimental means.

The transportation processes resulting from the excitation of a photo by laser pulses are due to the thermal effects caused by the pulses. The rapid movement of electrons and the generation of plasma waves (carrier density) in semiconductor materials are a direct result of the thermal excitation caused by the laser pulses. Laser pulses are intense bursts of light energy that are generated by a laser device. These pulses are created by an optical amplification process. Laser pulses have some unique characteristics that make them incredibly useful in various fields of science, technology, and medicine. These characteristics include high intensity, short duration, and high precision. Laser pulses are used in a wide range of applications, including telecommunications, spectroscopy, microscopy, and even in scientific research to create and study extreme conditions like high temperature and high pressure plasma. They have revolutionized various industries and

have undoubtedly become an indispensable tool in modern technology and scientific advancements. Photothermal refers to a process that involves the conversion of light into heat. Elastic medium refers to a substance or material that can deform when a force is applied to it and then return to its original shape when the force is removed. This particular process holds great significance in the microelectronics industry, as it contributes to the generation of plasma and elastic-thermal waves. In order to heat the surface of a semiconductor plane, a non-Gaussian laser beam is used when subjecting an intracavity sample of semiconductor to laser pulses or beams of laser light sources for photoacoustic spectroscopy analysis. In such cases, the photothermal theory is applied. Othman et al. [3] have conducted a study considering the impact of thermal laser pulses on wave propagation in generalized micro-elastic structures with a focus on energy dissipation. Additionally, Alshehri and Lotfy [4] have carried out an analysis of the photo-thermoelastic waves resulting from the interaction between electrons and holes in semiconductor materials under the influence of laser pulses.

Porosity in photothermal elastic medium refers to the presence of tiny pores or voids within the material. The term "photothermal" refers to the interaction between light (photons) and heat (thermal energy) in the material. Porosity affects the thermal and mechanical properties of the medium, making it a crucial parameter in various applications and industries. Porosity in photothermal elastic media plays a crucial role in their thermal and mechanical properties. It affects thermal conductivity, mechanical integrity, and can be manipulated for specific applications. Mastering porosity control is essential in the development of advanced photothermal materials for industries such as electronics, energy, and aerospace among others. The most basic idea behind the conventional theory of elasticity is the theory of an elastic solid with voids. In this theory, the presence of empty spaces or voids within the elastic material is considered. These voids, also known as pores, are a kinematic variable in the theory. Singh [5] discussed how waves propagate in a generalized thermoelastic material that includes voids, with the volume fraction of voids being one of the variables. Raddadi, Merfat H., et al. [6], studied a new model of a semiconductor material with pores, focusing on the excitation through photothermoelasticity and the influence of initial stress. Porous materials are widely used in various engineering disciplines, such as the petroleum industry, material science, and biology. Kilany et al. [7] demonstrated the photothermal and void effects in a semiconductor rotational medium based on the Lord-Shulman theory. Two computational algorithms for the numerical solution for system of fractional differential equations are studied by Khader et al. [8]. Mahdy et al. [9] produced a numerical method for solving the nonlinear equations of Emden-Fowler models. Mahdy [10] done stability, existence, and uniqueness for solving fractional glioblastoma multiforme using a Caputo-Fabrizio derivative. Gepreel et al. [11] demonstrated optimal control, signal flow graph, and system electronic circuit realization for nonlinear Anopheles mosquito model. Optimal and memristor-based control of a nonlinear fractional tumor-immune model studied by Mahdy et al. [12]. Use of optimal control in studying the dynamical behaviors of fractional financial awareness models done by Mahdy et al. [13]. General fractional financial models of awareness with Caputo-Fabrizio derivative used by Mahdy et al. [14]. A numerical method for solving the Rubella ailment disease model introduced by Mahdy et al. [15]. Numerical solution and dynamical behaviors for solving fractional nonlinear Rubella ailment disease model demonstrated by Mahdy et al. [16]. Mahdy et al. [17] produced numerical, approximate solutions, and optimal control on the deathly Lassa hemorrhagic fever disease in pregnant women.

Laser pulses have significant effects on porous semiconductor elastic mediums through a photothermal process. This process involves the conversion of light energy into thermal energy, leading to various physical and chemical changes within the medium. When a laser beam interacts with the surface of the medium, it is absorbed by the material, causing localized heating. This results in a temperature increase near the irradiated region, which can induce thermal expansion and stress within the porous structure. The photothermal process can also lead to changes in the material's properties. For instance, laser-induced heating can cause the evaporation or desorption of volatile components, modifying the chemical composition and structure of the semiconductor. Furthermore, laser pulses can induce mechanical stress in the porous semiconductor. The rapid heating and expansion of the material can create pressure waves, leading to acoustic vibrations or even shock waves. These mechanical effects can cause structural deformations, cracking, or fragmentation of the porous semiconductor elastic mediums through a photothermal process. They induce temperature rise, which can lead to thermal expansion, stress generation, and chemical changes within the material. These effects can be utilized for various applications, ranging from microfabrication to the synthesis and modification of porous semiconductors.

Currently, we are addressing the issue of 2-dimensional deformation in generalized thermoelasticity using photothermal process transportation with porosity, while taking into account the impact of laser pulses. These pulses are directed at the surface of a semiconductor medium, leading to electron excitation. To observe the analytical solution of physical fields, we employ normal mode analysis. Furthermore, we have graphically examined and discussed the numerical findings alongside some comparisons.

II. MATERIALS AND METHODS

Basic equations

The laser pulse interacts with a semiconductor medium [18] and is transformed into thermal energy, resulting in a thermal effect as described by the given equation.

$$Q = I_0 f(t)g(z)h(x) .$$

The temporal profile f(t) is given as,

$$f(t) = \frac{t}{t_0^2} \exp(-\frac{t}{t_0}) .$$
⁽²⁾

The Gaussian pulse spatial profile in z -direction, takes the form

$$g(z) = \frac{\gamma'}{2\pi r^2} \exp(-\frac{z^2}{r^2}).$$
 (3)

The heat deposition caused by the laser pulse in a semiconductor medium can be expressed in an exponential manner.

$$h(x) = \gamma' \exp(-\gamma' x) . \tag{4}$$

Hence, equation (1) takes the form

$$Q = \frac{I_0 \gamma' t}{2\pi r^2 t_0^2} \exp(-\frac{z^2}{r^2} - \frac{t}{t_0} - \gamma' x),$$
(5)

Where the absorbed energy is I_0 , the pulse rise time is t_0 , the radius of the beam defined by r and y' is the heating energy absorption at depth (z).

The main four variables are $N(\vec{r},t), T(\vec{r},t), \vec{u}(\vec{r},t)$ and $\varphi(\vec{r},t)(\vec{r}$ is the space vector and the time is t), the carrier density, temperature distribution, elastic displacement, and heat conductivity temperature can be described as follows. In a thermal scenario, the heat flux at the surface leads to two temperature effects. It is assumed that the overall charge carrier density remains constant, which allows for a linear relationship between temperature, displacement, and carrier density.

The interconnectedness of plasma waves and thermal waves can be expressed through the governing equations in this study [19-22].

$$\frac{\partial N(\vec{r},t)}{\partial t} = D_E \nabla^2 N(\vec{r},t) - \frac{N(\vec{r},t)}{\tau} + \kappa T(\vec{r},t) .$$
(6)

Motion equation:

$$\rho \frac{\partial^2 \vec{u}(\vec{r},t)}{\partial t^2} = (\mu+k)\nabla^2 \vec{u}(\vec{r},t) + (\mu+\lambda)\nabla(\nabla \cdot \vec{u}(\vec{r},t)) - \gamma(1+v_0\frac{\partial}{\partial t})\nabla T(\vec{r},t) - \delta_n \nabla N(\vec{r},t) + \lambda_0 \nabla \phi.$$
(7)

Heat conduction equation:

$$K\nabla^{2}T - \rho c_{E}(n_{1} + \tau_{0}\frac{\partial}{\partial t})\frac{\partial T(\vec{r},t)}{\partial t} + \frac{E_{g}}{\tau}N(\vec{r},t) - \gamma T_{0}\frac{\partial\phi}{\partial t} - \gamma T_{0}(n_{1} + n_{0}\tau_{0}\frac{\partial}{\partial t})(\frac{\partial}{\partial t}\nabla.\vec{u}(\vec{r},t) - \rho Q) = 0.$$
(8)

Porous (voids) equation can be given as [23]:

$$\alpha \nabla^2 \phi - \lambda_o e - \varsigma_1 \phi - \omega_o \dot{\phi} + mT = \rho \psi \ddot{\phi}.$$

The constitutive relations in tensor form:

$$\sigma_{XX} = (2\mu + \lambda + k)\frac{\partial u}{\partial x} + \lambda \frac{\partial w}{\partial z} + (3\lambda + 2\mu)d_nN + \lambda_o\phi - \gamma(1 + \nu_0\frac{\partial}{\partial t})T, \qquad (10)$$

$$\sigma_{ZZ} = (2\mu + \lambda) \frac{\partial w}{\partial z} + \lambda \frac{\partial u}{\partial x} + (3\lambda + 2\mu)d_n N + \lambda_0 \phi - \gamma (1 + v_0 \frac{\partial}{\partial t})T, \qquad (11)$$

$$\sigma_{XZ} = (k+\mu)\frac{\partial w}{\partial x} + \mu\frac{\partial u}{\partial z}.$$
(12)

Where, at temperature *T*, in general case the thermal activation coupling parameter is non-zero and takes the form $\kappa = \frac{\partial \lambda_0}{\partial T} \frac{T}{\tau}$ [24-25]. The laser pulse with heat effect can be expressed by *Q* and the thermal viscosity takes

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(9)

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(1)

the form $\frac{\rho C_e}{K} = \frac{1}{k}$, *k* is the diffusivity [26-27]. Solving this problem in two dimensions (2D), therefor the displacement vector can be expressed in the form: $\vec{u} = (u, 0, w)$, where u(x, z, t) and w(x, z, t).

Formulation of the problem

We can be introduced the dimensionless of two scalar potential functions $\Pi(x, z, t)$ and $\psi(x, z, t)$, which can be defined as follows:

$$u = \frac{\partial \Pi}{\partial x} - \frac{\partial \psi}{\partial z}, \qquad \qquad w = \frac{\partial \Pi}{\partial z} + \frac{\partial \psi}{\partial x}.$$

To simplify the governing equations, we can introduce new variables in a dimensionless form.

$$(x', z', u', w') = \frac{(x, z, u, w)}{C_T t^*} , (t', v'_0) = \frac{(t, v_0)}{t}, \quad T' = \frac{\gamma T}{2\mu + \lambda},$$

$$h' = \frac{\rho c_E h}{\mu_o H_o \sigma_o k}, \sigma'_{ij} = \frac{\sigma_{ij}}{\mu}, \quad N' = \frac{\delta_n N}{2\mu + \lambda}, \quad E'_i = \frac{\rho c_E E_i}{\mu_o^2 H_o \sigma_o k c_T},$$

$$t^* = \frac{k}{\rho c_E c_T^2}, \quad \varphi' = \frac{\psi \omega^{*2}}{C_2^2} \varphi, \quad c_T^2 = \frac{2\mu + \lambda}{\rho}, \quad Q' = \frac{Q}{\rho c_E t^*}.$$

$$(13)$$

To rephrase, when we apply the equation (13) that deals with dimensionless variables and the two scalar potential functions (Π, ψ) in the governing equations (6-12), we can derive the following equations by removing the dash:

$$(\nabla^2 - q_1 - q_2 \frac{\partial}{\partial t})N + \varepsilon_3 T = 0 \quad , \tag{14}$$

$$(\nabla^2 - a_{17} \frac{\partial^2}{\partial t^2})\Pi - a_{18}T - a_{19}N + b_4\phi = 0,$$
(15)

$$(\nabla^2 - a_{22} \frac{\partial^2}{\partial t^2})\psi = 0, \tag{16}$$

$$(\nabla^2 - a_8 \frac{\partial}{\partial t})T - a_9 \frac{\partial}{\partial t} \nabla^2 \Pi + a_{10}N - b_5 \frac{\partial \phi}{\partial t} = -\varepsilon_4 (n_1 t + n_0 \tau_0 \{1 - \frac{t}{t_0}\}) \frac{Q}{t},$$
(17)

$$(\nabla^2 - b_6)\phi - b_7 \nabla^2 \Pi + b_8 T = 0, \tag{18}$$

$$\sigma_{XX} = a_{11} \frac{\partial^2 \Pi}{\partial x^2} + a_{12} \frac{\partial^2 \Pi}{\partial z^2} + (a_{12} - a_{11}) \frac{\partial^2 \psi}{\partial x \partial z} - a_{13} T + a_{14} N + b_9 \phi,$$
(19)

$$\sigma_{ZZ} = a_{11} \frac{\partial^2 \Pi}{\partial z^2} + a_{12} \frac{\partial^2 \Pi}{\partial x^2} + (a_{11} - a_{12}) \frac{\partial^2 \psi}{\partial x \partial z} - a_{13} T + a_{14} N + b_9 \phi,$$
(20)

$$\sigma_{XZ} = b_{11} \frac{\partial^2 \psi}{\partial x^2} + b_{10} \frac{\partial^2 \Pi}{\partial x \partial z} - \frac{\partial^2 \psi}{\partial z^2} .$$
(21)

Solution of the problem

We have the option to utilize the normal mode analysis, which enables us to represent fundamental variables in an expressed manner.

$$\left. \begin{array}{c} (\Pi, \psi, T, \phi, \sigma_{ij}, N)(x, z, t) = \\ & * & * & * & * \\ (\Pi^{'}(x), \psi^{'}(x), T^{'}(x), \phi^{'}(x), \sigma_{ij}^{'}(x), N^{'}(x),) \exp(\omega t + ibz) \end{array} \right\}$$

$$(22)$$

Where ω the complex time is constant, *i* is the imaginary unit part, *b* is the wave number and $[\Pi^*(x), \psi^*(x), T^*(x), \phi^*(x), \sigma_{ij}^*(x), N^*(x)]$ are Applying the normal mode analysis technique equation (22) to equations (14-21) results in obtaining the rephrased values of the primary variables' amplitudes. $(D^2 - \alpha_1)N^* + \varepsilon_3 T^* = 0$, (23)

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$$(D^{2} - \alpha_{2})\Pi^{*} - a_{18}T^{*} - a_{19}N^{*} + b_{4}\phi^{*} = 0,$$
(24)

$$(D^2 - \alpha_3)\psi^* = 0, (25)$$

$$(D^{2} - \alpha_{4})T^{*} - \alpha_{5}(D^{2} - b^{2})\Pi^{*} + a_{10}N^{*} - \alpha_{6}\phi^{*} = -g(z, t)\exp(-\gamma'x),$$
(26)

$$(D^{2} - \alpha_{9})\phi^{*} - b_{7}(D^{2} - b^{2})\Pi^{*} + b_{8}T^{*} = 0.$$
⁽²⁷⁾

Solving equations (23, 24, 26 and 27) in the variable $\prod^{*}(x)$ can be observed as homogenous ordinary differential equation (Elimination method):

$$[D^{8} - A_{1}D^{6} + A_{2}D^{4} - A_{3}D^{2} + A_{4}]\prod^{*}(x) = L_{1}g(z,t)\exp(-\gamma'x).$$
⁽²⁸⁾

Equations (25) and (28) can be put in the characteristic form as:

$$\begin{pmatrix} D^2 - k_1^2 \end{pmatrix} \psi^* = 0,$$

$$\begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix} \begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix} \begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix} \begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix} \begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix} (-2 & -2 & -2 \end{pmatrix}$$
(29)

$$\left(D^2 - k_2^2\right) \left(D^2 - k_3^2\right) \left(D^2 - k_4^2\right) \left(D^2 - k_5^2\right) \left(\Pi^*, \phi^*, \hat{T}^*, N^*\right) = 0.$$

$$(30)$$

Where, k_n^{-} ; n = 2,3,4,5 are the roots of equation (30) and k_1^{-} are the root of equation (29). The solution of equation (29) is given by:

$$\psi^{*}(x) = M_{1}(b,\omega) \exp(-k_{1}x).$$
 (31)

The solution of homogenous equation (30) is given by

$$\Pi^{*}(x) = \sum_{n=2}^{5} M_{n}(b,\omega) \exp(-k_{n}x) + L_{2} \exp(-\gamma' x),$$
(32)

In same manner,

$$T^{*}(x) = \sum_{n=2}^{5} M'_{n}(b,\omega) \exp(-k_{n}x) + L_{3} \exp(-\gamma'x),$$
(33)

$$N^{*}(x) = \sum_{n=2}^{5} M_{n}''(b,\omega) \exp(-k_{n}x) + L_{4} \exp(-\gamma' x),$$
(34)

$$\phi^{*}(x) = \sum_{n=2}^{5} M_{n}^{"'}(b,\omega) \exp(-k_{n}x) + L_{5} \exp(-\gamma'x).$$
(35)

The displacement components can be obtained as:

$$u^{*}(x) = D\Pi^{*} - i b\psi^{*},$$
(36)

$$w'(x) = i b \Pi' + D \psi'.$$
 (37)

After substituting the expressions from equations (31) and (32) into equations (36) and (37), we can now determine the amplitude of the displacement components u and w.

$$u^{*}(x) = -\sum_{n=2}^{5} M_{n}(b,\omega) k_{n} e^{-k_{n}x} + L_{2} \exp(-\gamma x) - ibM_{1}(b,\omega) e^{-k_{1}x},$$
(38)

$$w^{*}(x) = ib \sum_{n=2}^{5} M_{n}(b,\omega) e^{-k_{n}x} + L_{2} \exp(-\gamma'x) - ibM_{1}(b,\omega) e^{-k_{1}x}.$$
(39)

Where M_n , M'_n , M''_n , M''', M''' and M_1 are the unknown parameters, in which depend on b and ω .

We can obtain the relations between the parameters M_n , M'_n , M''_n , M''', M''' as following:

$$M'_{n}(b,\omega) = H_{1n}M_{n}(b,\omega), \qquad n = 2,3,4,5$$
 (40)

$$M''_{n}(b,\omega) = H_{2n}M_{n}(b,\omega), \qquad n = 2,3,4,5$$
 (41)

$$M_n'''(b,\omega) = H_{3n}M_n(b,\omega), \qquad n = 2,3,4,5$$
 (42)

Where,

$$H_{2n} = \begin{cases} (k_n^6 \alpha_8 + k_n^4 b_7 b_4 - k_n^2 b^2 b_7 b_4 - k_n^4 \alpha_1 - k_n^4 \alpha_2 - k_n^4 \alpha_9 - k_n^2 \alpha_9 - k_n^2 \alpha_9 \alpha_2 - \alpha_1 \alpha_9 \alpha_2) \div \\ k_{2n} = \begin{cases} k_n^2 \alpha_1 b_7 b_4 + b^2 \alpha_1 b_7 b_4 + k_n^2 \alpha_1 \alpha_2 + k_n^2 \alpha_9 \alpha_2 - \alpha_1 \alpha_9 \alpha_2) \div \\ (k_n^4 a_{18} - k_n^2 a_{18} \alpha_1 - k_n^2 a_{18} \alpha_9 - k_n^2 a_{19} \varepsilon_3 + k_n^2 b_8 b_4 + k_n^2 \alpha_9 \alpha_9 - \alpha_1 b_8 b_4), & n = 2,3,4,5. \end{cases}$$

$$(43)$$

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$$\left. \left. \begin{array}{c} \left(-\varepsilon_{3}(k_{n}^{4} + k_{n}^{2}b_{4}b_{7} - b^{2}b_{4}b_{7} - k_{n}^{2}\alpha_{2} - k_{n}^{2}\alpha_{9} + \alpha_{2}\alpha_{9}) \right) \div \\ H_{3n} = \left(k_{n}^{6}a_{18} - k_{n}^{2}a_{18}\alpha_{1} - k_{n}^{2}a_{18}\alpha_{9} - k_{n}^{2}a_{19}\varepsilon_{3} + k_{n}^{2}b_{4}b_{8} + a_{18}\alpha_{1}\alpha_{9} + a_{19}\alpha_{9}\varepsilon_{3} - \alpha_{1}b_{4}b_{8} \right), \quad n = 2,3,4,5. \end{array} \right\},$$

$$\left. \left. \begin{array}{c} \left(44 \right) \\ \left(44 \right)$$

$$H_{4n} = \begin{cases} (k_n^4 a_{18}b_7 - k_n^2 b^2 a_{18}b_7 - k_n^4 b_8 - k_n^2 a_{18}\alpha_1 b_7 - k_n^2 a_{19}b_7 \varepsilon_3 + b^2 \alpha_1 b_7 a_{18} + b_8 a_{18} a_{18}$$

We can discuss the parameters in terms of the physical quantities they represent. M_n Only as:

$$T^{*}(x) = \sum_{n=2}^{5} H_{2n} M_{n}(b, \omega) \exp(-k_{n} x) + L_{3} \exp(-\gamma' x),$$
(46)

$$N^{*}(x) = \sum_{n=2}^{5} H_{3n} M_{n}(b, \omega) \exp(-k_{n} x) + L_{4} \exp(-\gamma' x),$$
(47)

$$\phi^{*}(x) = \sum_{n=2}^{5} H_{4n} M_n(b, \omega) \exp(-k_n x) + L_5 \exp(-\gamma' x).$$
(48)

The stress-strain relations can be expressed in terms of
$$M_n$$
 only, as

$$\sigma_{xx}^{*} = \sum_{n=2}^{5} h_n M_n(b,\omega) \exp(-k_n x) + \chi_1 \exp(-\gamma' x) - 2ibk_1 M_1 \exp(-k_1 x),$$
(49)

$$\sigma_{zz}^{*} = \sum_{n=2}^{5} h'_{n} M_{n}(b,\omega) \exp(-k_{n}x) + \chi_{2} \exp(-\gamma'x) - 2ibk_{1}M_{1} \exp(-k_{1}x), \qquad (50)$$

$$\sigma_{xz}^* = \sum_{n=2}^5 h_n'' M_n(b,\omega) \exp(-k_n x) + \chi_3 \exp(-\gamma' x) + (b_{11}k_1^2 + b^2) M_1 \exp(-k_1 x) .$$
(51)
Applications

Applications

In this part, we can obtain the parameters M_n (n = 1,2,3,4,5,6). In physical problems, we typically assume that exponential functions with positive exponents grow indefinitely as the input approaches infinity.

To determine the unknown parameters M_n (n = 1,2,3,4,5,6), the free surface semiconductor medium is subjected to boundary conditions involving mechanical, thermal, porosity, and plasma. y = 0:

(I) the rewritten form of the plasma boundary condition at the free surface involves the incorporation of carrier density diffusion, transportation, and photogeneration during recombination processes.

$$\frac{\partial N(0,t,z)}{\partial x} = \frac{s}{D_E} N(0,t,z),$$
(52)

On the other hand obtained the following relation:

$$\sum_{n=2}^{5} C_n M_n e^{-k_n x} = -(1+\gamma')L_4 , \qquad (53)$$

(II) The thermal shock scenario requires imposing an isothermal boundary condition at the exposed surface. y = 0 as:

$$T(0,t,z) = 0,$$
 (54)

Therefore,
$$\sum_{n=2}^{\Sigma} H_{2n} M_n = -L_3,$$
(55)

(III) The mechanical boundary conditions for the free surface involve stress conditions that are described as traction-free loads. This means that there are no normal or tangential stresses acting on the surface. y = 0 yields:

$$\rightarrow \sigma_{XX}(0,t,z) = \sigma_o,$$
(56)

So,
$$\sum_{n=1}^{\infty} h_n M_n - ibk_1(a_{12} - a_{11})M_1 = -\chi_1 + \sigma_o^2$$
, (57)

$$\to \sigma_{\chi Z}(0,t,z) = 0, \tag{58}$$

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So,
$$\sum_{n=1}^{5} h_n^{"} M_n + (b_{11}k_1^2 + b^2)M_1 = -\chi_3,$$
 (59)

(v) Reword the boundary condition that governs the alteration in the volume fraction distribution at the exposed surface. y = 0, yields:

$$\rightarrow \frac{\partial \phi}{\partial z} = 0,$$
So, $\sum_{n=2}^{5} H_{4n} M_n e^{-k_n x} = -L_5.$
(60)
(61)

By solving the above equations in terms of the parameters M_n , the unknown parameters M_n can be obtained.

III. Numerical results and discussion

We can take the Silicon (Si) material for the computational and numerical simulation. The physical constants of the Silicon (in SI unit) are taken in [28-29]:

$$\begin{split} \lambda &= 3.64 \times 10^{10} \ N/m^2, \quad \mu = 5.46 \times 10^{10} \ N/m^2, \quad \rho = 2330 \ kg/m^3, \quad T_0 = 800 \ \text{K}, \\ \tau &= 5 \times 10^{-5} \ \text{sec}, \quad d_n = -9 \times 10^{-31} \ m^3, \quad D_E = 2.5 \times 10^{-3} \ m^2/\text{sec}, \quad E_g = 1.11 \ eV, \\ s &= 2 \ m/\text{sec}, \quad \alpha_t = 4.14 \times 10^{-6} \ \text{K}^{-1}, \quad K = 150 \ \text{Wm}^{-1} \ \text{K}^{-1}, \quad C_e = 695 \ J/(kg \ \text{K}), \\ j &= 0.2 \times 10^{-19} \ m^2, \quad \gamma = 0.779 \times 10^{-9} \ \text{N}, \quad k = 10^{10} \ \text{Nm}^{-2}, \quad t = 0.001 \text{sec}, \quad z = 1, \\ \tau_0 &= 0.00005 \ s, \quad v_0 = 0.0005 \ s, \quad n_0 = 10^{20} \ m^{-3}, \quad \psi = 1.753 \times 10^{-15} \ m^2, \quad \alpha = 3.688 \times 10^{-5} \ \text{N}, \\ \lambda_o &= 1.13849 \times 10^{10} \ \text{Nm}^{-2}, \quad m = 2 \times 10^{6} \ \text{Nm}^{-2} \ \text{deg}^{-1}, \quad \varsigma_1 = 1.475 \times 10^{10} \ \text{Nm}^{-2}, \quad \omega_o = 2.4 \ rad \ \text{sec}, \end{split}$$

 $\zeta = 0.9 \ rad / sec$, $k_o = 386 \ N / K. sec$, b = 0.9.

During the previously discussed numerical approach phase, the actual elements of the task were outlined as the elastic wave (*u*), temperature (*T*), stress tensor components (σ_{xx}, σ_{xz}), the change in volume fraction field (ϕ), and photo-electronic or plasma (carrier density, *N*) are the primary areas of concentration for us. By

employing various laser pulse and porosity parameters, we can effectively ascertain the spatial arrangement of these components. These values are then visually represented on a graph, where overlapping curves in the sub-figures highlight subtle variations. This approach enables us to better comprehend intricate details on a small scale, as the overlapping curves offer a more distinct representation of any inconsistencies.

Conclusion

In this research, we investigate the impact of laser pulses containing porosity on a generalized photothermo-elastic solid medium. We focus on analyzing various physical quantities of interest in this study. In the first group (described in Figure 1), the aim of this study is to observe the changes in dimensionless physical variables as influenced by porosity impact and porosity parameters. The data obtained showcases that the main fields display higher values when porosity parameters are taken into account as compared to when there is no porosity. This effect is notable in the presence of a magnetic field, except for the carrier density and normal stress which exhibit the opposite behavior. In conclusion, the porosity parameters have a significant influence on the main fields. The second category (described in Fig. 2), the numerical calculations of the volume fraction field and stress distributions (both normal and tangential) were observed as the horizontal distance varied. These calculations were performed with and without a laser pulse. It was found that the laser pulse had a positive impact on all the physical fields, as the curves for both with and without the laser pulse matched closely in all aspects. Additionally, the porosity parameters and laser pulse had a significant influence on the distribution of these physical fields in the photo-thermo-elastic model. These research findings have potential implications for enhancing the functionality and long-term durability of electronic devices, energy harvesting systems, sensors, materials used in structural health monitoring, and overall infrastructure. The impact of varying levels of the laser pulse and porosity on these applications was studied in this research.

Disclosure statement

The authors have disclosed that they do not have any conflicts of interest to declare.

Authors' contributions

Kh. Lotfy: Conceptualization, Methodology, Anwer: Software, Writing- Original draft preparation. Saleh: Software, Validation. A. El-Bary: Writing- Reviewing and Editing, Data curation.

Availability of data and materials

https://bfszu.journals.ekb.eg/journal

The information applied in this research is ready from the authors at request. Acknowledgment

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Appendix

The main parameters in equations (14-21) can be expressed as:

$$\begin{split} q_{1} &= \frac{kt^{*}}{\rho c_{E} D_{E}}, \quad q_{2} = \frac{k}{\rho c_{E} D_{E}}, \quad \varepsilon_{3} = \frac{\kappa K \delta_{n} t^{*}}{\gamma \rho c_{E} D_{E}}, \quad \varepsilon_{4} = \rho T_{0} c_{E} t^{*^{2}}, \quad a_{1} = \frac{\mu + \lambda}{\rho c_{T}^{2}}, \quad a_{2} = \frac{k + \mu}{\rho c_{T}^{2}}, \\ a_{3} &= \frac{2\mu + \lambda}{\rho c_{T}^{2}} (1 + v_{o} \omega), \quad a_{4} = \frac{2\mu + \lambda}{\rho c_{T}^{2}}, \quad a_{8} = \frac{\rho c_{E}}{c_{T}^{2} t^{*^{3}} K} (n_{1} + \tau_{o} \omega), \\ a_{9} &= \frac{\gamma T_{0}}{(2\mu + \lambda) t^{*^{3}} c_{T}^{2} K} (n_{1} + n_{o} \tau_{o} \omega), \quad a_{10} = \frac{E_{g} \gamma}{t^{*^{3}} c_{T}^{2} \tau \delta_{n} K}, \\ a_{12} &= \frac{\lambda}{\mu}, \quad a_{13} = \frac{2\mu + \lambda}{\mu} (1 + v_{o} \omega), \quad a_{14} = \frac{d_{n} (2\mu + \lambda)}{\mu \delta_{n}} (2\mu + 3\lambda), \\ b_{2} &= \frac{\lambda_{o}}{\psi \rho t^{*^{*}}}, \quad b_{4} = \frac{b_{2}}{a_{1} + a_{2}}, \quad b_{5} = \frac{\gamma^{2} T_{0}}{(2\mu + \lambda) K t^{*^{5}} \psi}, \quad b_{8} = \frac{(2\mu + \lambda) \lambda_{o} \psi t^{*^{4}}}{\alpha}, \\ b_{6} &= \frac{\rho \psi^{2} t^{*^{2}}}{\alpha} \omega^{2} + \frac{\omega_{o} \psi t^{*^{3}}}{\alpha} \omega - \frac{\zeta_{1} \psi t^{*^{4}}}{\alpha}, \quad b_{7} = \frac{\lambda_{o} \psi t^{*^{4}}}{\alpha}, \quad b_{9} = \frac{\lambda_{o} c_{T}^{2}}{\mu \psi t^{*^{2}}}, \\ b_{10} &= 1 + \frac{\mu + k}{\mu}, \quad b_{11} = \frac{\mu + k}{\mu}, \quad b_{12} = b_{5} \omega, \quad \alpha_{10} = b^{2} + b_{6}, \\ a_{17} &= \frac{1}{a_{1} + a_{2}}, \quad a_{18} = \frac{a_{3}}{a_{1} + a_{2}}, \quad a_{2} = b^{2} + a_{17} \omega^{2}, \quad \alpha_{3} = b^{4} + b^{2} a_{20} \omega + b^{2} a_{17} \omega^{2}, \\ a_{22} &= \frac{\rho}{\mu}, \quad \alpha_{1} = b^{2} + q_{1} + q_{2} \omega, \quad \alpha_{2} = b^{2} + a_{17} \omega^{2}, \quad \alpha_{3} = b^{4} + b^{2} a_{20} \omega + b^{2} a_{17} \omega^{2}, \\ a_{4} &= b^{2} + a_{8} \omega, \quad \alpha_{5} = a_{9} \omega, \quad \alpha_{6} = b_{5} \omega. \end{split}$$

The main parameters in equations (53-61) can be expressed as:

$$\begin{aligned} \sigma_{0}^{*} &= \sigma_{0} e^{-(\omega t + ibz)}, \\ h_{n} &= a_{11}k_{n}^{2} - a_{12}b^{2} - a_{13}H_{2n} + a_{14}H_{3n} + b_{9}H_{4n} \quad ;n = 2.3.4.5 \\ h_{n}^{'} &= -a_{11}b^{2} + a_{12}k_{n}^{2} - a_{13}H_{2n} + a_{14}H_{3n} + b_{9}H_{4n} \quad ;n = 2.3.4.5 \\ h_{n}^{''} &= -ibb_{10}k_{n}, \quad \chi_{1} &= a_{11}\gamma^{'2}L_{2} + a_{12}b^{2}L_{2} - a_{13}L_{3} + a_{14}L_{4} + b_{9}L_{5}, \\ \chi_{2} &= a_{12}\gamma^{'2}L_{2} - a_{11}b^{2}L_{2} - a_{13}L_{3} + a_{14}L_{4} + b_{9}L_{5}, \quad \chi_{3} &= -ibL_{2}b_{10}\gamma', \\ L_{1} &= -\gamma^{'4}a_{18} - (a_{18}a_{1} + a_{18}a_{9} + a_{19}c_{3})\gamma^{'2} - a_{18}a_{1}a_{9} - a_{19}c_{3}a_{9} + \alpha_{1}b_{4}b_{8}, \\ L_{2} &= \frac{L_{18}(z,t)}{\left[\gamma'^{8} - A_{1}\gamma'^{6} + A_{2}\gamma'^{4} - A_{3}\gamma'^{2} + A_{4}\right]}, \\ \gamma'^{6}a_{8} + \gamma'^{4}b_{7}b_{4} - \gamma'^{2}b^{2}b_{7}b_{4} - \gamma'^{4}a_{1} - \gamma'^{4}a_{2} - \gamma'^{4}a_{9} - \right] \\ L_{3} &= L_{2} \left(\gamma'^{6}a_{18} - \gamma'^{2}a_{18}a_{1} - \gamma'^{2}a_{18}a_{9} - \gamma'^{2}a_{19}c_{3} + \gamma'^{2}b_{8}b_{4} + a_{18}a_{1}a_{9} + a_{19}a_{9}c_{3} - \alpha_{1}b_{8}b_{4}), \quad n = 2.3.4.5. \right], \\ L_{4} &= L_{2} \left((\gamma'^{6}a_{18} - \gamma'^{2}a_{18}a_{1} - \gamma'^{2}a_{18}a_{9} - \gamma'^{2}a_{19}c_{3} + \gamma'^{2}b_{4}b_{8} + a_{18}a_{1}a_{9} + a_{19}a_{9}c_{3} - \alpha_{1}b_{8}b_{4}), \quad n = 2.3.4.5. \right], \\ L_{4} &= L_{2} \left((\gamma'^{6}a_{18} - \gamma'^{2}a_{18}a_{1} - \gamma'^{2}a_{18}a_{9} - \gamma'^{2}a_{19}c_{3} + \gamma'^{2}b_{4}b_{8} + a_{18}a_{1}a_{9} + a_{19}a_{9}c_{3} - \alpha_{1}b_{4}b_{8}), \quad n = 2.3.4.5. \right], \\ (\gamma'^{4}a_{18}b_{7} - \gamma'^{2}b^{2}a_{18}b_{7} - \gamma'^{4}b_{8} - \gamma'^{2}a_{18}a_{1}b_{7} - \gamma'^{2}a_{19}b_{7}c_{3} + b^{2}a_{1}b_{7}a_{18} + a_{18}a_{9}a_{1} + \alpha_{19}a_{9}c_{3} - \alpha_{1}b_{4}b_{8}), \quad n = 2.3.4.5. \right], \\ (\gamma'^{4}a_{18}b_{7} - \gamma'^{2}b^{2}a_{18}b_{7} - \gamma'^{2}a_{18}a_{9} - \gamma'^{2}a_{19}c_{3} + \gamma'^{2}b_{8}b_{4} + a_{18}a_{9}a_{1} + \alpha_{19}a_{9}c_{3} - \alpha_{1}b_{8}b_{4}), \quad n = 2.3.4.5. \right], \\ (\gamma'^{4}a_{18} - \gamma'^{2}a_{18}a_{1} - \gamma'^{2}a_{18}a_{9} - \gamma'^{2}a_{19}c_{3} + \gamma'^{2}b_{8}b_{4} + a_{18}a_{9}a_{1} + \alpha_{19}a_{9}c_{3} - \alpha_{1}b_{8}b_{4}), \quad n = 2.3.4.5. \right],$$

 $C_n = (\frac{3}{D_E} + k_n)H_{3n}$; n = 2,3,4,5.

Coefficients of equation (36):

$$\begin{split} A_{1} &= a_{18}\alpha_{5} - b_{4}b_{7} + \alpha_{1} + \alpha_{2} + \alpha_{4} + \alpha_{9}, \\ A_{2} &= b^{2}a_{18}\alpha_{5} - b^{2}b_{4}b_{7} + a_{18}\alpha_{1}\alpha_{5} + a_{18}\alpha_{5}\alpha_{9} - a_{18}\alpha_{6}b_{7} + a_{19}\alpha_{5}\varepsilon_{3} - \alpha_{1}b_{4}b_{7} \\ &- \alpha_{4}b_{4}b_{7} - \alpha_{5}b_{4}b_{8} - a_{10}\varepsilon_{3} + \alpha_{1}\alpha_{2} + \alpha_{1}\alpha_{4} + \alpha_{1}\alpha_{9} + \alpha_{2}\alpha_{4} + \alpha_{2}\alpha_{9} + \alpha_{4}\alpha_{9} + \alpha_{6}b_{8}, \\ A_{3} &= b^{2}a_{19}\alpha_{5}\varepsilon_{3} + a_{10}b_{4}b_{7}\varepsilon_{3} + a_{18}\alpha_{1}\alpha_{5}\alpha_{9} - a_{18}\alpha_{1}\alpha_{6}b_{7} + a_{19}\alpha_{5}\alpha_{9}\varepsilon_{3} - a_{19}\alpha_{6}b_{7}\varepsilon_{3} - \\ b_{4}b_{7}\alpha_{1}\alpha_{4} - \alpha_{1}\alpha_{5}b_{4}b_{8} - b^{2}\alpha_{5}b_{4}b_{8} - b^{2}\alpha_{4}b_{4}b_{7} - b^{2}a_{18}\alpha_{6}b_{7} + b^{2}a_{18}\alpha_{5}\alpha_{9} + b^{2}a_{18}\alpha_{1}\alpha_{5} \\ &- b^{2}\alpha_{1}b_{4}b_{7} + \alpha_{1}\alpha_{2}\alpha_{4} + \alpha_{1}\alpha_{2}\alpha_{9} - \alpha_{1}\alpha_{4}\alpha_{9} + \alpha_{2}\alpha_{4}\alpha_{9} + \alpha_{2}\alpha_{6}b_{8} + \alpha_{1}\alpha_{4}\alpha_{9} + \alpha_{1}\alpha_{6}b_{8} \\ &- a_{10}\alpha_{9}\varepsilon_{3} - a_{10}\alpha_{2}\varepsilon_{3}, \\ A_{4} &= b^{2}a_{10}b_{4}b_{7}\varepsilon_{3} + b^{2}a_{18}\alpha_{1}\alpha_{5}\alpha_{9} - b^{2}a_{18}\alpha_{1}\alpha_{6}b_{7} + b^{2}a_{19}\alpha_{5}\alpha_{9}\varepsilon_{3} - b^{2}a_{19}\alpha_{6}b_{7}\varepsilon_{3} - \\ &b^{2}b_{4}b_{7}\alpha_{1}\alpha_{4} - b^{2}\alpha_{1}\alpha_{5}b_{4}b_{8} - a_{10}\alpha_{2}\alpha_{10}\varepsilon_{3} - a_{10}\alpha_{2}\alpha_{9}\varepsilon_{3} + \alpha_{4}\alpha_{1}\alpha_{2}\alpha_{9} + \alpha_{1}\alpha_{2}b_{8}\alpha_{6}, \end{split}$$







Figure (1): Variation of physical quantities against the distance x with porosity and without porosity









Figure (2): Variation of physical quantities against the distance x with Laser Pulse and without Laser Pulse.

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