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THERMOELASTOPLASTIC AND VIBRATION CHARACTERISTICS OF COMPOSITES WITH INTERPHASES

Y. M. Shabana[*], E. M. Rabeih[**] and E. I. Morgan[*]

ABSTRACT

This paper addresses the problem of predicting the behaviors of a composite material, which consists of a matrix and a spherical inclusion coated by a multi-layered interphase, under thermal and/or mechanical loading variations and based on the micromechanics principles. The multi-layered interphase, which in general includes different properties for each layer, is modeled by applying the multi-inclusion model. The considered damage mode, which is represented by the progressive debonding of the particle from the interphase, is assumed to be controlled by a critical energy criterion and the Weibull's distribution function. The effects of the interphase parameters such as its thickness and the properties of each layer on the effective thermomechanical properties and the vibration characteristics of composites with multi-layered interphases are presented and discussed. The natural frequencies give information about the resonance avoidance whereas mode shapes give information about observability and controllability of different structures. Therefore, the natural frequencies and mode shapes as vibration characteristics are investigated based on Euler and Timoshenko theories.

KEYWORDS

Particulate composites, Multi-layered interphase, Micromechanics, Properties, Vibration characteristics.

[*] Mechatronics Department, Faculty of Engineering and Materials Science, German University in Cairo, New Cairo city, Egypt.

[**] Mechanical Design Department, Faculty of Engineering, El-Mataria, Helwan University, P.O. Box 11718, Cairo, Egypt.

INTRODUCTION

For a composite without interphase between the reinforcement and the matrix, internal stresses develop as a result of the mismatch in properties of the reinforcement and the matrix. To reduce these internal stresses, coated-reinforcements are proposed and analysed concluding that the particle-matrix interphases play an important role from the point of view of the material performance and significantly affect the effective properties and dynamic behaviors of composites.

An interphase between a matrix and a reinforcement is either homogeneous or inhomogeneous. For the homogeneous interphase, Sarvestani [1] proposed a micromechanical approach to estimate the global mechanical properties of linear elastic composites containing finite concentrations of thickly coated spherical particles. Jiang et al. [2] introduced an incremental constitutive model for composites taking into account the evolution of debonding damage, particle size, interphase thickness and matrix elastoplasticity.

For the inhomogeneous interphase and based on the strain Green's function and together with the Mori-Tanaka frame and double-inclusion model, a micromechanics model was developed [3] to study the effective elastic properties of particulate composites. Using the generalized self consistent method as well as introducing an inhomogeneous comparison material, a pair of explicit expressions of the upper and lower bounds of the effective elastic moduli of the particle-filled composites was obtained [4]. They found that the inhomogeneous interphase affects considerably the effective moduli of the composite. For composites that are subjected to thermal loading conditions, Li [5] developed a multi-inclusion model and obtained explicit expressions to study the effective thermoelastic moduli and thermoelastic field. The properties of the interphase were considered to vary in an arbitrary manner and there could be a thermal gradient in the interphase.

In this paper, the three-phase incremental damage theory [2] for a composite containing matrix, particle and homogeneous interphase is modified to be valid for a composite contains multi-layered inhomogeneous interphase. Also, the effects of the thermal loading conditions are added so that the present constitutive laws can predict the macroscopic and microscopic behaviors of composites containing multi-layered interphases when subjected to thermal and mechanical loading conditions either individually or in combination. The multi-inclusion model is considered and the particle and all layers of the interphase are assumed to be spherical and coaxial. Since functionally graded materials (FGMs) show, in general, better behaviors relative to traditional composites, the properties of the interphase layers are controlled so that composites with functionally graded interphases are investigated. However the interfaces between the various layers and also between the outermost layer and the matrix are assumed to be perfectly bonded, the particle may debond from the interphase due to severe loading conditions. The effects of different microstructural parameters such as the interphase thickness and number of layers are presented and discussed. For the sake of comparison and in order to demonstrate the accuracy of the present analysis, theoretical and experimental findings of some of the previously reported problems, which are special cases encompassed by the present investigation, are re-examined and it is revealed that the proposed microstructure-based model is applicable for not only micro-composites but also nano-composites. Moreover, the vibration characteristics of a cantilever beam made of the proposed class of composites and a homogeneous material are compared. These analyses are carried out based on Euler and Timoshenko theories.

PROPERTIES OF THE CONSTITUENT MATERIALS

In the composite system, the matrix undergoes an elastic-plastic deformation while other phases of the particle and interphase deform elastically. The elastic incremental stress-strain relation of all phases, considering temperature change, is given by Hook's law.

$$d\sigma_i = L_i(E_i, v_i)[d\varepsilon_i - N_i dT] \tag{1}$$

where the subscript i = 0 refers to the matrix, while i = p refers to the particle, and i = 1, 2,…n refers to the layer number of the interphase starting from the innermost layer. $L_i(E_i, v_i)$ is the elastic stiffness tensor described by the elastic modulus E_i and Poisson ratio v_i . dT is the temperature change. $\boldsymbol{N_i}$ is a vector containing the coefficients of thermal expansion (CTE) and its transpose is given as

$$N_i^T = \lceil \alpha_i \quad \alpha_i \quad \alpha_i \quad 0 \quad 0 \quad 0 \rceil \tag{2}$$

When the matrix plastifies and the elastic-plastic deformation occurs, the Prandtl-Reuss equation (J2-flow theory) is approximated by the same form as the elastic relation (Eqn. 1). In that case, the equivalent elastic modulus and equivalent Poisson ratio are used instead of the traditional ones to calculate the equivalent tangent modulus [6].

INCREMENTAL CONSTITUTIVE LAWS

Following Eshelby's equivalence principle combined with Mori-Tanaka mean field concept, the incremental stresses in the intact particle and its associated k^{th} layer of the interphase, ($d\sigma^p$ and $d\sigma_{Ik}$) under thermo-mechanical loading are given by

$$\begin{aligned} d\sigma^p &= d\sigma + d\tilde{\sigma} + d\sigma_{Ip}^{pt} \\ &= L_p(d\varepsilon_0 - N_p\, dT + d\tilde{\varepsilon} + d\varepsilon_{Ip}^{pt}) \\ &= L_0(d\varepsilon_0 - N_0 dT + d\tilde{\varepsilon} + d\varepsilon_{Ip}^{pt} - d\varepsilon_{Ip}^{*}) \end{aligned} \tag{3}$$

$$\begin{aligned} d\sigma_{Ik} &= d\sigma + d\tilde{\sigma} + d\sigma_{Ik}^{pt} \\ &= L_k(d\varepsilon_0 - N_k\, dT + d\tilde{\varepsilon} + d\varepsilon_{Ik}^{pt}) \\ &= L_0(d\varepsilon_0 - N_0 dT + d\tilde{\varepsilon} + d\varepsilon_{Ik}^{pt} - d\varepsilon_{Ik}^{*}) \end{aligned} \tag{4}$$

It is assumed that the debonding damage occurs between the particle and the interphase. Therefore, the Eshelby's equivalence principle for the debonded reinforcement and its associated k^{th} layer of the interphase may be written as

$$\begin{aligned} 0 &= d\sigma + d\tilde{\sigma} + d\sigma_{2p}^{pt} \\ &= L_0(d\varepsilon_0 - N_0 dT + d\tilde{\varepsilon} + d\varepsilon_{2p}^{pt} - d\varepsilon_{2p}^{*}) \end{aligned} \tag{5}$$

$$\begin{aligned} d\sigma_{2k} &= d\sigma + d\tilde{\sigma} + d\sigma_{2k}^{pt} \\ &= L_k(d\varepsilon_0 - N_k\, dT + d\tilde{\varepsilon} + d\varepsilon_{2k}^{pt}) \\ &= L_0(d\varepsilon_0 - N_0 dT + d\tilde{\varepsilon} + d\varepsilon_{2k}^{pt} - d\varepsilon_{2k}^{*}) \end{aligned} \tag{6}$$

Furthermore, the following equations are obtained for the reinforcement in the debonding process and its associated k^{th} layer of the interphase

$$\begin{aligned} -\sigma^{p} &= d\sigma + d\tilde{\sigma} + \sigma_{3p}^{pt} \\ &= L_0(d\varepsilon_0 - N_0 dT + d\tilde{\varepsilon} + \varepsilon_{3p}^{pt} - \varepsilon_{3p}^{*}) \end{aligned} \tag{7}$$

$$\begin{aligned} d\sigma_{3k} &= d\sigma + d\tilde{\sigma} + d\sigma_{3k}^{pt} \\ &= L_k(d\varepsilon_0 - N_k\, dT + d\tilde{\varepsilon} + d\varepsilon_{3k}^{pt}) \\ &= L_0(d\varepsilon_0 - N_0 dT + d\tilde{\varepsilon} + d\varepsilon_{3k}^{pt} - d\varepsilon_{3k}^{*}) \end{aligned} \tag{8}$$

For one layer interphase, solving Eqns. (3-8) yields the macroscopic strain-stress constitutive law of the composite as

$$\begin{aligned} d\varepsilon &= (I+Z)L_0^{-1} d\sigma + \left[Z - (S-I)^{-1}\right] L_0^{-1} df_p \sigma^{p} \\ &+ \begin{Bmatrix} Z(-df_p N_0 - D) + f_p^{d}(S-I)^{-1}(N_0 - Q) \\ + df_p (S-I)^{-1} N_0 - f_{10} B^{-1}(R-Q) \end{Bmatrix} dT \end{aligned} \tag{9}$$

The definitions of all symbols exist in [7]. In Eq. (9), the first, second and third terms represent the strain increments due to the applied stress increment, the debonding damage, and the temperature change, respectively. The effective CTE of the composite can be extracted from the third term as

$$\begin{aligned} \alpha &= Z(-df_p N_0 - D) + f_p^{d}(S-I)^{-1}(N_0 \\ &- Q) + df_p (S-I)^{-1} N_0 - f_{10} B^{-1}(R-Q) \end{aligned} \tag{10}$$

Also, the microscopic stresses in the matrix, intact particle and interphase can be evaluated.

VIBRATION CHARACTERISTICS

It is necessary to look at the vibration characteristics, which include the natural frequencies and mode shapes, in order to use this class of composites in different engineering applications. The natural frequencies give information about the resonance avoidance. Mode shapes, on the other hand, give information about observability and controllability, where control units are put at the nodes positions to avoid vibration of such units, while vibration instruments are put at the anti-nodes position for good observation of the vibration level measurements.

As beams are one of the most important structural elements in mechanical systems, a cantilever beam made of this class of composites is considered to look at its vibration characteristics. The beam is modeled by the finite element method using MATLAB package and based on both Euler and Timoshenko beam theories, [8].

RESULTS AND DISCUSSION

Figure 1 shows a comparison of the predicted normalized elastic modulus (NEM) by the present model with the experimental findings by Shen et al. [9] and the theoretical results by Boutaleb et al. [10]. Also, the case without interphase is shown in the figure. The considered particle radius and interphase thickness are 40 *nm* and 20 *nm*, respectively. It can be seen from Fig. 1 that the predictions of both the present model and Boutaleb et al. [10] are well matched irrespective of whether the interphase exists or not. Moreover, the predictions of the current model agree well with the experimental findings by Shen et al. [9]. Since Boutaleb et al. [10] and Shen et al. [9] studied nano-composite materials, it is quite worth mentioning that the current proposed constitutive laws are applicable for predicting the elastic modulus of not only the micro-composites but also nano-composites.

Figure 2 shows the variation of the NCTE with f_p for different values of n. It can be noted that the NCTE increases with the increase of n.

Figure 3 shows the variation of the NCTE with f_p for different values of the interphase thickness (t in mm). At a certain value of f_p, increasing t results in an increase of f_I and consequently an equal decrease of the matrix volume fraction. Therefore, the contribution of the particle CTE increases when t increases and that leads to a decrease in the NCTE. In addition, these results, based on the considered values of the constituent materials, show that t is a dominant parameter and this is consistent with the results of Sevostianov and Kachanov [11].

Figures 4 and 5 show the first four non-dimensional natural frequencies of a cantilever beam as functions of f_p and based on the Euler and Timoshenko beams, respectively, note that the elastic modulus and Poisson's ratio are changed with f_p. It can be seen, as well known, that Euler predictions are in general higher than those of Timoshenko ones and the difference is increasing as we move up to the higher frequencies. However, the effects of f_p on the non-dimensional natural frequencies are negligible as shown in Figs. 4 and 5. The percentage deviation of those values between the extreme values of f_p (0 and 1) is calculated as 1.74%.

Figures 6 and 7 show the first three mode shapes of the cantilever beam based on Euler and Timoshenko theories, respectively. These modes have been drawn for extreme values of f_p. They look identical for booth values of f_p.

CONCLUSIONS

A micromechanical model is proposed for predicting different behaviors of composite materials containing multi-layered interphases subjected to thermomechanical loading conditions. The effects of the geometrical parameters and material parameters of the constituents on the effective properties and the vibration characteristics are investigated. Therefore, this investigation provides a helpful tool for optimizing material design and manufacture of the particle, matrix, and interphase in order to maintain the performance of composites in service and the production of advanced composites with superior thermo-mechanical properties.

The numerical results illustrate that the proposed microstructure-based model is applicable for thickly coated composites and can be used for not only micro-composites but also nano-composites as the results agree well with those of Shen et al. [9] and Boutaleb et al. [10]. Moreover, the interphase has negligible effect on the vibration characteristics within the investigated domain.

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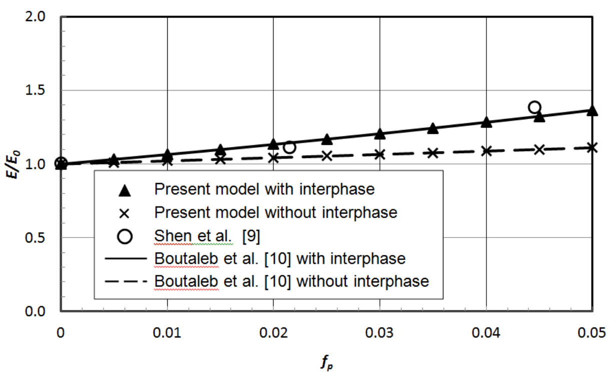

Fig. 1. Comparison of the predicted normalized Young's modulus with other results in the literature.

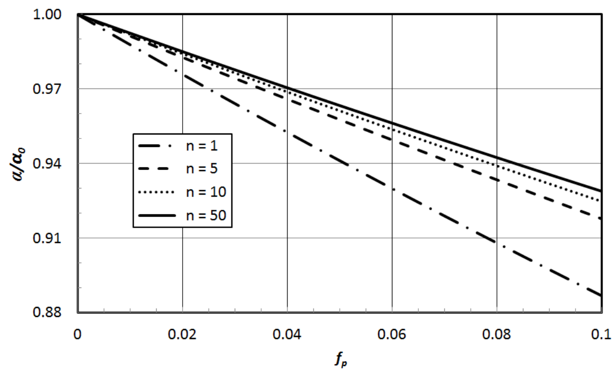

Fig. 2. Variation of the NCTE with f_p for different numbers of layers.

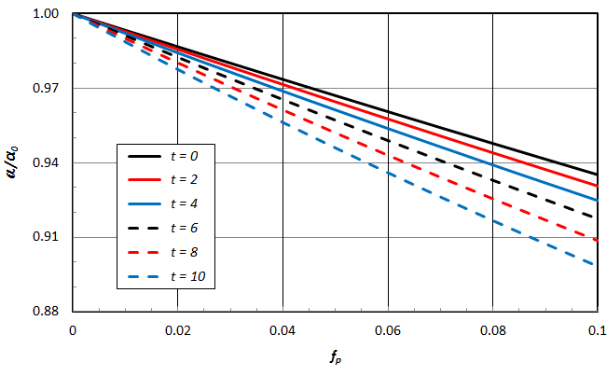

Fig. 3. Variation of the normalized CTE with f_p for different values of the interphase thickness.

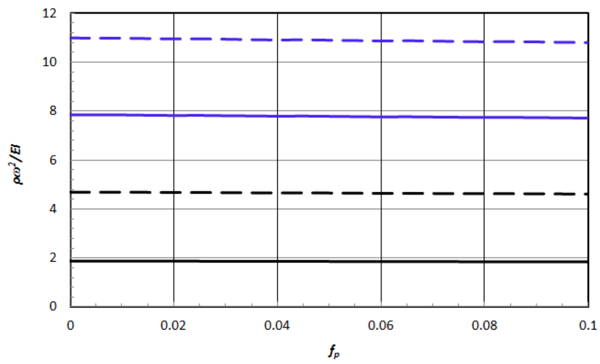

Fig. 4. The first four non-dimensional natural frequencies based on Euler theory.

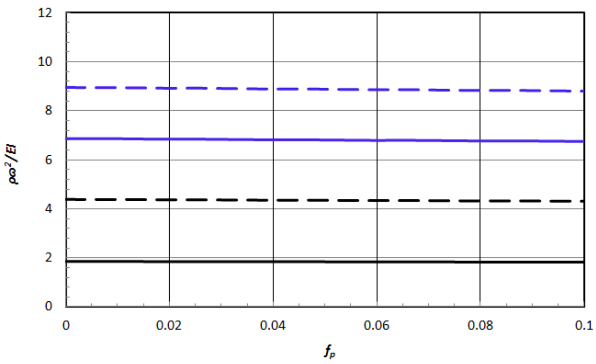

Fig. 5. The first four non-dimensional natural frequencies based on Timoshenko theory.

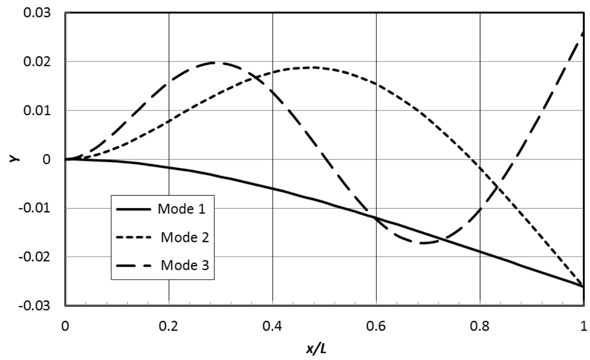

Fig. 6. The first three mode shapes based on Euler theory (for extreme values of f_p).

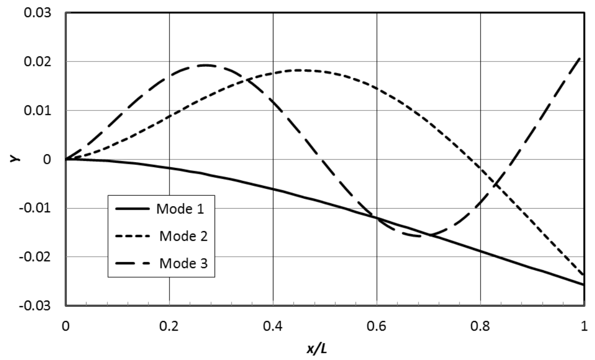

Fig. 7. The first three mode shapes based on Timoshenko theory (for extreme values of f_p).