Military Technical College
Kobry El-Kobbah,
Cairo, Egypt.

16th International Conference
on Applied Mechanics and
Mechanical Engineering.

DISTRIBUTION OF SUBHARMONIC AND ULTRAHARMONIC WAVES WITH THE NONLINEAR MECHANISM OF DISSIPATION OF ENERGY

A. N. Tyurekhojayev* and M.Zh. Sergaziyev**

ABSTRACT

The statics and dynamics of mechanical systems with contact dry friction, representing the nonlinear mechanism of the energy dissipation, is a new actual direction in the nonlinear mechanics of deformable solids. Damping effects of this sort can appear on the boundaries of rough layered environments or in relative movements of one body along a surface of another. Such kind of tasks come to consideration of the hyperbolic type equations of a nonlinear system and connected with distribution and attenuation of nonlinear waves. Analytical results for the distribution of nonlinear waves in the system with contact dry friction under the influence of cyclic loads were obtained. The class of loads under which the system shows subharmonic and ultraharmonic oscillations was determined. Based on the results of obtained decisions the following conclusion has been made: there is a class of cyclic loads with the frequency to arbitrary integer times which differs from the frequency of its own fluctuation of system under the action of which the system performs established or resonant fluctuations.

KEY WORDS

Nonlinear elastic waves, Dry contact friction, Hyperbolic type equation, Subharmonic and ultraharmonic oscillations.

---

* Prof. Dr , Institute of Industrial Engineering, Kazakh National Technical University after K. Satpaev, Almaty, Republic of Kazakhstan.

** Associate prof. PhD, Faculty of Information Technologies, International Information Technologies University, Almaty, Republic of Kazakhstan.

NOMENCLATURE

a	Velocity of wave propagation in the rod
E	Elasticity modulus
H(z)	Heaviside's unit function
q	Friction force
U(x, t)	Displacement of sections of the rod
$\upsilon(x,t)$	Velocity function
$\sigma(x,t)$	Normal stress on section x
ρ	Density of the material

INTRODUCTION

The propagation of nonlinear waves in a mechanical system with dry friction under cyclic loading with a period that is integrally greater or less than the system's natural period of oscillation was investigated analytically. Problems of this sort reduce to an investigation of nonlinear systems of hyperbolic-type equations and are connected to the investigation of the propagation and attenuation of nonlinear waves [1, 2]. Nonlinearity is due to the presence of dry contact friction. In the case of dynamic deformation, the nonlinearity of the dissipation mechanism provides an a priori unknown nonlinear velocity function. In the case of movement, the friction assumes a maximum value with a plus or minus sign; at standstill, it takes any value between its positive or negative maximum. The main complexity consists of determining the expression of the friction's sign function, which significantly depends on both the boundary and initial conditions, of the law of dry friction. The dependence domain for resolving problems of this sort is determined by the kappa-function method of Nikitin-Turekhodjayev [3]. Applying the kappa-function method in many problems of this sort can determine the nonlinear function of friction and record it as an infinite sum of Heaviside functions with shifted arguments. Then, the nonlinear velocity function becomes a function of independent arguments, and the problem becomes linear. Thus, it can be resolved using one of the standard methods for solving linear equations. Analytical results were obtained for a class of problems wherein the frequency of the external load is **n** times greater or less than the system's free frequency. The analysis of results obtained for $n = 1; 2; 3; 4$ allowed us to construct solutions at the whole area of the dependence domain of the problem solution ($0 \le t < \infty, 0 \le x \le \ell$). The general solution of the problem is recorded by progressive waves that covered the travel way. The record of solutions in characteristic regions gives a pictorial view of the functions of displacement, stress and velocity.

NONLINEAR STEADY OSCILLATIONS OF A MECHANICAL SYSTEM WITH DRY FRICTION UNDER CYCLIC LOADING

An analytical investigation was performed on the pattern of nonlinear steady wave propagation in mechanical systems caused by the presence of a nonlinear mechanism of energy dissipation. Analytical results were obtained on the longitudinal oscillations of a terminal flexible rod, the surface of which interacted with the environment by Coulomb's dry friction law, during dynamic agitation as a "rectangular harmonic load" with a frequency integrally greater than the system's free frequencies.

The specificity of the investigation is that the problems with contact dry friction are incorrect to the extent that nonlinearity does not allow us to generalize the results of one problem into a class of problems. Thus, each type of loadings must be investigated separately. We shall demonstrate the method of solution with an example investigating longitudinal oscillations of a terminal flexible rod under the effect of an oscillatory step load with a frequency four times greater than the system's free frequencies.

Suppose that we have a rod, the end $x=\ell$ of which is plugged, and a periodic stress is applied on the end $x=0$ (Fig. 1).

$$\sigma(0,t)=\sigma_0\left\{H(t)-2\sum_{k=0}^{\infty}(-1)^k H(t-\frac{1}{2}k\ell/a)\right\} \tag{1}$$

Normal stress $\sigma(0,t)$ is pictured in the Fig. 1.

At the initial instant, the rod is assumed to be at rest and unstressed:

$$\vartheta=0, \qquad \sigma=0, \ t\leq 0. \tag{2}$$

The equation of motion and Hooke's law can be generally recorded as follows:

$$\frac{\partial\sigma}{\partial x}=\rho\frac{\partial\upsilon}{\partial t}+\aleph\left(\upsilon/|\upsilon|, \partial\upsilon/\partial t/|\partial\upsilon/\partial t|\right)\cdot q,$$
$$\frac{\partial\sigma}{\partial t}=E\frac{\partial\upsilon}{\partial x}, \tag{3}$$

where the sign of the velocity $\aleph=\mathrm{sign}(\vartheta)$ if $\vartheta\neq 0$ and $\aleph\in[-1;1]$ if motion occurs with stops.

The system (3) is highly nonlinear due to presence of the function $\aleph$. The difficulty in solving this nonlinear problem can be overcome because the frictional force, in the investigated problem, being passive, cannot change the sign of the velocity. In the result of the counting direct and reflex waves we get the following result for the function $\aleph$:

$$\aleph(x,t)=\sum_{k=0}^{\infty}\{H(t-x/a-4k\ell/a)-H(t-x/a-(4k+\tfrac{1}{2})\ell/a)\}\,(H(x)-H(x-\tfrac{3}{4}\ell))+$$
$$+\sum_{k=0}^{\infty}\{H(t-x/a-4k\ell/a)-H(t+x/a-2(2k+1)\ell/a)\}\cdot H(x-\tfrac{3}{4}\ell)-$$
$$-\sum_{k=0}^{\infty}\{H(t-x/a-(4k+\tfrac{1}{2})\ell/a)-H(t-x/a-(4k+1)\ell/a)\}\cdot(H(x)-H(x-\tfrac{3}{4}\ell))-$$
$$-\sum_{k=0}^{\infty}\{H(t-x/a-(4k+\tfrac{1}{2})\ell/a)-H(t+x/a-(4k+\tfrac{5}{2})\ell/a)\}\cdot H(x-\tfrac{3}{4}\ell)+$$

$$
\begin{aligned}
&+\sum_{k=0}^{\infty}\left\{H\left(t-x/a-(4k+1)\,\ell/a\right)-H\left(t-x/a-\left(4k+\tfrac{3}{2}\right)\ell/a\right)\right\}\cdot\left(H(x)-H\left(x-\tfrac{1}{4}\ell\right)\right)+\\
&+\sum_{k=0}^{\infty}\left\{H\left(t-x/a-(4k+1)\,\ell/a\right)-H\left(t+x/a-2(2k+1)\ell/a\right)\right\}\left(H\left(x-\tfrac{1}{4}\ell\right)-H\left(x-\tfrac{1}{2}\ell\right)\right)+\\
&+\sum_{k=0}^{\infty}\left\{H\left(t+x/a-\left(4k+\tfrac{5}{2}\right)\ell/a\right)-H\left(t-x/a-\left(4k+\tfrac{3}{2}\right)\ell/a\right)\right\}\left(H\left(x-\tfrac{1}{2}\ell\right)-H\left(x-\tfrac{3}{4}\ell\right)\right)+\\
&+\sum_{k=0}^{\infty}\left\{H\left(t-x/a-(4k+1)\,\ell/a\right)-H\left(t+x/a-(4k+3)\ell/a\right)\right\}\cdot H\left(x-\tfrac{3}{4}\ell\right)-\\
&-\sum_{k=0}^{\infty}\left\{H\left(t-x/a-\left(4k+\tfrac{3}{2}\right)\ell/a\right)-H\left(t+x/a-\left(4k+\tfrac{5}{2}\right)\ell/a\right)\right\}\cdot\left(H(x)-H\left(x-\tfrac{1}{2}\ell\right)\right)+\\
&+\sum_{k=0}^{\infty}\left\{H\left(t+x/a-\left(4k+\tfrac{5}{2}\right)\ell/a\right)-H\left(t+x/a-(4k+3)\ell/a\right)\right\}\cdot\left(H(x)-H\left(x-\tfrac{1}{4}\ell\right)\right)+\\
&+\sum_{k=0}^{\infty}\left\{H\left(t-x/a-2(2k+1)\,\ell/a\right)-H\left(t+x/a-(4k+3)\ell/a\right)\right\}\left(H\left(x-\tfrac{1}{4}\ell\right)-H\left(x-\tfrac{1}{2}\ell\right)\right)-\\
&-\sum_{k=0}^{\infty}\left\{H\left(t+x/a-(4k+3)\,\ell/a\right)-H\left(t+x/a-\left(4k+\tfrac{7}{2}\right)\ell/a\right)\right\}\cdot\left(H(x)-H\left(x-\tfrac{3}{4}\ell\right)\right)-\\
&-\sum_{k=0}^{\infty}\left\{H\left(t-x/a-\left(4k+\tfrac{3}{2}\right)\ell/a\right)-H\left(t+x/a-\left(4k+\tfrac{7}{2}\right)\ell/a\right)\right\}\cdot H\left(x-\tfrac{3}{4}\ell\right)+\\
&+\sum_{k=0}^{\infty}\left\{H\left(t+x/a-\left(4k+\tfrac{7}{2}\right)\ell/a\right)-H\left(t+x/a-4(k+1)\ell/a\right)\right\}\cdot\left(H(x)-H\left(x-\tfrac{3}{4}\ell\right)\right)+\\
&+\sum_{k=0}^{\infty}\left\{H\left(t-x/a-2(2k+1)\,\ell/a\right)-H\left(t+x/a-4(k+1)\ell/a\right)\right\}\cdot H\left(x-\tfrac{3}{4}\ell\right)\ .
\end{aligned}
\tag{4}
$$

where a is the velocity of wave propagation in the rod.

Figure 2 provides the signs of velocities in areas restricted by wave fronts. Substitution of (4) into the system (3) reduces the initial nonlinear problem to a linear equation

$$E\frac{\partial^2 U}{\partial x^2}=\rho\frac{\partial^2 U}{\partial t^2}+q\cdot\aleph(x,t) \tag{5}$$

where U(x, t) is the displacement of sections of the rod.

We can use the Laplace transform to solve this equation. After transformation, taking zero initial conditions into account, equation (2) and the boundary conditions assume the following form:

$$\frac{d^2\overline{U}}{dx^2}-\frac{p^2}{a^2}\overline{U}-\frac{q}{E}\cdot\overline{\aleph}(p,x)=0, \tag{6}$$

$$\overline{U}=0 \quad \text{on} \quad x=\ell, \tag{7}$$

$$\frac{d\overline{U}}{dx}=\frac{\sigma_0}{E}\left\{1-2\sum_{k=0}^{\infty}(-1)^k e^{-p\frac{1}{2}k\ell/a}\right\} \quad \text{при} \quad x=0 \tag{8}$$

where p is transformation parameter.

Solution of the problem (6) - (8) in images has the following form:

$$\overline{U}(p,x)=\frac{a\sigma_0}{pE}\left\{1-2\sum_{k=0}^{\infty}(-1)^k e^{-p\frac{1}{2}k\ell/a}\right\}\left[\sum_{k=0}^{\infty}(-1)^k e^{p(x-2(k+1)\ell)/a}-\right.$$

$$\left.-\sum_{k=0}^{\infty}(-1)^k e^{-p(x+2k\ell)/a}\right]+\frac{aq}{2pE}\sum_{k=0}^{\infty}e^{-2pk\ell/a}\left[-\left(\frac{a}{p}-3\ell\right)\sum_{k=0}^{\infty}e^{p\left(x-\left(4k+\frac{9}{2}\right)\ell\right)/a}+\right.$$

$$\left(\frac{a}{p}-3\ell\right)\sum_{k=0}^{\infty}e^{p(x-(4k+5)\ell)/a}+\left(\frac{a}{2p}-\frac{3}{2}\ell\right)\sum_{k=0}^{\infty}e^{p(x-4(k+1)\ell)/a}-\left(\frac{a}{p}-3\ell\right)\sum_{k=0}^{\infty}e^{p\left(x-\left(4k+\frac{11}{2}\right)\ell\right)/a}+$$

$$+\frac{a}{2p}\sum_{k=0}^{\infty}e^{p(x-2(2k+1)\ell)/a}+\frac{a}{p}\sum_{k=0}^{\infty}e^{p(x-(4k+3)\ell)/a}-\frac{a}{p}\sum_{k=0}^{\infty}e^{p\left(x-\left(4k+\frac{5}{2}\right)\ell\right)/a}--\frac{a}{p}\sum_{k=0}^{\infty}e^{p\left(x-\left(4k+\frac{7}{2}\right)\ell\right)/a}-$$

$$+\frac{3}{2}\ell\sum_{k=0}^{\infty}e^{p(x-2(2k+3)\ell)/a}-\left(\frac{a}{2p}-\frac{3}{2}\ell\right)\sum_{k=0}^{\infty}e^{-p(x+2(2k+1)\ell)/a}+\left(\frac{a}{p}-3\ell\right)H(x)\sum_{k=0}^{\infty}e^{-p\left(x+\left(4k+\frac{5}{2}\right)\ell\right)/a}-$$

$$-\left(\frac{a}{p}-3\ell\right)\sum_{k=0}^{\infty}e^{-p(x+(4k+3)\ell)/a}+\left(\frac{a}{p}-3\ell\right)H(x)\sum_{k=0}^{\infty}e^{-p\left(x+\left(4k+\frac{7}{2}\right)\ell\right)/a}--\frac{a}{2p}\sum_{k=0}^{\infty}e^{-p(x+4k\ell)/a}-$$

$$\left.\frac{a}{p}\sum_{k=0}^{\infty}e^{-p(x+(4k+1)\ell)/a}+\frac{a}{p}\sum_{k=0}^{\infty}e^{-p\left(x+\left(4k+\frac{1}{2}\right)\ell\right)/a}+\frac{a}{p}\sum_{k=0}^{\infty}e^{-p\left(x+\left(4k+\frac{3}{2}\right)\ell\right)/a}+\frac{3}{2}\ell\sum_{k=0}^{\infty}e^{-p(x+4(k+1)\ell)/a}\right]+$$

$$+\frac{aq}{2pE}\left[\left(\frac{a}{2p}+x\right)H(x)\sum_{k=0}^{\infty}e^{-p(x+4k\ell)/a}+\sum_{k=0}^{\infty}e^{-p\left(x+\left(4k+\frac{1}{2}\right)\ell\right)/a}\left\{\frac{a}{p}+2x-H\left(x-\tfrac{3}{4}\ell\right)\left(\frac{a}{p}+\left(x-\frac{3}{4}\ell\right)\right)\right\}+\right.$$

$$+\sum_{k=0}^{\infty}e^{p(x-2(2k+1)\ell)/a}\left\{-\frac{a}{2p}+\frac{3}{2}\ell-H\left(x-\tfrac{1}{2}\ell\right)\times\right.$$

$$\left.\times\left(\frac{a}{p}-\left(x-\frac{1}{2}\ell\right)\right)+H\left(x-\tfrac{1}{4}\ell\right)\left(\frac{a}{p}-\left(x-\frac{1}{4}\ell\right)\right)+H\left(x-\tfrac{3}{4}\ell\right)\left(\frac{a}{p}-\left(x-\frac{3}{4}\ell\right)\right)\right\}-$$

$$-\sum_{k=0}^{\infty}e^{-p(x+(4k+1)\ell)/a}\left\{H(x)\left(\frac{a}{p}+2x\right)-H\left(x-\tfrac{1}{2}\ell\right)\left(\frac{a}{p}+\left(x-\frac{1}{2}\ell\right)\right)\right\}+ \tag{9}$$

$$+\sum_{k=0}^{\infty}e^{p\left(x-\left(4k+\frac{5}{2}\right)\ell\right)/a}\left\{H(x)\left(-\frac{a}{p}+2x-3\ell\right)+H\left(x-\tfrac{1}{4}\ell\right)\left(\frac{a}{p}-\left(x-\frac{1}{4}\ell\right)\right)\right\}+$$

$$+\sum_{k=0}^{\infty}e^{-p\left(x+\left(4k+\frac{3}{2}\right)\ell\right)/a}\left\{H(x)\left(\frac{a}{p}+2x\right)-H\left(x-\tfrac{1}{4}\ell\right)\left(\frac{a}{p}+\left(x-\frac{1}{2}\ell\right)\right)\right\}-$$

$$-\sum_{k=0}^{\infty} e^{p(x-(4k+3)\ell)/a}\left\{H(x)\left(-\frac{a}{p}+2x-3\ell\right)+H(x-\tfrac{1}{2}\ell)\left(\frac{a}{p}-\left(x-\frac{1}{2}\ell\right)\right)\right\}+$$

$$+\sum_{k=0}^{\infty} e^{-p(x+2(2k+1)\ell)/a}\left\{H(x-\tfrac{1}{2}\ell)\left(\frac{a}{p}+\left(x-\frac{1}{2}\ell\right)\right)-H(x-\tfrac{1}{4}\ell)\left(\frac{a}{p}+\left(x-\frac{1}{4}\ell\right)\right)-\right.$$

$$\left.-H(x-\tfrac{3}{4}\ell)\left(\frac{a}{p}+\left(x-\frac{3}{4}\ell\right)\right)\right\}+\sum_{k=0}^{\infty} e^{p\,(x-(4k+\frac{7}{2})\ell)/a}\left\{H(x)\left(-\frac{a}{p}+2x-3\ell\right)+\right.$$

$$\left.\left.+H(x-\tfrac{3}{4}\ell)\left(\frac{a}{p}-\left(x-\frac{3}{4}\ell\right)\right)\right\}+\sum_{k=0}^{\infty} e^{p\,(x-4(k+1)\ell)/a}\left(\frac{3}{2}\ell+\frac{a}{p}-x\right)\right].$$

This bulky generalized solution of the problem acquires very compact expressions if they are recorded in appropriate characteristic areas. On the basis of (9), solutions for a number of the first specific areas – in the case of this problem, for 51 areas – were recorded, and then, using the method of mathematical induction, solutions for all areas covering semi-infinite strips $0 \le x \le \ell,\ t > 0$ were determined. Investigation of the solutions allows us to distinguish 26 areas that characterize one full oscillation of the system. Solutions of problems on the areas that characterize the two subsequent full oscillations were obtained. They are omitted for the sake of brevity. Solution of the problem at $0 < t < \infty$ is written in subsequent specific areas. In general view, let us represent them correspondingly as $26k+1,\ 26k+2,\ \ldots, 26(k+1)\quad (k = 0, 1, 2, \ldots)$, see Fig.3.

Then, for trapezoidal areas $26k+1$:

$at < 2(2k+1)\,\ell - x$, $x < at < \left(\frac{1}{2}\ell + 4k\right) + x$, $\quad 0 \le x \le \ell$

stresses and velocities will have the following expressions:

$$\sigma_{26k+1} = -\sigma_0 + \frac{q}{2}x, \qquad \vartheta_{26k+1} = \vartheta_0 - \frac{aq}{2E}(at - 4k\ell).$$

In specific areas $26k + 2$ bounded by characteristics

$x + \left(4k + \frac{1}{2}\ell\right) < at < x + (4k+1)\ell,\ \ at < 2(2k+1)\ell, \qquad 0 \le x \le \frac{3}{4}\ell,$

Solutions have the following expressions

$$\sigma_{26k+2} = \sigma_0 - \frac{q}{2}x, \qquad \vartheta_{26k+2} = -\vartheta_0 + \frac{aq}{E}\left(\frac{at}{2} - \left(4k + \frac{1}{2}\right)\ell\right).$$

In regions $26k+3$:

$x+(4k+\ell)<at<x+\left(4k+\frac{3}{2}\right)\ell,\ \ at<2(2k+1)\ell$, $\quad 0\le x\le \frac{1}{2}\ell$

we obtain

$$\sigma_{26k+3}=-\sigma_0+\frac{q}{2}x \ , \qquad \vartheta_{26k+3}=\vartheta_0-\frac{aq}{2E}\left(at-(4k+1)\ell\right) \ .$$

In areas $26k+4$:

$x+\left(4k+\frac{3}{2}\right)\ell<at<2(2k+1)\ell-x$, $\quad 0\le x\le \frac{1}{4}\ell$

we have

$$\sigma_{26k+4}=\sigma_0-\frac{q}{2}x \ , \qquad \vartheta_{26k+4}=-\vartheta_0+\frac{aq}{E}\left(\frac{at}{2}-(4k+1)\ell\right) .$$

In areas $26k+5$:

$x+\left(4k+\frac{3}{2}\right)\ell<at<x+2(2k+1)\ell$, $\quad 2(2k+1)\ell-x<at<\left(4k+\frac{5}{2}\right)\ell-x$

solutions have the following expressions

$$\sigma_{26k+5}=-\frac{q}{2}\left(x-\frac{3}{2}\ell\right) , \qquad \vartheta_{26k+5}=-2\vartheta_0+\frac{aq}{2E}\left(at-\left(4k+\frac{1}{2}\right)\ell\right) .$$

In areas $26k+6$:

$x+2(2k+1)\ell<at<\left(4k+\frac{5}{2}\right)\ell-x$, $\quad 0\le x\le \frac{1}{4}\ell$,

we have

$$\sigma_{26k+6}=-\sigma_0-\frac{q}{2}x \ , \qquad \vartheta_{26k+6}=-\vartheta_0+\frac{aq}{2E}\left(at+(4k+1)\ell\right) .$$

In areas $26k+7$ bounded by characteristics

$x+2(2k+1)\ell<at<x+(4k+\frac{5}{2})\ell$, $\quad \left(4k+\frac{5}{2}\right)\ell-x<at<(4k+3)\ell-x$

solutions are

$$\sigma_{26k+7}=\sigma_0+\frac{q}{2}(x-3\ell) \ , \qquad \vartheta_{26k+7}=\vartheta_0-\frac{aq}{2E}\left(at-(4k+3)\ell\right).$$

In areas $26k+8$

$$\sigma_{26k+8}=\sigma_0+\frac{q}{2}x \ , \qquad \vartheta_{26k+8}=\vartheta_0-\frac{aq}{2E}\left(at-4k\,\ell\right) .$$

In areas $26k+9$

$$\sigma_{26k+9}=-\sigma_0-\frac{q}{2}(x-3\ell) \ , \quad \vartheta_{26k+9}=-\vartheta_0+\frac{aq}{2E}\left(at-(4k+3)\ell\right).$$

In areas $26k+10$ we have

$$\sigma_{26k+10} = -\sigma_0 - \frac{q}{2}x, \quad \vartheta_{26k+10} = -\vartheta_0 + \frac{aq}{2E}(at - 4k\ell).$$

Analysis of the solution in areas $26k+11$ gives:

$$\sigma_{26k+11} = \sigma_0 + \frac{q}{2}(x - 3\ell), \qquad \vartheta_{26k+11} = \vartheta_0 - \frac{aq}{2E}(at - 4(k+1)\ell).$$

In areas $26k+12$ we have

$$\sigma_{26k+12} = \sigma_0 + \frac{q}{2}x, \qquad \vartheta_{26k+12} = \vartheta_0 - \frac{aq}{2E}(at - (4k+1)\ell).$$

In specific areas $26k+13$ the required functions are

$$\sigma_{26k+13} = -2\sigma_0 + \frac{7}{8}q\ell, \qquad \vartheta_{26k+13} = 0.$$

In areas $26k+14$:

$$\sigma_{26k+14} = 2\sigma_0 - \frac{5}{8}q\ell, \qquad \vartheta_{26k+14} = 0.$$

In areas $26k+15$, solutions are defined by the following expressions:

$$\sigma_{26k+15} = -\sigma_0 - \frac{q}{2}x, \qquad \vartheta_{26k+15} = -\vartheta_0 + \frac{aq}{2E}(at - 4k\ell).$$

In areas $26k+16$ we obtain

$$\sigma_{26k+16} = \sigma_0 + \frac{q}{2}x, \qquad \vartheta_{26k+16} = \vartheta_0 - \frac{aq}{2E}(at - 4(k+1)\ell).$$

In characteristic areas $26k+17$ solutions have the following expressions

$$\sigma_{26k+17} = -\frac{q}{2}x + \frac{5}{8}q\ell, \qquad \vartheta_{26k+17} = -2\vartheta_0 + \frac{aq}{2E}\left(at - \left(4k - \frac{1}{4}\right)\ell\right).$$

In areas $26k+18$

$$\sigma_{26k+18} = \frac{q}{2}(x - \ell), \qquad \vartheta_{26k+18} = 2\vartheta_0 - \frac{aq}{2E}\left(at + \left(4k + \frac{1}{2}\right)\ell\right).$$

In areas $26k+19$ solutions are

$$\sigma_{26k+19} = -\frac{q}{2}\left(x - \frac{5}{4}\ell\right), \quad \vartheta_{26k+19} = -2\vartheta_0 + \frac{aq}{2E}\left(at - \left(4k + \frac{5}{4}\right)\ell\right).$$

In areas $26k+20$:

$$\sigma_{26k+20} = \sigma_0 + \frac{q}{2}(x - 3\ell), \qquad \vartheta_{26k+20} = \vartheta_0 - \frac{aq}{2E}(at - 4(k+1)\ell).$$

In $26k+21$ the required functions are

$\sigma_{26k+21} = -2\sigma_0 + q\ell$, $\vartheta_{26k+21} = 0$.

In areas $26k + 22$

$\sigma_{26k+22} = 2\sigma_0 - \frac{3}{4}q\ell$, $\vartheta_{26k+22} = 0$.

In areas $26k + 23$

$\sigma_{26k+23} = -2\sigma_0 + \frac{3}{4}q\ell$, $\vartheta_{26k+23} = 0$.

In areas $26k + 24$ we have

$\sigma_{26k+24} = 2\sigma_0 - \frac{1}{2}q\ell$, $\vartheta_{26k+24} = 0$.

In areas $26k + 25$ solutions have the following expressions

$\sigma_{26k+25} = -\frac{3}{2}q\ell$, $\vartheta_{26k+25} = 0$.

Further analysis of the problem shows that in the trapezoidal areas $26(k+1)$ the system turns out to be at rest:

$\sigma_{26(k+1)} = 0$, $\vartheta_{26(k+1)} = 0$.

In Figure 4, the oscillation curve of the end point of the rod is shown. The resulting solution of the nonlinear problem of wave propagation in the system under consideration under cyclic step load (1) with a frequency four times greater than the rod's free frequencies indicate that the system with dry friction under investigation shows steady periodic oscillations with a period $4\ell/a$.

CONCLUSION

Based on the results of the solutions obtained, the following conclusion can be drawn: there is a class of cyclic loads with a frequency that is an arbitrary integer times greater than the frequency of the fluctuation of the system:

$$\sigma(0,t) = \sigma_0\left\{H(t) - 2\sum_{k=0}^{\infty}(-1)^k H(t - \frac{2}{n}k\ell/a)\right\},$$

where n is positive integer.

Under the action of such loadings the system will have steady ultraharmonic oscillations with two frequencies. In addition, one oscillation coincides with the frequency of the oscillation of the system, the other with the frequency of external loading. When **n** is even the system performs steady oscillations so that for each subsequent time range which is equal to ***n*** periods of external loading, the system begins oscillations from the rest and undisturbed state under the load identical to the load at $0 < t < 4\ell/a$.

REFERENCES

[1] Tyurekhodgaev A.N., Nikitin L.V., "Wave Propagation and Vibration of Elastic Rods with Interfacial Frictional Slip", Wave Motion No.12, pp. 513-526 (1990).
[2] Tyurekhodjayev A.N., "Action of a Shock Wave in a Ground on the Pipeline of Final Length", Proceedings of the European Conference on Geomechanics, Amsterdam (1999).
[3] Tyurekhodjayev A.N., Ibraev A.G., Sergaziyev M.Zh., "Dynamics of Nonlinear Mechanical Systems with External Dry Friction", Proceedings of the International Conference MEATIP4, Assiut, Egypt (2006).

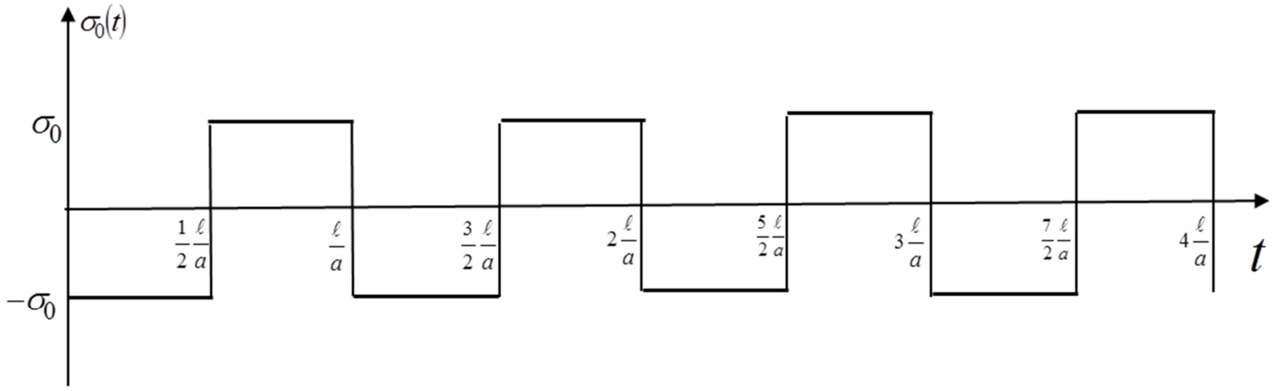

Fig. 1. Normal stress at the end x=0 of the rod.

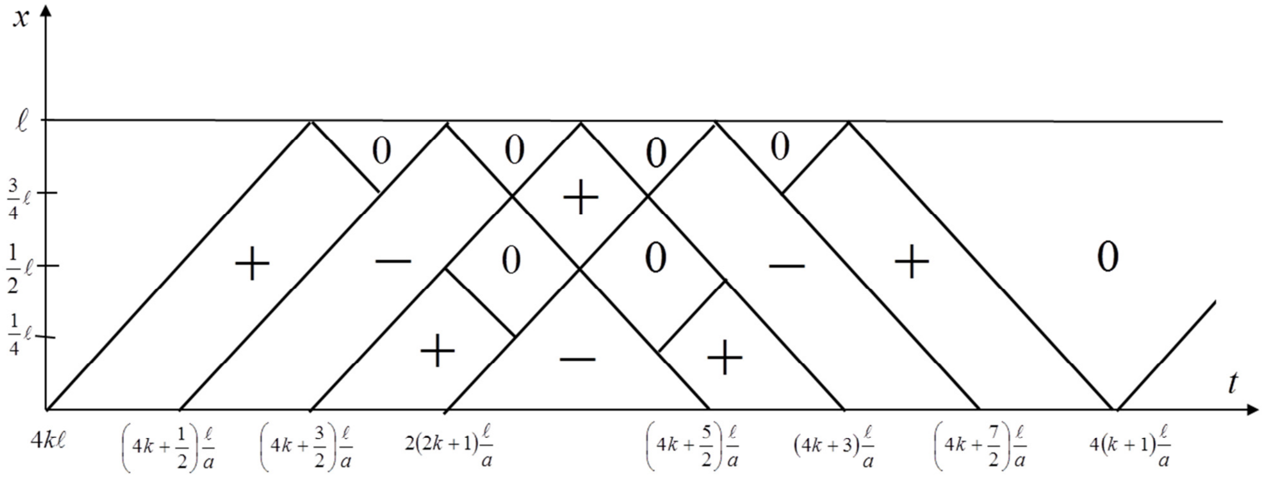

Fig. 2. Signs of velocities in characteristic areas of motion.

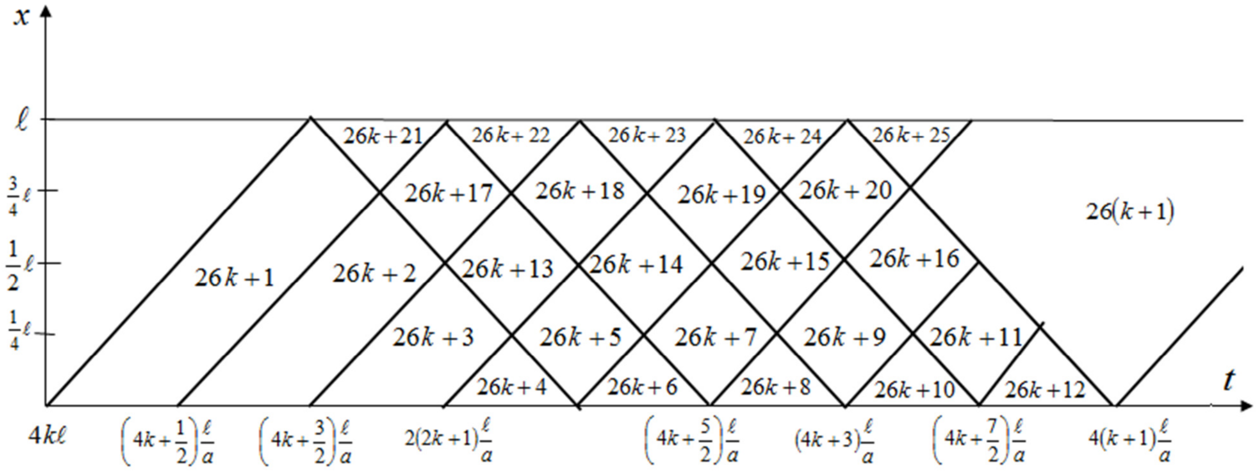

Fig. 3. Characteristic areas of the motion plane.

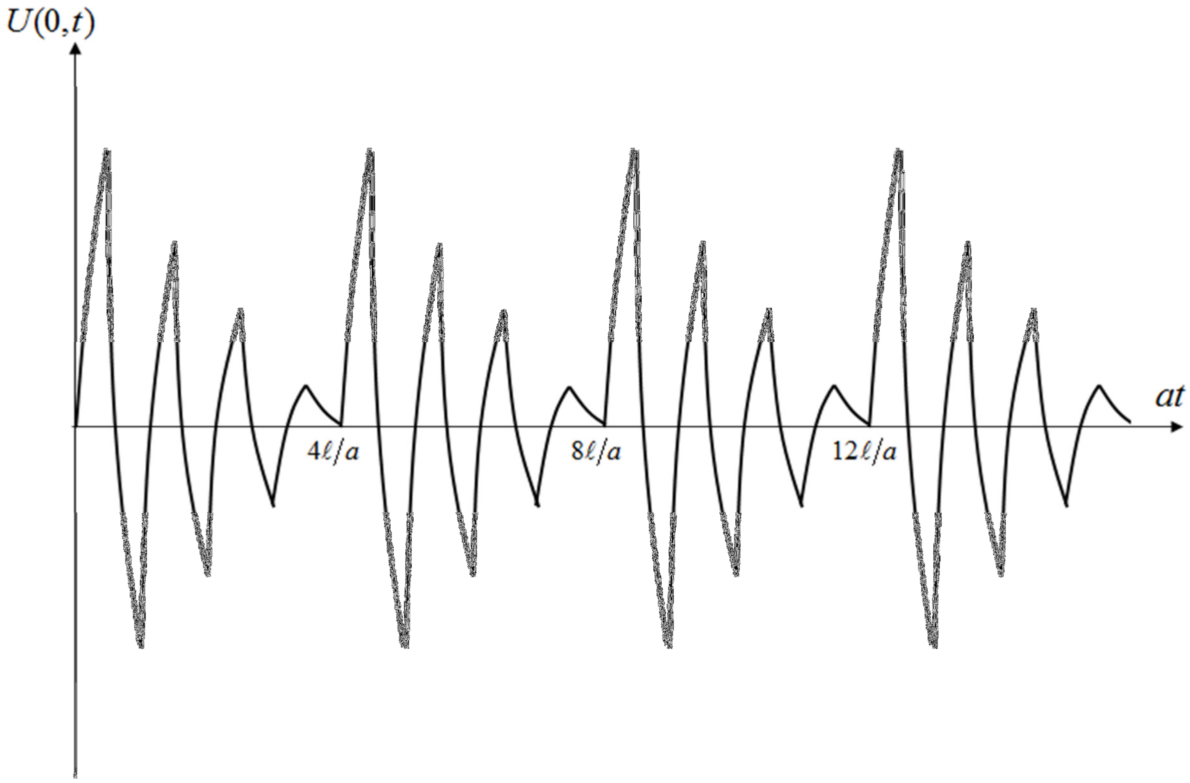

Fig. 4. Oscillation curve of the end point of the rod.